Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/22-1.1.2.5-a+b-x^2-^p-c+d-x^2-^q-e+f-x^2-^r

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [115]. This is test number [22].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.13 (114)	0.87 (1)
Mathematica	99.13 (114)	0.87 (1)
Maple	93.04 (107)	6.96 (8)
Fricas	59.13 (68)	40.87 (47)
Giac	26.96 (31)	73.04 (84)
Mupad	23.48 (27)	76.52 (88)
Sympy	22.61 (26)	77.39 (89)
Maxima	13.04 (15)	86.96 (100)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

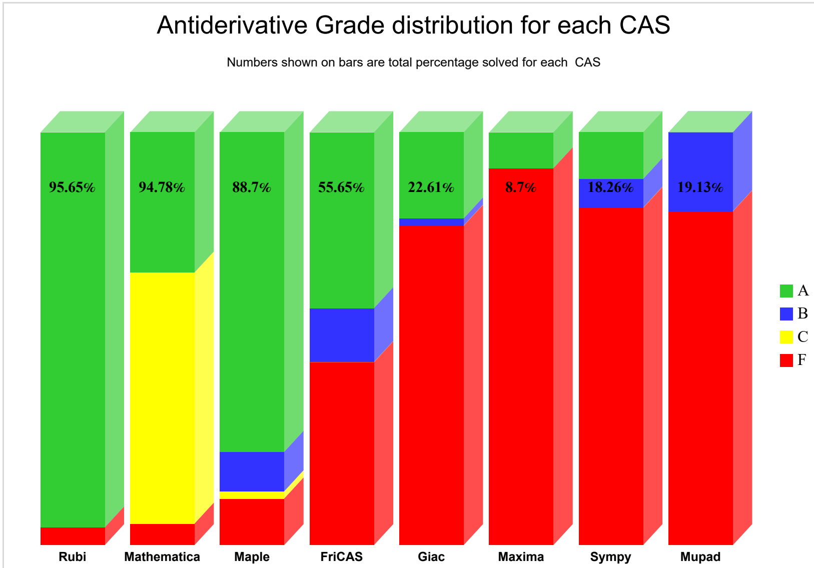
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

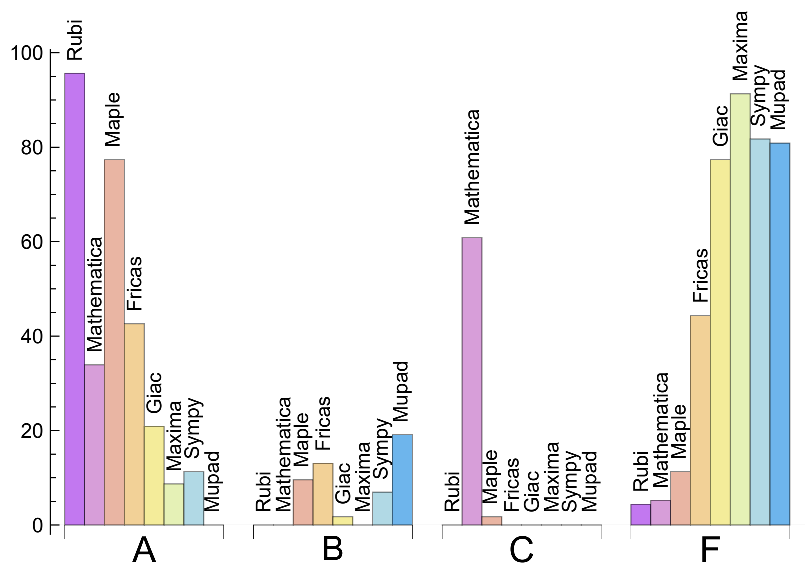
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.783	0.000	0.000	5.217
Maple	77.391	9.565	1.739	11.304
Fricas	42.609	13.043	0.000	44.348
Mathematica	33.913	0.000	60.870	5.217
Giac	20.870	1.739	0.000	77.391
Sympy	11.304	6.957	0.000	81.739
Maxima	8.696	0.000	0.000	91.304
Mupad	0.000	19.130	0.000	80.870

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	8	100.00	0.00	0.00
Fricas	47	25.53	74.47	0.00
Giac	84	95.24	0.00	4.76
Mupad	88	0.00	100.00	0.00
Sympy	89	92.13	7.87	0.00
Maxima	100	87.00	0.00	13.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Rubi	0.45
Giac	0.47
Fricas	2.86
Mupad	3.54
Mathematica	3.83
Maple	4.25
Sympy	5.22

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	121.27	0.99	100.00	1.02
Mupad	184.81	1.13	158.00	1.05
Giac	199.97	1.28	182.00	1.27
Sympy	251.08	1.60	211.00	1.35
Mathematica	258.54	0.95	210.00	0.99
Rubi	284.37	1.01	255.00	1.00
Fricas	488.59	2.31	330.00	1.27
Maple	498.62	1.66	334.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

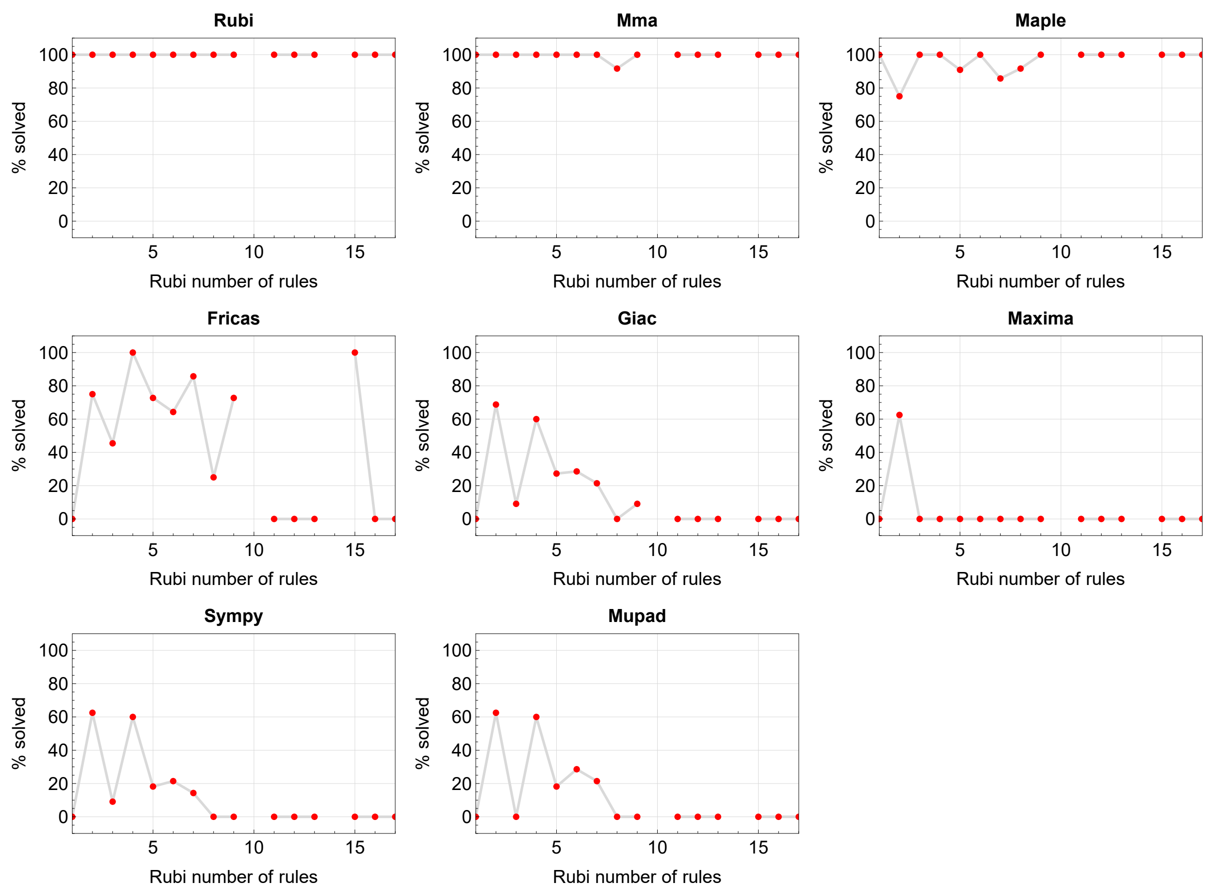


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

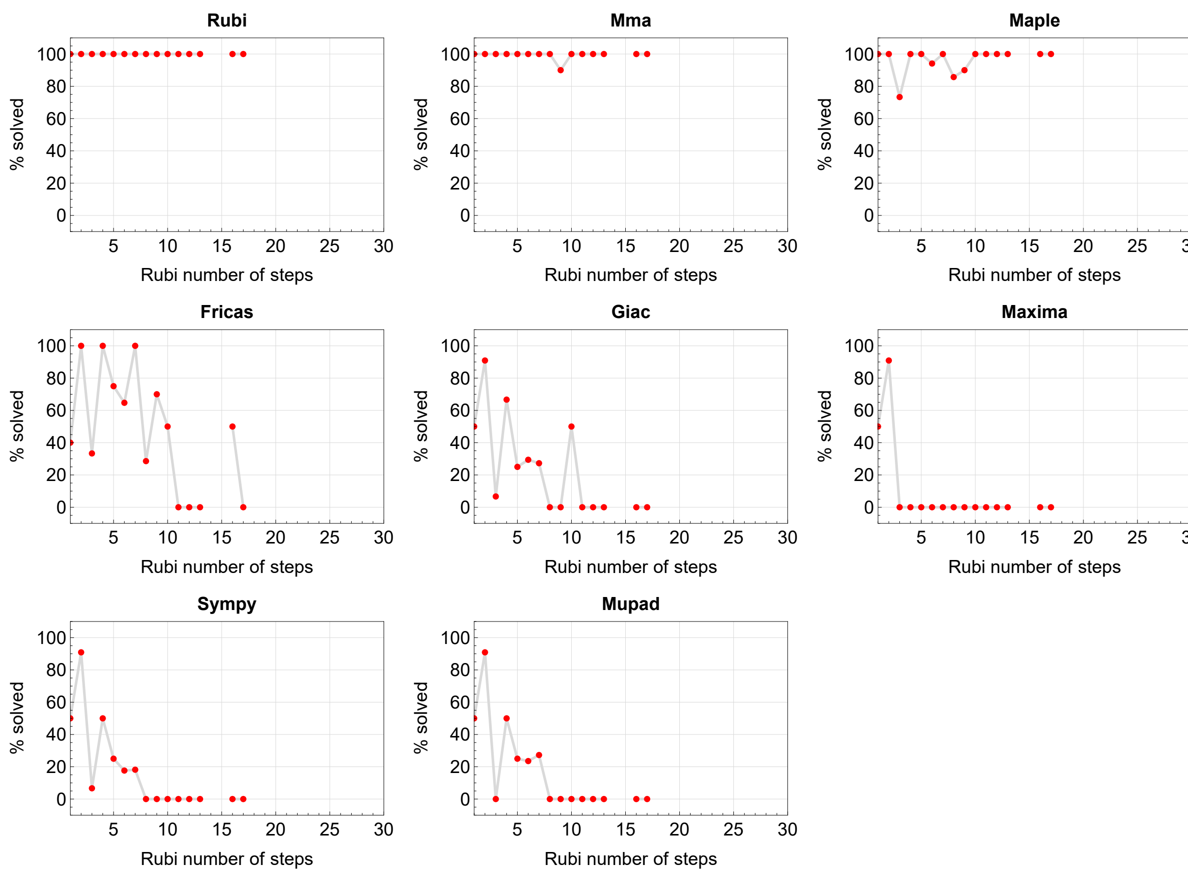


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

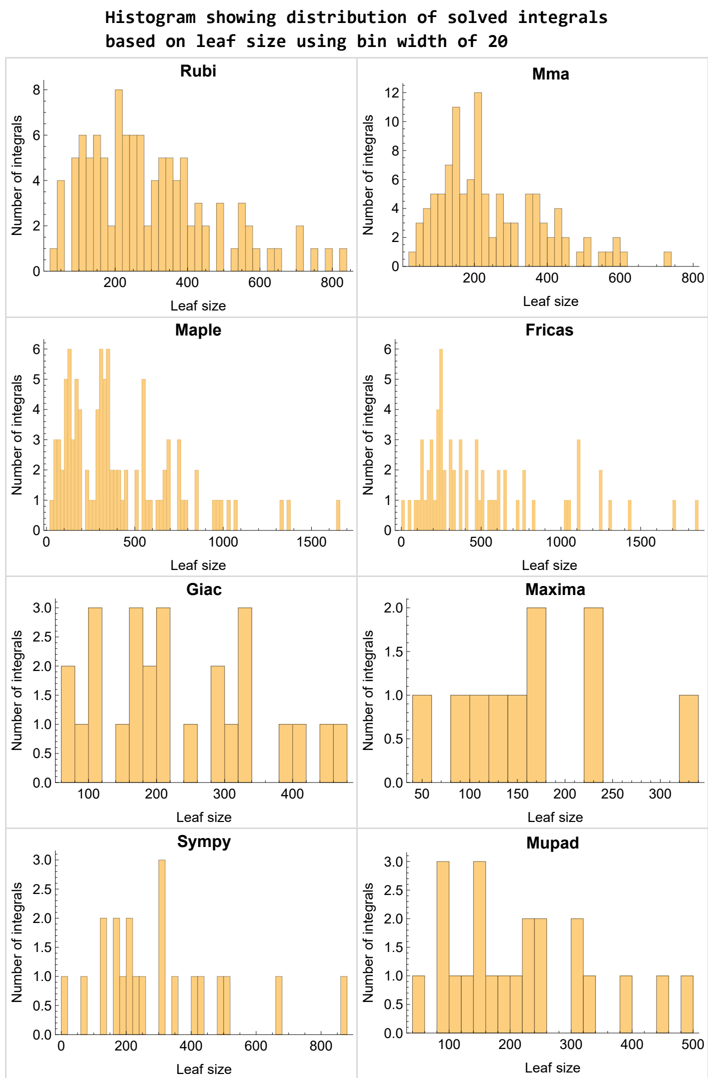


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

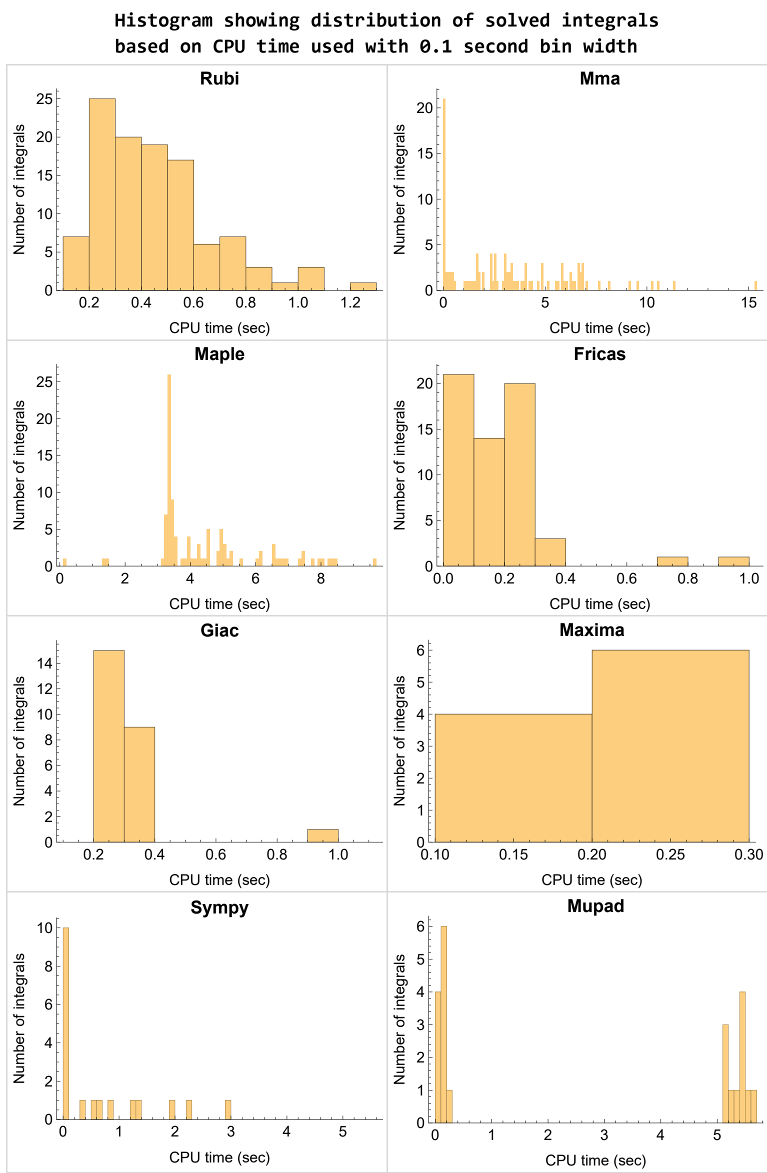


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

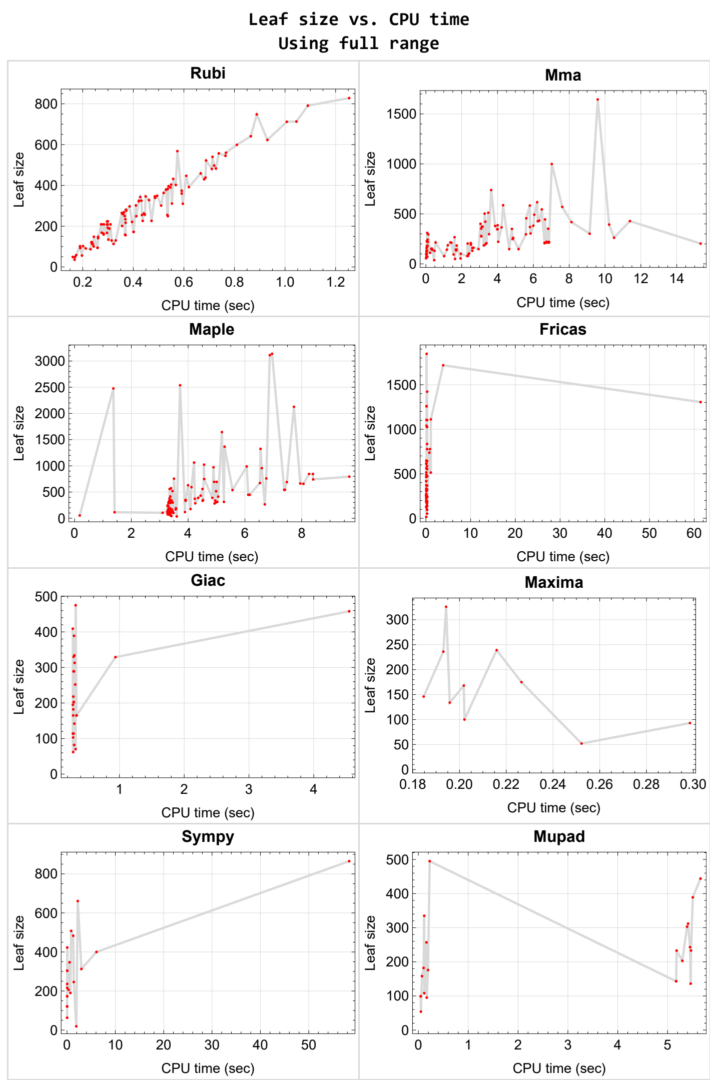


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{103, 107, 110, 112, 115\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{55, 56\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

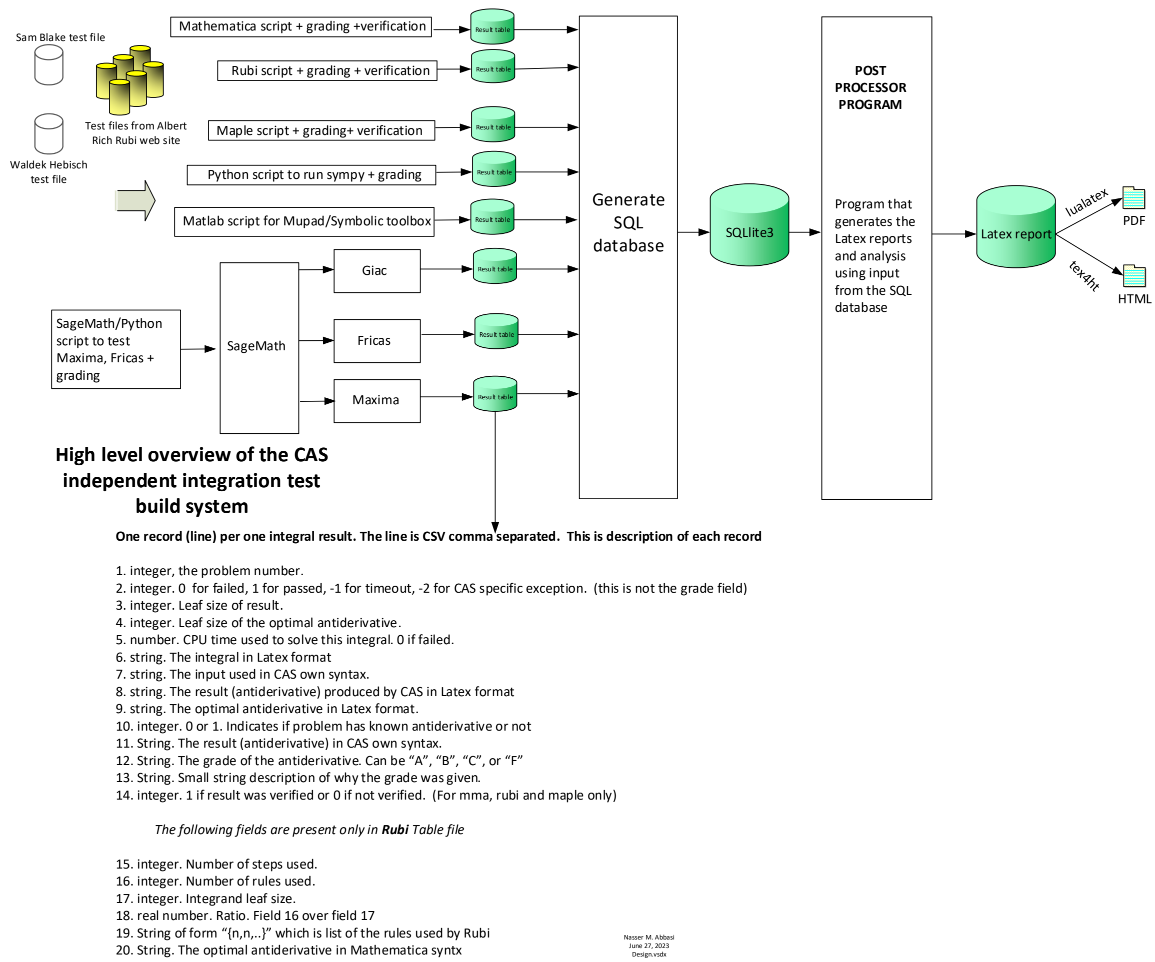
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

B grade { }

C grade { }

F normal fail { 30 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 57, 58, 59, 60, 61, 62, 63, 96, 97, 98, 104, 105, 106, 109, 111, 113, 114 }

B grade { }

C grade { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102 }

F normal fail { 108 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 77, 78, 79, 80, 81, 83, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102 }

B grade { 55, 56, 69, 70, 74, 75, 76, 82, 84, 88, 101 }

C grade { 89, 90 }

F normal fail { 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 35, 36, 37, 38, 39, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 97 }

B grade { 8, 15, 22, 28, 33, 34, 40, 41, 46, 47, 55, 60, 61, 62, 98 }

C grade { }

F normal fail { 89, 90, 91, 92, 93, 94, 95, 104, 106, 108, 109, 114 }

F(-1) timedout fail { 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 96, 99, 100, 101, 102, 103, 105, 111, 113 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 9, 10, 11, 16, 17, 18 }

B grade { }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 22, 57 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 61, 62 }

B grade { 63, 98 }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 58, 59, 60 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 }

C grade { }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 16, 17, 18, 97 }

B grade { 5, 7, 12, 13, 14, 19, 20, 21 }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { 15, 22, 34, 47, 75, 76, 93 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	175	175	236	218	182
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.37	1.27	1.06
time (sec)	N/A	0.378	0.058	4.084	0.227	0.288	0.033	0.287	0.105

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	134	134	173	165	143
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.33	1.27	1.10
time (sec)	N/A	0.319	0.043	3.295	0.196	0.270	0.028	0.285	5.171

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	93	93	121	114	99
N.S.	1	1.00	1.02	1.00	0.99	0.99	1.29	1.21	1.05
time (sec)	N/A	0.257	0.025	3.378	0.299	0.273	0.024	0.280	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	52	52	63	62	54
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.12	1.11	0.96
time (sec)	N/A	0.201	0.009	0.183	0.252	0.271	0.020	0.285	0.051

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	87	72	74	0	191	206	82	108
N.S.	1	1.07	0.89	0.91	0.00	2.36	2.54	1.01	1.33
time (sec)	N/A	0.226	0.036	3.279	0.000	0.291	0.315	0.300	0.118

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	95	97	0	318	190	103	95
N.S.	1	1.03	0.88	0.90	0.00	2.94	1.76	0.95	0.88
time (sec)	N/A	0.238	0.044	3.272	0.000	0.292	0.685	0.285	0.169

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	141	130	132	0	471	246	142	136
N.S.	1	1.08	1.00	1.02	0.00	3.62	1.89	1.09	1.05
time (sec)	N/A	0.272	0.053	3.270	0.000	0.314	1.389	0.303	5.463

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	168	171	163	0	642	313	195	176
N.S.	1	0.98	1.00	0.95	0.00	3.75	1.83	1.14	1.03
time (sec)	N/A	0.297	0.067	3.295	0.000	0.290	2.979	0.280	0.194

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	236	236	304	289	233
N.S.	1	1.00	1.00	1.05	1.04	1.04	1.35	1.28	1.03
time (sec)	N/A	0.446	0.056	3.304	0.193	0.257	0.035	0.290	5.470

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	168	168	216	202	158
N.S.	1	1.00	1.00	1.07	1.06	1.06	1.37	1.28	1.00
time (sec)	N/A	0.362	0.038	3.426	0.202	0.249	0.031	0.295	0.073

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	100	100	100	121	114	99
N.S.	1	1.00	1.02	1.06	1.06	1.06	1.29	1.21	1.05
time (sec)	N/A	0.272	0.021	3.306	0.202	0.250	0.024	0.292	0.047

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	156	115	170	0	366	347	182	203
N.S.	1	1.10	0.81	1.20	0.00	2.58	2.44	1.28	1.43
time (sec)	N/A	0.384	0.041	3.303	0.000	0.273	0.523	0.287	5.295

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	172	134	182	0	552	483	203	257
N.S.	1	1.05	0.82	1.11	0.00	3.37	2.95	1.24	1.57
time (sec)	N/A	0.388	0.063	3.314	0.000	0.296	1.278	0.293	0.164

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	221	183	230	0	777	400	252	243
N.S.	1	1.07	0.88	1.11	0.00	3.75	1.93	1.22	1.17
time (sec)	N/A	0.398	0.081	3.282	0.000	0.280	6.092	0.317	5.450

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	262	242	289	0	1024	0	330	303
N.S.	1	1.09	1.01	1.20	0.00	4.27	0.00	1.38	1.26
time (sec)	N/A	0.463	0.097	3.313	0.000	0.274	0.000	0.293	5.390

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	326	326	423	409	335
N.S.	1	1.00	1.00	1.09	1.05	1.05	1.36	1.32	1.08
time (sec)	N/A	0.591	0.076	3.387	0.194	0.244	0.041	0.279	0.118

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	244	239	239	304	289	233
N.S.	1	1.00	1.00	1.08	1.06	1.06	1.35	1.28	1.03
time (sec)	N/A	0.443	0.057	3.275	0.216	0.327	0.045	0.298	5.181

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	144	146	146	173	165	143
N.S.	1	1.00	1.00	1.11	1.12	1.12	1.33	1.27	1.10
time (sec)	N/A	0.328	0.034	3.332	0.185	0.254	0.028	0.338	5.172

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	249	179	300	0	586	508	313	312
N.S.	1	1.10	0.79	1.32	0.00	2.58	2.24	1.38	1.37
time (sec)	N/A	0.550	0.059	3.307	0.000	0.264	0.850	0.307	5.411

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	255	176	308	0	834	661	334	389
N.S.	1	1.05	0.73	1.27	0.00	3.45	2.73	1.38	1.61
time (sec)	N/A	0.552	0.088	3.428	0.000	0.270	2.225	0.304	5.506

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	311	219	344	0	1102	865	389	495
N.S.	1	1.07	0.75	1.18	0.00	3.79	2.97	1.34	1.70
time (sec)	N/A	0.552	0.106	3.354	0.000	0.287	58.357	0.299	0.227

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	374	295	417	0	1422	0	475	444
N.S.	1	1.07	0.85	1.20	0.00	4.09	0.00	1.36	1.28
time (sec)	N/A	0.596	0.132	3.372	0.000	0.293	0.000	0.323	5.660

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	481	373	695	0	463	0	0	0
N.S.	1	0.88	0.69	1.28	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.729	3.132	5.006	0.000	0.104	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	346	267	431	0	305	0	0	0
N.S.	1	0.91	0.70	1.13	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.487	1.602	4.441	0.000	0.107	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	268	212	312	0	191	0	0	0
N.S.	1	0.95	0.75	1.10	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.367	1.422	5.026	0.000	0.099	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	260	192	328	0	223	0	0	0
N.S.	1	0.96	0.71	1.21	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.357	2.513	3.328	0.000	0.099	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	276	297	518	0	515	0	0	0
N.S.	1	1.01	1.08	1.89	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.378	3.527	3.450	0.000	0.101	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	404	379	756	0	1105	0	0	0
N.S.	1	1.05	0.98	1.96	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.563	3.864	3.502	0.000	0.133	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	483	372	695	0	465	0	0	0
N.S.	1	0.89	0.69	1.28	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.742	3.165	4.915	0.000	0.098	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	0	275	448	0	318	0	0	0
N.S.	1	0.00	0.69	1.12	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.000	3.099	6.167	0.000	0.095	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	339	248	544	0	416	0	0	0
N.S.	1	0.92	0.67	1.47	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.502	4.841	7.384	0.000	0.099	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	349	296	559	0	561	0	0	0
N.S.	1	0.94	0.79	1.50	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.523	5.581	4.504	0.000	0.107	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	390	382	749	0	1040	0	0	0
N.S.	1	1.04	1.02	1.99	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.568	5.997	4.566	0.000	0.138	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	560	545	1023	0	1847	0	0	0
N.S.	1	1.05	1.03	1.93	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.787	6.470	4.561	0.000	0.174	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	497	386	691	0	489	0	0	0
N.S.	1	0.90	0.70	1.25	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.732	4.004	7.475	0.000	0.102	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	363	279	448	0	334	0	0	0
N.S.	1	0.92	0.70	1.13	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.531	3.086	6.117	0.000	0.094	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	265	215	312	0	201	0	0	0
N.S.	1	0.94	0.76	1.11	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.365	1.372	5.260	0.000	0.095	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	207	131	158	0	130	0	0	0
N.S.	1	1.00	0.64	0.77	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.299	2.589	3.357	0.000	0.085	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	254	0	0	0
N.S.	1	1.00	0.99	1.60	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.279	6.623	3.911	0.000	0.088	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	297	302	540	0	611	0	0	0
N.S.	1	1.05	1.06	1.90	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.380	9.140	5.563	0.000	0.110	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	432	393	760	0	1257	0	0	0
N.S.	1	1.08	0.98	1.90	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.570	10.226	6.752	0.000	0.142	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	459	369	794	0	648	0	0	0
N.S.	1	0.92	0.74	1.58	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.679	5.810	9.676	0.000	0.101	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	340	260	543	0	418	0	0	0
N.S.	1	0.95	0.73	1.52	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.500	4.875	7.410	0.000	0.093	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	260	208	378	0	239	0	0	0
N.S.	1	1.01	0.81	1.47	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.363	2.507	3.346	0.000	0.088	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	258	0	0	0
N.S.	1	1.00	1.01	1.67	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.283	6.870	3.906	0.000	0.099	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	281	262	515	0	618	0	0	0
N.S.	1	1.03	0.96	1.89	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.385	10.501	4.985	0.000	0.107	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	396	428	673	0	1259	0	0	0
N.S.	1	1.06	1.14	1.79	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.560	11.387	6.528	0.000	0.152	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	260	0	0	0
N.S.	1	1.00	1.01	1.67	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.285	6.628	3.912	0.000	0.094	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	257	220	349	0	257	0	0	0
N.S.	1	1.04	0.89	1.41	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.434	6.872	4.525	0.000	0.092	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	249	213	333	0	254	0	0	0
N.S.	1	1.05	0.90	1.41	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.412	6.753	4.510	0.000	0.092	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	255	221	338	0	257	0	0	0
N.S.	1	1.05	0.91	1.40	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.433	6.742	4.953	0.000	0.094	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	192	81	105	0	124	0	0	0
N.S.	1	1.01	0.42	0.55	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.290	2.358	3.100	0.000	0.087	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	250	142	282	0	181	0	0	0
N.S.	1	0.95	0.54	1.08	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.354	1.172	4.930	0.000	0.088	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	328	186	390	0	274	0	0	0
N.S.	1	0.92	0.52	1.10	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.448	1.223	4.852	0.000	0.094	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	104	2538	0	369	0	0	0
N.S.	1	1.00	0.92	22.46	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.317	2.417	3.717	0.000	0.110	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	526	557	100	2477	0	372	0	0	0
N.S.	1	1.06	0.19	4.71	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.740	2.387	1.368	0.000	0.099	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	134	141	115	0	777	0	0	0
N.S.	1	1.05	1.10	0.90	0.00	6.07	0.00	0.00	0.00
time (sec)	N/A	0.305	0.356	1.410	0.000	0.904	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	301	214	191	0	1718	0	0	0
N.S.	1	0.99	0.70	0.63	0.00	5.65	0.00	0.00	0.00
time (sec)	N/A	0.532	0.544	3.571	0.000	3.868	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	167	151	132	0	1111	0	0	0
N.S.	1	1.01	0.91	0.80	0.00	6.69	0.00	0.00	0.00
time (sec)	N/A	0.310	0.297	3.457	0.000	1.115	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	111	76	0	737	0	0	0
N.S.	1	1.00	1.22	0.84	0.00	8.10	0.00	0.00	0.00
time (sec)	N/A	0.213	0.231	3.349	0.000	0.788	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	41	0	241	0	70	0
N.S.	1	1.00	1.43	0.84	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.172	0.081	3.412	0.000	0.302	0.000	0.323	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	153	120	0	1305	0	165	0
N.S.	1	1.00	1.25	0.98	0.00	10.70	0.00	1.35	0.00
time (sec)	N/A	0.253	0.330	3.433	0.000	61.426	0.000	0.337	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	201	203	169	0	0	0	458	0
N.S.	1	0.99	1.00	0.83	0.00	0.00	0.00	2.26	0.00
time (sec)	N/A	0.367	15.335	3.564	0.000	0.000	0.000	4.547	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	713	456	657	0	0	0	0	0
N.S.	1	1.17	0.75	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.069	5.603	8.061	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	383	346	845	0	0	0	0	0
N.S.	1	0.96	0.86	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	3.985	8.397	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	325	184	340	0	0	0	0	0
N.S.	1	1.01	0.57	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	1.720	3.360	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	1.714	3.365	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	347	390	0	0	0	0	0
N.S.	1	1.00	1.66	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	4.002	4.353	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	402	427	1366	0	0	0	0	0
N.S.	1	1.00	1.06	3.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.572	6.245	5.285	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	641	584	3138	0	0	0	0	0
N.S.	1	1.02	0.93	4.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.893	5.802	6.956	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	712	445	663	0	0	0	0	0
N.S.	1	1.08	0.68	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	6.650	7.950	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	380	739	845	0	0	0	0	0
N.S.	1	0.94	1.83	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	3.644	8.258	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	325	184	300	0	0	0	0	0
N.S.	1	0.99	0.56	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	3.232	3.457	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	492	630	0	0	0	0	0
N.S.	1	1.00	2.20	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	6.050	3.997	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	392	999	1645	0	0	0	0	0
N.S.	1	1.00	2.55	4.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.629	7.026	5.190	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	623	570	3112	0	0	0	0	0
N.S.	1	0.97	0.89	4.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.952	7.609	6.876	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	599	350	741	0	0	0	0	0
N.S.	1	0.96	0.56	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	4.802	8.404	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	322	197	341	0	0	0	0	0
N.S.	1	1.01	0.62	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	3.328	3.417	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	1.669	3.339	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	118	0	0	0	0	0
N.S.	1	1.00	1.01	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	1.955	3.887	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	343	365	413	0	0	0	0	0
N.S.	1	1.00	1.06	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	4.222	5.058	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	438	433	1325	0	0	0	0	0
N.S.	1	1.01	1.00	3.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.692	6.330	6.551	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	980	828	352	1063	0	0	0	0	0
N.S.	1	0.84	0.36	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.268	6.826	4.210	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	304	594	0	0	0	0	0
N.S.	1	1.00	1.36	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	5.843	4.118	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	207	285	0	0	0	0	0
N.S.	1	1.00	0.99	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	3.384	4.250	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	346	221	303	0	0	0	0	0
N.S.	1	1.01	0.64	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	4.036	4.983	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	522	418	956	0	0	0	0	0
N.S.	1	0.97	0.78	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	8.126	6.595	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	814	791	1645	2127	0	0	0	0	0
N.S.	1	0.97	2.02	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.102	9.591	7.726	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	234	204	265	0	0	0	0	0
N.S.	1	0.97	0.84	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	2.334	6.699	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	188	78	121	0	0	0	0	0
N.S.	1	0.98	0.41	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	1.020	3.326	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	64	0	0	0	0	0
N.S.	1	1.00	0.93	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	1.938	3.344	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	78	147	0	0	0	0	0
N.S.	1	1.00	0.64	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	2.311	3.347	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	217	353	372	0	0	0	0	0
N.S.	1	1.01	1.64	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	3.084	4.234	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	302	134	293	0	0	0	0	0
N.S.	1	1.01	0.45	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	1.690	3.382	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0	0
N.S.	1	1.00	1.01	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	1.549	3.355	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	53	0	0	0	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	1.625	3.389	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	14	19	0	0
N.S.	1	1.00	1.03	0.97	0.00	0.39	0.53	0.00	0.00
time (sec)	N/A	0.181	0.464	3.599	0.000	0.080	1.919	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	131	100	0	513	0	329	0
N.S.	1	1.00	1.16	0.88	0.00	4.54	0.00	2.91	0.00
time (sec)	N/A	0.242	0.478	3.483	0.000	1.109	0.000	0.938	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	360	422	574	0	0	0	0	0
N.S.	1	1.00	1.18	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	3.318	3.397	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	378	401	559	0	0	0	0	0
N.S.	1	0.99	1.05	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	3.047	3.349	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	430	617	989	0	0	0	0	0
N.S.	1	1.01	1.45	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	6.210	6.064	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	447	587	973	0	0	0	0	0
N.S.	1	0.92	1.21	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	4.310	4.894	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	0	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	0.00	0.91	0.88	0.88
time (sec)	N/A	0.177	18.211	0.020	0.239	0.000	12.864	0.351	5.900

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	545	545	503	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	3.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	162	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	2.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	4.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	50	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	1.47	0.91	0.88	0.88
time (sec)	N/A	0.181	20.482	0.084	0.273	54.452	17.592	0.404	6.217

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	484	748	0	0	0	0	0	0	0
N.S.	1	1.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.850	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	568	148	0	0	0	0	0	0
N.S.	1	1.78	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	5.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	111	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	3.26	0.91	0.88	0.88
time (sec)	N/A	0.180	21.111	0.087	0.265	0.261	11.111	0.359	6.260

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	541	540	512	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	3.484	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	53	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	1.56	0.91	0.88	0.88
time (sec)	N/A	0.181	5.068	0.088	0.267	60.517	9.861	0.367	7.785

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	159	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	2.599	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	2.939	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	127	32	30	30
N.S.	1	1.00	1.06	0.82	0.88	3.74	0.94	0.88	0.88
time (sec)	N/A	0.183	10.594	0.088	0.285	0.278	7.327	0.352	9.043

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [64] had the largest ratio of [.5312500000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	24	0.083
2	A	2	2	1.00	24	0.083
3	A	2	2	1.00	24	0.083
4	A	2	2	1.00	22	0.091
5	A	4	4	1.07	24	0.167
6	A	4	4	1.03	24	0.167
7	A	4	4	1.08	24	0.167
8	A	5	5	0.98	24	0.208
9	A	2	2	1.00	26	0.077
10	A	2	2	1.00	26	0.077
11	A	2	2	1.00	24	0.083
12	A	6	6	1.10	26	0.231
13	A	6	6	1.05	26	0.231
14	A	5	5	1.07	26	0.192
15	A	6	6	1.09	26	0.231
16	A	2	2	1.00	26	0.077
17	A	2	2	1.00	26	0.077
18	A	2	2	1.00	24	0.083
19	A	7	7	1.10	26	0.269
20	A	7	7	1.05	26	0.269
21	A	6	6	1.07	26	0.231
22	A	7	7	1.07	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	8	0.88	30	0.267
24	A	7	7	0.91	30	0.233
25	A	6	6	0.95	30	0.200
26	A	6	6	0.96	30	0.200
27	A	5	5	1.01	30	0.167
28	A	7	7	1.05	30	0.233
29	A	9	9	0.89	30	0.300
30	F	0	0	N/A	0.000	N/A
31	A	7	7	0.92	30	0.233
32	A	9	9	0.94	30	0.300
33	A	7	7	1.04	30	0.233
34	A	9	9	1.05	30	0.300
35	A	9	9	0.90	30	0.300
36	A	7	7	0.92	30	0.233
37	A	6	6	0.94	30	0.200
38	A	4	4	1.00	30	0.133
39	A	3	3	1.00	30	0.100
40	A	5	5	1.05	30	0.167
41	A	7	7	1.08	30	0.233
42	A	9	9	0.92	30	0.300
43	A	8	8	0.95	30	0.267
44	A	6	6	1.01	30	0.200
45	A	3	3	1.00	30	0.100
46	A	5	5	1.03	30	0.167
47	A	7	7	1.06	30	0.233
48	A	3	3	1.00	30	0.100
49	A	9	9	1.04	31	0.290
50	A	8	8	1.05	31	0.258
51	A	9	9	1.05	32	0.281
52	A	4	4	1.01	30	0.133
53	A	6	6	0.95	30	0.200
54	A	7	7	0.92	30	0.233
55	A	2	2	1.00	87	0.023

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	5	5	1.06	81	0.062
57	A	8	7	1.05	28	0.250
58	A	16	15	0.99	30	0.500
59	A	10	9	1.01	30	0.300
60	A	6	5	1.00	28	0.179
61	A	3	2	1.00	21	0.095
62	A	4	3	1.00	30	0.100
63	A	10	9	0.99	30	0.300
64	A	17	17	1.17	32	0.531
65	A	8	8	0.96	32	0.250
66	A	6	6	1.01	32	0.188
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	8	8	1.00	32	0.250
70	A	12	12	1.02	32	0.375
71	A	16	16	1.08	32	0.500
72	A	8	8	0.94	32	0.250
73	A	6	6	0.99	32	0.188
74	A	3	3	1.00	32	0.094
75	A	8	8	1.00	32	0.250
76	A	12	12	0.97	32	0.375
77	A	13	13	0.96	32	0.406
78	A	6	6	1.01	32	0.188
79	A	1	1	1.00	32	0.031
80	A	3	3	1.00	32	0.094
81	A	5	5	1.00	32	0.156
82	A	9	9	1.01	32	0.281
83	A	17	17	0.84	32	0.531
84	A	3	3	1.00	32	0.094
85	A	3	3	1.00	32	0.094
86	A	5	5	1.01	32	0.156
87	A	9	9	0.97	32	0.281
88	A	13	13	0.97	32	0.406
89	A	8	8	0.97	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	6	6	0.98	28	0.214
91	A	1	1	1.00	28	0.036
92	A	3	3	1.00	28	0.107
93	A	8	8	1.01	28	0.286
94	A	6	6	1.01	32	0.188
95	A	1	1	1.00	32	0.031
96	A	1	1	1.00	32	0.031
97	A	3	3	1.00	30	0.100
98	A	6	5	1.00	28	0.179
99	A	11	11	1.00	33	0.333
100	A	8	8	0.99	32	0.250
101	A	11	11	1.01	33	0.333
102	A	8	8	0.92	32	0.250
103	N/A	1	0	1.00	34	0.000
104	A	8	7	1.00	34	0.206
105	A	3	2	1.00	34	0.059
106	A	3	2	1.00	34	0.059
107	N/A	1	0	1.00	34	0.000
108	A	9	8	1.55	34	0.235
109	A	6	5	1.78	34	0.147
110	N/A	1	0	1.00	34	0.000
111	A	8	7	1.00	34	0.206
112	N/A	1	0	1.00	34	0.000
113	A	3	2	1.00	34	0.059
114	A	3	2	1.00	34	0.059
115	N/A	1	0	1.00	34	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx$

3.1.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx = {} & ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3 \\ & +\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5 \\ & +\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7 \\ & +\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9 \\ & +\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}\end{aligned}$$

output
```
a*c*e^4*x+1/3*e^3*(4*a*c*f+a*d*e+b*c*e)*x^3+1/5*e^2*(2*a*f*(3*c*f+2*d*e)+b
*e*(4*c*f+d*e))*x^5+2/7*e*f*(a*f*(2*c*f+3*d*e)+b*e*(3*c*f+2*d*e))*x^7+1/9*
f^2*(a*f*(c*f+4*d*e)+2*b*e*(2*c*f+3*d*e))*x^9+1/11*f^3*(a*d*f+b*c*f+4*b*d*
e)*x^11+1/13*b*d*f^4*x^13
```

3.1.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx = ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3 +\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5 +\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7 +\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9 +\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

output
```
a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3
*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d
*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9
)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

3.1.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(e^3x^2(4acf+ade+bce)+e^2x^4(2af(3cf+2de)+be(4cf+de))+f^3x^{10}(adf+bcf+4bde)+f^2x^8(af(cf+4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

output
```
a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3
*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d
*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9
)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

3.1.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

method	result
default	$\frac{bd\,f^4x^{13}}{13}+\frac{((ad+bc)f^4+4bde\,f^3)x^{11}}{11}+\frac{(ac\,f^4+4(ad+bc)e\,f^3+6bd\,e^2f^2)x^9}{9}+\frac{(4ace\,f^3+6(ad+bc)e^2f^2+4bd\,e^3f)x^7}{7}+$
norman	$\frac{bd\,f^4x^{13}}{13}+\left(\frac{1}{11}ad\,f^4+\frac{1}{11}bc\,f^4+\frac{4}{11}bde\,f^3\right)x^{11}+\left(\frac{1}{9}ac\,f^4+\frac{4}{9}ade\,f^3+\frac{4}{9}bce\,f^3+\frac{2}{3}bd\,e^2f^2\right)x^9$
gosper	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$
risch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$
parallelrisch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*f^4+4*(a*
d+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+b*c)*e^2*f^2+4*b*d
*e^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*f+b*d*e^4)*x^5+1/3*(4*a*c*e
^3*f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = \frac{1}{13}bdf^4x^{13} + \frac{1}{11}\left(4\,bdef^3 + (bc+ad)f^4\right)x^{11} + \frac{1}{9}\left(6\,bde^2f^2 + acf^4 + 4\,(bc+ad)ef^3\right)x^9 + \frac{2}{7}\left(2\,bde^3f + 2\,acef^3 + 3\,(bc+ad)e^2f^2\right)x^7 + ace^4x + \frac{1}{5}\left(bde^4 + 6\,ace^2f^2 + 4\,(bc+ad)e^3f\right)x^5 + \frac{1}{3}\left(4\,ace^3f + (bc+ad)e^4\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d
*e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e
*f^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f
^2 + 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.37

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = ace^4x + \frac{bdf^4x^{13}}{13} + x^{11}\left(\frac{adf^4}{11} + \frac{bcf^4}{11} + \frac{4bdef^3}{11}\right) + x^9\left(\frac{acf^4}{9} + \frac{4adef^3}{9} + \frac{4bcef^3}{9} + \frac{2bde^2f^2}{3}\right) + x^7 \cdot \left(\frac{4acef^3}{7} + \frac{6ade^2f^2}{7} + \frac{6bce^2f^2}{7} + \frac{4bde^3f}{7}\right) + x^5 \cdot \left(\frac{6ace^2f^2}{5} + \frac{4ade^3f}{5} + \frac{4bce^3f}{5} + \frac{bde^4}{5}\right) + x^3 \cdot \left(\frac{4ace^3f}{3} + \frac{ade^4}{3} + \frac{bce^4}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

output
```
a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11 + 4*b*d*
e*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e*f**3/9 + 2*b*d*e*
*2*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*f**2/7 + 6*b*c*e**2*f**2/7
+ 4*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*
f/5 + b*d*e**4/5) + x**3*(4*a*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (a+bx^2)\,(c+dx^2)\,(e+fx^2)^4\,dx &= \frac{1}{13}\,bdf^4x^{13} + \frac{1}{11}\,(4\,bdef^3 + (bc+ad)f^4)x^{11} \\ &\quad + \frac{1}{9}\,(6\,bde^2f^2 + acf^4 + 4\,(bc+ad)ef^3)x^9 \\ &\quad + \frac{2}{7}\,(2\,bde^3f + 2\,acef^3 + 3\,(bc+ad)e^2f^2)x^7 \\ &\quad + ace^4x + \frac{1}{5}\,(bde^4 + 6\,ace^2f^2 + 4\,(bc+ad)e^3f)x^5 \\ &\quad + \frac{1}{3}\,(4\,ace^3f + (bc+ad)e^4)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d
*e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e
*f^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f
^2 + 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (a+bx^2)\,(c+dx^2)\,(e+fx^2)^4\,dx &= \frac{1}{13}\,bdf^4x^{13} + \frac{4}{11}\,bdef^3x^{11} + \frac{1}{11}\,bcf^4x^{11} + \frac{1}{11}\,adf^4x^{11} \\ &\quad + \frac{2}{3}\,bde^2f^2x^9 + \frac{4}{9}\,bcef^3x^9 + \frac{4}{9}\,adef^3x^9 + \frac{1}{9}\,acf^4x^9 \\ &\quad + \frac{4}{7}\,bde^3fx^7 + \frac{6}{7}\,bce^2f^2x^7 + \frac{6}{7}\,ade^2f^2x^7 + \frac{4}{7}\,acef^3x^7 \\ &\quad + \frac{1}{5}\,bde^4x^5 + \frac{4}{5}\,bce^3fx^5 + \frac{4}{5}\,ade^3fx^5 + \frac{6}{5}\,ace^2f^2x^5 \\ &\quad + \frac{1}{3}\,bce^4x^3 + \frac{1}{3}\,ade^4x^3 + \frac{4}{3}\,ace^3fx^3 + ace^4x\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/13*b*d*f^4*x^13 + 4/11*b*d*e*f^3*x^11 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4
*x^11 + 2/3*b*d*e^2*f^2*x^9 + 4/9*b*c*e*f^3*x^9 + 4/9*a*d*e*f^3*x^9 + 1/9*
a*c*f^4*x^9 + 4/7*b*d*e^3*f*x^7 + 6/7*b*c*e^2*f^2*x^7 + 6/7*a*d*e^2*f^2*x^
7 + 4/7*a*c*e*f^3*x^7 + 1/5*b*d*e^4*x^5 + 4/5*b*c*e^3*f*x^5 + 4/5*a*d*e^3*
f*x^5 + 6/5*a*c*e^2*f^2*x^5 + 1/3*b*c*e^4*x^3 + 1/3*a*d*e^4*x^3 + 4/3*a*c*
e^3*f*x^3 + a*c*e^4*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = x^3\left(\frac{a\,d\,e^4}{3}+\frac{b\,c\,e^4}{3}+\frac{4\,a\,c\,e^3\,f}{3}\right) + x^{11}\left(\frac{a\,d\,f^4}{11}+\frac{b\,c\,f^4}{11}+\frac{4\,b\,d\,e\,f^3}{11}\right) + x^5\left(\frac{b\,d\,e^4}{5}+\frac{4\,a\,d\,e^3\,f}{5}+\frac{4\,b\,c\,e^3\,f}{5}+\frac{6\,a\,c\,e^2\,f^2}{5}\right) + x^9\left(\frac{a\,c\,f^4}{9}+\frac{4\,a\,d\,e\,f^3}{9}+\frac{4\,b\,c\,e\,f^3}{9}+\frac{2\,b\,d\,e^2\,f^2}{3}\right) + \frac{2\,e\,f\,x^7\,(2\,a\,c\,f^2+2\,b\,d\,e^2+3\,a\,d\,e\,f+3\,b\,c\,e\,f)}{7} + a\,c\,e^4\,x + \frac{b\,d\,f^4\,x^{13}}{13}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x)
```

output
```
x^3*((a*d*e^4)/3 + (b*c*e^4)/3 + (4*a*c*e^3*f)/3) + x^11*((a*d*f^4)/11 + (
b*c*f^4)/11 + (4*b*d*e*f^3)/11) + x^5*((b*d*e^4)/5 + (4*a*d*e^3*f)/5 + (4*
b*c*e^3*f)/5 + (6*a*c*e^2*f^2)/5) + x^9*((a*c*f^4)/9 + (4*a*d*e*f^3)/9 + (
4*b*c*e*f^3)/9 + (2*b*d*e^2*f^2)/3) + (2*e*f*x^7*(2*a*c*f^2 + 2*b*d*e^2 +
3*a*d*e*f + 3*b*c*e*f))/7 + a*c*e^4*x + (b*d*f^4*x^13)/13
```

3.2 $\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx$

3.2.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = {} & ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3 \\ & +\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5 \\ & +\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7 \\ & +\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}\end{aligned}$$

output
```
a*c*e^3*x+1/3*e^2*(3*a*c*f+a*d*e+b*c*e)*x^3+1/5*e*(3*a*f*(c*f+d*e)+b*e*(3*
c*f+d*e))*x^5+1/7*f*(3*b*e*(c*f+d*e)+a*f*(c*f+3*d*e))*x^7+1/9*f^2*(a*d*f+b
*c*f+3*b*d*e)*x^9+1/11*b*d*f^3*x^11
```

3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = {} & ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3 \\ & +\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5 \\ & +\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7 \\ & +\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}\end{aligned}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f)
+ b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x
^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

3.2.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^3 dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(e^2x^2(3acf+ade+bce)+f^2x^8(adf+bcf+3bde)+fx^6(af(cf+3de)+3be(cf+de))+ex^4(3af(cf+de)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f)
+ b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x
^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

3.2.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

method	result
default	$\frac{bd\,f^3x^{11}}{11}+\frac{((ad+bc)f^3+3bde\,f^2)x^9}{9}+\frac{(ac\,f^3+3(ad+bc)e\,f^2+3bd\,e^2f)x^7}{7}+\frac{(3ace\,f^2+3(ad+bc)e^2f+bd\,e^3)x^5}{5}+\frac{(3ac\,e}{}$
norman	$\frac{bd\,f^3x^{11}}{11}+\left(\frac{1}{9}ad\,f^3+\frac{1}{9}bc\,f^3+\frac{1}{3}bde\,f^2\right)x^9+\left(\frac{1}{7}ac\,f^3+\frac{3}{7}ade\,f^2+\frac{3}{7}bce\,f^2+\frac{3}{7}bd\,e^2f\right)x^7+\left(\frac{3}{5}\right.$
gosper	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$
risch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$
parallelrisch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^3+3*(a*d+
b*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)*e^2*f+b*d*e^3)*x^
5+1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx =& \frac{1}{11}bdf^3x^{11}+\frac{1}{9}\left(3\,bdef^2+(bc+ad)f^3\right)x^9\\&+\frac{1}{7}\left(3\,bde^2f+acf^3+3\,(bc+ad)ef^2\right)x^7\\&+ace^3x+\frac{1}{5}\left(bde^3+3\,acef^2+3\,(bc+ad)e^2f\right)x^5\\&+\frac{1}{3}\left(3\,ace^2f+(bc+ad)e^3\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e
^2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a
*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*
x^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = ace^3x + \frac{bdf^3x^{11}}{11} + x^9\left(\frac{adf^3}{9} + \frac{bcf^3}{9} + \frac{bdef^2}{3}\right) + x^7\left(\frac{acf^3}{7} + \frac{3adef^2}{7} + \frac{3bcef^2}{7} + \frac{3bde^2f}{7}\right) + x^5\cdot\left(\frac{3acef^2}{5} + \frac{3ade^2f}{5} + \frac{3bce^2f}{5} + \frac{bde^3}{5}\right) + x^3\left(ace^2f + \frac{ade^3}{3} + \frac{bce^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

output
```
a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 + b*d*e*f**
2/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/7 + 3*b*d*e**2*f/7
) + x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b*c*e**2*f/5 + b*d*e**3/5) +
 x**3*(a*c*e**2*f + a*d*e**3/3 + b*c*e**3/3)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = \frac{1}{11}\,bdf^3x^{11} + \frac{1}{9}\left(3\,bdef^2 + (bc+ad)f^3\right)x^9 + \frac{1}{7}\left(3\,bde^2f + acf^3 + 3\,(bc+ad)ef^2\right)x^7 + ace^3x + \frac{1}{5}\left(bde^3 + 3\,acef^2 + 3\,(bc+ad)e^2f\right)x^5 + \frac{1}{3}\left(3\,ace^2f + (bc+ad)e^3\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e
^2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a
*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*
x^3
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = \frac{1}{11}bdf^3x^{11} + \frac{1}{3}bdef^2x^9 + \frac{1}{9}bcf^3x^9 + \frac{1}{9}adf^3x^9 + \frac{3}{7}bde^2fx^7 + \frac{3}{7}bcef^2x^7 + \frac{3}{7}adef^2x^7 + \frac{1}{7}acf^3x^7 + \frac{1}{5}bde^3x^5 + \frac{3}{5}bce^2fx^5 + \frac{3}{5}ade^2fx^5 + \frac{3}{5}acef^2x^5 + \frac{1}{3}bce^3x^3 + \frac{1}{3}ade^3x^3 + ace^2fx^3 + ace^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/11*b*d*f^3*x^11 + 1/3*b*d*e*f^2*x^9 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9
+ 3/7*b*d*e^2*f*x^7 + 3/7*b*c*e*f^2*x^7 + 3/7*a*d*e*f^2*x^7 + 1/7*a*c*f^3*
x^7 + 1/5*b*d*e^3*x^5 + 3/5*b*c*e^2*f*x^5 + 3/5*a*d*e^2*f*x^5 + 3/5*a*c*e*
f^2*x^5 + 1/3*b*c*e^3*x^3 + 1/3*a*d*e^3*x^3 + a*c*e^2*f*x^3 + a*c*e^3*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = x^5\left(\frac{b\,d\,e^3}{5}+\frac{3\,a\,c\,e\,f^2}{5}+\frac{3\,a\,d\,e^2\,f}{5}+\frac{3\,b\,c\,e^2\,f}{5}\right) + x^7\left(\frac{a\,c\,f^3}{7}+\frac{3\,a\,d\,e\,f^2}{7}+\frac{3\,b\,c\,e\,f^2}{7}+\frac{3\,b\,d\,e^2\,f}{7}\right) + x^3\left(\frac{a\,d\,e^3}{3}+\frac{b\,c\,e^3}{3}+a\,c\,e^2\,f\right) + x^9\left(\frac{a\,d\,f^3}{9}+\frac{b\,c\,f^3}{9}+\frac{b\,d\,e\,f^2}{3}\right) + a\,c\,e^3\,x + \frac{b\,d\,f^3\,x^{11}}{11}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

output
```
x^5*((b*d*e^3)/5 + (3*a*c*e*f^2)/5 + (3*a*d*e^2*f)/5 + (3*b*c*e^2*f)/5) +
x^7*((a*c*f^3)/7 + (3*a*d*e*f^2)/7 + (3*b*c*e*f^2)/7 + (3*b*d*e^2*f)/7) +
x^3*((a*d*e^3)/3 + (b*c*e^3)/3 + a*c*e^2*f) + x^9*((a*d*f^3)/9 + (b*c*f^3)
/9 + (b*d*e*f^2)/3) + a*c*e^3*x + (b*d*f^3*x^11)/11
```

3.2. $\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx$

3.3 $\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx$

3.3.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3 + \frac{1}{5}(af(2de+cf)+be(de+2cf))x^5 + \frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

output
```
a*c*e^2*x+1/3*e*(2*a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*f*(c*f+2*d*e)+b*e*(2*c*f+
d*e))*x^5+1/7*f*(a*d*f+b*c*f+2*b*d*e)*x^7+1/9*b*d*f^2*x^9
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3 + \frac{1}{5}\left(bde^2+2bcef+2adef+acf^2\right)x^5 + \frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b*c*e*f +
2*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f
^2*x^9)/9
```

3.3.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(fx^6(adf+bcf+2bde)+x^4(af(cf+2de)+be(2cf+de))+ex^2(2acf+ade+bce)+ace^2+bdf^2x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*
e*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x
^9)/9
```

3.3.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
default	$\frac{bd\,f^{2}x^{9}}{9}+\frac{\left(\left(ad+bc\right)f^{2}+2ebdf\right)x^{7}}{7}+\frac{\left(ac\,f^{2}+2\left(ad+bc\right)ef+bd\,e^{2}\right)x^{5}}{5}+\frac{\left(2acef+\left(ad+bc\right)e^{2}\right)x^{3}}{3}+ac\,e^{2}x$
norman	$\frac{bd\,f^{2}x^{9}}{9}+\left(\frac{1}{7}ad\,f^{2}+\frac{1}{7}bc\,f^{2}+\frac{2}{7}ebdf\right)x^{7}+\left(\frac{1}{5}ac\,f^{2}+\frac{2}{5}adef+\frac{2}{5}bcef+\frac{1}{5}bd\,e^{2}\right)x^{5}+\left(\frac{2}{3}acef+\right.$
gosper	$\frac{1}{9}bd\,f^{2}x^{9}+\frac{1}{7}x^{7}ad\,f^{2}+\frac{1}{7}x^{7}bc\,f^{2}+\frac{2}{7}x^{7}ebdf+\frac{1}{5}x^{5}ac\,f^{2}+\frac{2}{5}x^{5}adef+\frac{2}{5}x^{5}bcef+\frac{1}{5}x^{5}bd\,e^{2}+$
risch	$\frac{1}{9}bd\,f^{2}x^{9}+\frac{1}{7}x^{7}ad\,f^{2}+\frac{1}{7}x^{7}bc\,f^{2}+\frac{2}{7}x^{7}ebdf+\frac{1}{5}x^{5}ac\,f^{2}+\frac{2}{5}x^{5}adef+\frac{2}{5}x^{5}bcef+\frac{1}{5}x^{5}bd\,e^{2}+$
parallelrisch	$\frac{1}{9}bd\,f^{2}x^{9}+\frac{1}{7}x^{7}ad\,f^{2}+\frac{1}{7}x^{7}bc\,f^{2}+\frac{2}{7}x^{7}ebdf+\frac{1}{5}x^{5}ac\,f^{2}+\frac{2}{5}x^{5}adef+\frac{2}{5}x^{5}bcef+\frac{1}{5}x^{5}bd\,e^{2}+$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*e*b*d*f)*x^7+1/5*(a*c*f^2+2*(a*d+b*c)
*e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*e^2*x
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\left(2\,bdef+(bc+ad)f^2\right)x^7+\frac{1}{5}\left(bde^2+acf^2+2\,(bc+ad)ef\right)x^5+ace^2x+\frac{1}{3}\left(2\,acef+(bc+ad)e^2\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a
*c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)
*e^2)*x^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x+\frac{bdf^2x^9}{9}+x^7\left(\frac{adf^2}{7}+\frac{bcf^2}{7}+\frac{2bdef}{7}\right)+x^5\left(\frac{acf^2}{5}+\frac{2adef}{5}+\frac{2bcef}{5}+\frac{bde^2}{5}\right)+x^3\cdot\left(\frac{2acef}{3}+\frac{ade^2}{3}+\frac{bce^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

output
```
a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*b*d*e*f/7
) + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5) + x**3*(2*a
*c*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9+\frac{1}{7}(2\,bdef+(bc+ad)f^2)x^7+\frac{1}{5}(bde^2+acf^2+2\,(bc+ad)ef)x^5+ace^2x+\frac{1}{3}(2\,acef+(bc+ad)e^2)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a
*c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)
*e^2)*x^3
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{2}{7}bdefx^7 + \frac{1}{7}bcf^2x^7 + \frac{1}{7}adf^2x^7 + \frac{1}{5}bde^2x^5 + \frac{2}{5}bcefx^5 + \frac{2}{5}adefx^5 + \frac{1}{5}acf^2x^5 + \frac{1}{3}bce^2x^3 + \frac{1}{3}ade^2x^3 + \frac{2}{3}acefx^3 + ace^2x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/9*b*d*f^2*x^9 + 2/7*b*d*e*f*x^7 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 1/
5*b*d*e^2*x^5 + 2/5*b*c*e*f*x^5 + 2/5*a*d*e*f*x^5 + 1/5*a*c*f^2*x^5 + 1/3*
b*c*e^2*x^3 + 1/3*a*d*e^2*x^3 + 2/3*a*c*e*f*x^3 + a*c*e^2*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = x^5\left(\frac{a\,c\,f^2}{5}+\frac{b\,d\,e^2}{5}+\frac{2\,a\,d\,e\,f}{5}+\frac{2\,b\,c\,e\,f}{5}\right) + x^3\left(\frac{a\,d\,e^2}{3}+\frac{b\,c\,e^2}{3}+\frac{2\,a\,c\,e\,f}{3}\right) + x^7\left(\frac{a\,d\,f^2}{7}+\frac{b\,c\,f^2}{7}+\frac{2\,b\,d\,e\,f}{7}\right)+a\,c\,e^2\,x+\frac{b\,d\,f^2\,x^9}{9}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

output
```
x^5*((a*c*f^2)/5 + (b*d*e^2)/5 + (2*a*d*e*f)/5 + (2*b*c*e*f)/5) + x^3*((a*
d*e^2)/3 + (b*c*e^2)/3 + (2*a*c*e*f)/3) + x^7*((a*d*f^2)/7 + (b*c*f^2)/7 +
 (2*b*d*e*f)/7) + a*c*e^2*x + (b*d*f^2*x^9)/9
```

3.4 $\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$

3.4.1 Optimal result

Integrand size = 22, antiderivative size = 56

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex + \frac{1}{3}(bce+ade+acf)x^3 + \frac{1}{5}(bde+bcf+adf)x^5 + \frac{1}{7}bdfx^7$$

output
```
a*c*e*x+1/3*(a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*d*f+b*c*f+b*d*e)*x^5+1/7*b*d*f*
x^7
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex + \frac{1}{3}(bce+ade+acf)x^3 + \frac{1}{5}(bde+bcf+adf)x^5 + \frac{1}{7}bdfx^7$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/
5 + (b*d*f*x^7)/7
```

3.4.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(x^4(adf+bcf+bde)+x^2(acf+ade+bce)+ace+bdfx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/
5 + (b*d*f*x^7)/7
```

3.4.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{bdf\,x^7}{7}+\frac{((ad+bc)f+bde)x^5}{5}+\frac{(acf+(ad+bc)e)x^3}{3}+acex$	53
norman	$\frac{bdf\,x^7}{7}+\left(\frac{1}{5}adf+\frac{1}{5}bcf+\frac{1}{5}bde\right)x^5+\left(\frac{1}{3}acf+\frac{1}{3}ade+\frac{1}{3}bce\right)x^3+acex$	55
gosper	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
risch	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
parallelrisch	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)*x^3+a*c*
e*x
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7+\frac{1}{5}(bde+(bc+ad)f)x^5 + acex + \frac{1}{3}(acf+(bc+ad)e)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (
b*c + a*d)*e)*x^3
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex + \frac{bdfx^7}{7} + x^5\left(\frac{adf}{5} + \frac{bcf}{5} + \frac{bde}{5}\right) + x^3\left(\frac{acf}{3} + \frac{ade}{3} + \frac{bce}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

output
```
a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**3*(a*c*f/
3 + a*d*e/3 + b*c*e/3)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7 + \frac{1}{5}(bde + (bc+ad)f)x^5 + acex + \frac{1}{3}(acf + (bc+ad)e)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (
b*c + a*d)*e)*x^3
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7 + \frac{1}{5}bdex^5 + \frac{1}{5}bcfx^5 + \frac{1}{5}adfx^5 + \frac{1}{3}bcex^3 + \frac{1}{3}adex^3 + \frac{1}{3}acfx^3 + acex$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

output
```
1/7*b*d*f*x^7 + 1/5*b*d*e*x^5 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/3*b*c*e*
x^3 + 1/3*a*d*e*x^3 + 1/3*a*c*f*x^3 + a*c*e*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{b\,d\,f\,x^7}{7} + \left(\frac{a\,d\,f}{5} + \frac{b\,c\,f}{5} + \frac{b\,d\,e}{5}\right)x^5 + \left(\frac{a\,c\,f}{3} + \frac{a\,d\,e}{3} + \frac{b\,c\,e}{3}\right)x^3 + a\,c\,e\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2),x)
```

output
```
x^3*((a*c*f)/3 + (a*d*e)/3 + (b*c*e)/3) + x^5*((a*d*f)/5 + (b*c*f)/5 + (b*
d*e)/5) + a*c*e*x + (b*d*f*x^7)/7
```

3.5 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$

3.5.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

output
```
-1/3*(-2*a*d*f-3*b*c*f+3*b*d*e)*x/f^2+1/3*d*x*(b*x^2+a)/f+(-a*f+b*e)*(-c*f
+d*e)*arctan(x*f^(1/2)/e^(1/2))/f^(5/2)/e^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{(-bde+bcf+adf)x}{f^2} + \frac{bdx^3}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*f)*(d*e -
 c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

3.5.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}dx$$

$$\Big\downarrow \ 403$$

$$\frac{\int -\frac{(3bde-3bcf-2adf)x^2+a(de-3cf)}{fx^2+e}dx}{3f}+\frac{dx(a+bx^2)}{3f}$$

$$\Big\downarrow \ 25$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\int \frac{(3bde-3bcf-2adf)x^2+a(de-3cf)}{fx^2+e}dx}{3f}$$

$$\Big\downarrow \ 299$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\frac{x(-2adf-3bcf+3bde)}{f}-\frac{3(be-af)(de-cf)\int \frac{1}{fx^2+e}dx}{f}}{3f}$$

$$\Big\downarrow \ 218$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\frac{x(-2adf-3bcf+3bde)}{f}-\frac{3(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)}{\sqrt{e}f^{3/2}}}{3f}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(d*x*(a + b*x^2))/(3*f) - (((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/f - (3*(b*e -
 a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f)
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.5.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{1}{3} bdf\,x^{3}+adfx+bcfx-bdex}{f^{2}}+\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right) \arctan\left(\frac{fx}{\sqrt{ef}}\right)}{f^{2} \sqrt{ef}}$
risch	$\frac{bd\,x^{3}}{3f}+\frac{adx}{f}+\frac{bcx}{f}-\frac{bdex}{f^{2}}-\frac{\ln\left(fx+\sqrt{-ef}\right) ac}{2\sqrt{-ef}}+\frac{\ln\left(fx+\sqrt{-ef}\right) ade}{2f\sqrt{-ef}}+\frac{\ln\left(fx+\sqrt{-ef}\right) bce}{2f\sqrt{-ef}}-\frac{\ln\left(fx+\sqrt{-ef}\right) bd\,e^{2}}{2f^{2}\sqrt{-ef}}+\ln(-$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(1/3*b*d*f*x^3+a*d*f*x+b*c*f*x-b*d*e*x)+(a*c*f^2-a*d*e*f-b*c*e*f+b*d
*e^2)/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$$
$$= \left[\frac{2\,bdef^2x^3 - 3\,(bde^2+acf^2-(bc+ad)ef)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right) - 6\,(bde^2f-(bc+ad)ef^2)x}{6\,ef^3}, \frac{bde\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/6*(2*b*d*e*f^2*x^3 - 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*sqrt(-e*f)
*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^2*f - (b*c + a*d
)*e*f^2)*x)/(e*f^3), 1/3*(b*d*e*f^2*x^3 + 3*(b*d*e^2 + a*c*f^2 - (b*c + a*
d)*e*f)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^2*f - (b*c + a*d)*e*f^2
)*x)/(e*f^3)]
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{bdx^3}{3f} + x\left(\frac{ad}{f}+\frac{bc}{f}-\frac{bde}{f^2}\right) - \frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(-\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

output
```
b*d*x**3/(3*f) + x*(a*d/f + b*c/f - b*d*e/f**2) - sqrt(-1/(e*f**5))*(a*f -
 b*e)*(c*f - d*e)*log(-e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a
*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f -
b*e)*(c*f - d*e)*log(e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c
*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2
```

3.5.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{(bde^2 - bcef - adef + acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^2} + \frac{bdf^2x^3 - 3\,bdefx + 3\,bcf^2x + 3\,adf^2x}{3\,f^3}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
(b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f
^2) + 1/3*(b*d*f^2*x^3 - 3*b*d*e*f*x + 3*b*c*f^2*x + 3*a*d*f^2*x)/f^3
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = x\left(\frac{a\,d+b\,c}{f} - \frac{b\,d\,e}{f^2}\right) + \frac{b\,d\,x^3}{3\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,f^{5/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x)
```

output
```
x*((a*d + b*c)/f - (b*d*e)/f^2) + (b*d*x^3)/(3*f) + (atan((f^(1/2)*x*(a*f
- b*e)*(c*f - d*e))/(e^(1/2)*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)))*(a*
f - b*e)*(c*f - d*e))/(e^(1/2)*f^(5/2))
```

3.5. $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$

3.6 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx$

3.6.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx = \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

output
```
1/2*b*(-c*f+3*d*e)*x/e/f^2-1/2*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)-1/2*(b
*e*(-c*f+3*d*e)-a*f*(c*f+d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(5/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx = \frac{bdx}{f^2} + \frac{(be-af)(de-cf)x}{2ef^2(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
(b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) - ((b*e*(3
*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(
5/2))
```

3.6.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}dx$$

$$\Big\downarrow \; 401$$

$$-\frac{\int -\frac{b(3de-cf)x^2+a(de+cf)}{fx^2+e}dx}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b(3de-cf)x^2+a(de+cf)}{fx^2+e}dx}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{bx(3de-cf)}{f}-\frac{(be(3de-cf)-af(cf+de))\int \frac{1}{fx^2+e}dx}{f}}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{bx(3de-cf)}{f}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{\sqrt{e}f^{3/2}}}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
-1/2*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)) + ((b*(3*d*e - c*f)*x)/
f - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(S
qrt[e]*f^(3/2)))/(2*e*f)
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.6.4 Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result
default	$\frac{bdx}{f^2}+\frac{\frac{\left(ac\,f^2-adef-bcef+bd\,e^2\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(ac\,f^2+adef+bcef-3bd\,e^2\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}}{f^2}$
risch	$\frac{bdx}{f^2}+\frac{(ac\,f^2-adef-bcef+bd\,e^2)x}{2e\,f^2(f\,x^2+e)}-\frac{\ln(fx+\sqrt{-ef})ac}{4\sqrt{-ef}\,e}-\frac{\ln(fx+\sqrt{-ef})ad}{4f\sqrt{-ef}}-\frac{\ln(fx+\sqrt{-ef})bc}{4f\sqrt{-ef}}+\frac{3e\ln(fx+\sqrt{-ef})bd}{4f^2\sqrt{-ef}}+$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
b*d/f^2*x+1/f^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(
a*c*f^2+a*d*e*f+b*c*e*f-3*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.94

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,bde^2f^2x^3 + (3\,bde^3 - acef^2 - (bc+ad)e^2f + (3\,bde^2f - acf^3 - (bc+ad)ef^2)x^2)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-e}}{fx^2+e}\right.}{4\,(e^2f^4x^2+e^3f^3)}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*b*d*e^2*f^2*x^3 + (3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*
b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sq
rt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^
2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3), 1/2*(2*b*d*e^2*f^2*x^3 - (3*b*d*e^3 - a
*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)
*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^3*f + a*c*e*f^3 - (b*c +
a*d)*e^2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3)]
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$= \frac{bdx}{f^2} + \frac{x(acf^2 - adef - bcef + bde^2)}{2e^2f^2 + 2ef^3x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2 + adef + bcef - 3bde^2)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2 + adef + bcef - 3bde^2)\log\left(e^2f^2\sqrt{-\frac{1}{e^3f^5}} + x\right)}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*f**2 + 2*
e*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*
e**2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4 + sqrt(-1/(e**3*f**5))*(a
*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e**2)*log(e**2*f**2*sqrt(-1/(e**3*f**5
)) + x)/4
```

3.6.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2} - \frac{(3\,bde^2 - bcef - adef - acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\sqrt{ef}ef^2} + \frac{bde^2x - bcefx - adefx + acf^2x}{2\,(fx^2+e)ef^2}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b*d*x/f^2 - 1/2*(3*b*d*e^2 - b*c*e*f - a*d*e*f - a*c*f^2)*arctan(f*x/sqrt(
e*f))/(sqrt(e*f)*e*f^2) + 1/2*(b*d*e^2*x - b*c*e*f*x - a*d*e*f*x + a*c*f^2
*x)/((f*x^2 + e)*e*f^2)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{b\,d\,x}{f^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(a\,c\,f^2-3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{2\,e^{3/2}\,f^{5/2}} + \frac{x\,(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f)}{2\,e\,(f^3\,x^2+e\,f^2)}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

output
```
(b*d*x)/f^2 + (atan((f^(1/2)*x)/e^(1/2))*(a*c*f^2 - 3*b*d*e^2 + a*d*e*f +
b*c*e*f))/(2*e^(3/2)*f^(5/2)) + (x*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)
)/(2*e*(e*f^2 + f^3*x^2))
```

3.7 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx$

3.7.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx = -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

output
```
-1/4*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^2-1/8*(b*e*(c*f+3*d*e)-a*f*(3*c*
f+d*e))*x/e^2/f^2/(f*x^2+e)+1/8*(b*e*(c*f+3*d*e)+a*f*(3*c*f+d*e))*arctan(x
*f^(1/2)/e^(1/2))/e^(5/2)/f^(5/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx = \frac{(be-af)(de-cf)x}{4ef^2(e+fx^2)^2} + \frac{(be(-5de+cf)+af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d*e + c*f)
 + a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a
*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{b(3de+cf)x^2+a(de+3cf)}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(3de+cf)x^2+a(de+3cf)}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(af(3cf+de)+be(cf+3de))\int \frac{1}{fx^2+e}dx}{2ef}-\frac{x(be(cf+3de)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{2e^{3/2}f^{3/2}}-\frac{x(be(cf+3de)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
-1/4*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(3*d*e
+ c*f) - a*f*(d*e + 3*c*f))*x)/(e*f*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a
*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(3/2)))/(4*e*f
)
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.7.4 Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{\left(3ac\,f^{2}+adef+bcef-5bd\,e^{2}\right)x^{3}}{8e^{2}f}+\frac{\left(5ac\,f^{2}-adef-bcef-3bd\,e^{2}\right)x}{8e\,f^{2}}}{\left(f\,x^{2}+e\right)^{2}}+\frac{\left(3ac\,f^{2}+adef+bcef+3bd\,e^{2}\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8e^{2}f^{2}\sqrt{ef}}$
risch	$\frac{\frac{\left(3ac\,f^{2}+adef+bcef-5bd\,e^{2}\right)x^{3}}{8e^{2}f}+\frac{\left(5ac\,f^{2}-adef-bcef-3bd\,e^{2}\right)x}{8e\,f^{2}}}{\left(f\,x^{2}+e\right)^{2}}-\frac{3\ln\left(fx+\sqrt{-ef}\right)ac}{16\sqrt{-ef}\,e^{2}}-\frac{\ln\left(fx+\sqrt{-ef}\right)ad}{16\sqrt{-ef}\,fe}-\frac{\ln\left(fx+\sqrt{-ef}\right)bc}{16\sqrt{-ef}\,fe}-$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f^2-a*d*e*
f-b*c*e*f-3*b*d*e^2)/e/f^2*x)/(f*x^2+e)^2+1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f+3
*b*d*e^2)/e^2/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$$

$$= \Bigg[-\frac{2\,(5\,bde^3f^2 - 3\,acef^4 - (bc+ad)e^2f^3)x^3 + (3\,bde^4 + 3\,ace^2f^2 + (bc+ad)e^3f + (3\,bde^2f^2 + 3\,acf^4 +}{1\!\!\!\!\quad}$$

$$-\frac{(5\,bde^3f^2 - 3\,acef^4 - (bc+ad)e^2f^3)x^3 - (3\,bde^4 + 3\,ace^2f^2 + (bc+ad)e^3f + (3\,bde^2f^2 + 3\,acf^4 + (bc}{8\,(e^3f^5}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 + (3*b*d
*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4 + (b
*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f^2)
*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*
e^4*f - 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x
^2 + e^5*f^3), -1/8*((5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x
^3 - (3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a
*c*f^4 + (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*
d)*e^2*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^4*f - 5*a*c*e^
2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 + e^5*f^3)]
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(122) = 244$.

Time = 1.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$$

$$= -\frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2 + adef + bcef + 3bde^2)\log\left(-e^3f^2\sqrt{-\frac{1}{e^5f^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2 + adef + bcef + 3bde^2)\log\left(e^3f^2\sqrt{-\frac{1}{e^5f^5}} + x\right)}{16} + \frac{x^3\cdot(3acf^3 + adef^2 + bcef^2 - 5bde^2f) + x(5acef^2 - ade^2f - bce^2f - 3bde^3)}{8e^4f^2 + 16e^3f^3x^2 + 8e^2f^4x^4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
-sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(-e
**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f**5))*(3*a*c*f**2 +
 a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/1
6 + (x**3*(3*a*c*f**3 + a*d*e*f**2 + b*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c
*e*f**2 - a*d*e**2*f - b*c*e**2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f*
*3*x**2 + 8*e**2*f**4*x**4)
```

3.7.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx &= \frac{(3\,bde^2+bcef+adef+3\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\,\sqrt{ef}e^2f^2} \\ &\quad - \frac{5\,bde^2fx^3-bcef^2x^3-adef^2x^3-3\,acf^3x^3+3\,bde^3x+bce^2fx+ade^2fx-5\,acef^2x}{8\,(fx^2+e)^2e^2f^2}\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(3*b*d*e^2 + b*c*e*f + a*d*e*f + 3*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqr
t(e*f)*e^2*f^2) - 1/8*(5*b*d*e^2*f*x^3 - b*c*e*f^2*x^3 - a*d*e*f^2*x^3 - 3
*a*c*f^3*x^3 + 3*b*d*e^3*x + b*c*e^2*f*x + a*d*e^2*f*x - 5*a*c*e*f^2*x)/((
f*x^2 + e)^2*e^2*f^2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(3\,a\,c\,f^2+3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^{5/2}\,f^{5/2}} - \frac{\frac{x\,(3\,b\,d\,e^2-5\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e\,f^2}-\frac{x^3\,(3\,a\,c\,f^2-5\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^2\,f}}{e^2+2\,e\,f\,x^2+f^2\,x^4}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

output
```
(atan((f^(1/2)*x)/e^(1/2))*(3*a*c*f^2 + 3*b*d*e^2 + a*d*e*f + b*c*e*f))/(8
*e^(5/2)*f^(5/2)) - ((x*(3*b*d*e^2 - 5*a*c*f^2 + a*d*e*f + b*c*e*f))/(8*e*
f^2) - (x^3*(3*a*c*f^2 - 5*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*e^2*f))/(e^2 +
 f^2*x^4 + 2*e*f*x^2)
```

3.8 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$

3.8.1 Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

output
```
-1/6*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^3-1/24*(3*b*e*(c*f+d*e)-a*f*(5*c
*f+d*e))*x/e^2/f^2/(f*x^2+e)^2+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*x/e^3/
f^2/(f*x^2+e)+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*arctan(x*f^(1/2)/e^(1/2
))/e^(7/2)/f^(5/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(be-af)(de-cf)x}{6ef^2(e+fx^2)^3} + \frac{(be(-7de+cf)+af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d*e + c*f)
 + a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) +
a*f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(
d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

3.8.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {401, 25, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{3b(de+cf)x^2+a(de+5cf)}{(fx^2+e)^3}dx}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3b(de+cf)x^2+a(de+5cf)}{(fx^2+e)^3}dx}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{3(af(5cf+de)+be(cf+de))\int \frac{1}{(fx^2+e)^2}dx}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3(af(5cf+de)+be(cf+de))\left(\frac{\int \frac{1}{fx^2+e}dx}{2e}+\frac{x}{2e(e+fx^2)}\right)}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}\sqrt{f}}+\frac{x}{2e\left(e+fx^2\right)}\right)(af(5cf+de)+be(cf+de))}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(a+bx^2\right)(de-cf)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
-1/6*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^3) + (-1/4*((3*b*e*(d*e
+ c*f) - a*f*(d*e + 5*c*f))*x)/(e*f*(e + f*x^2)^2) + (3*(b*e*(d*e + c*f) +
 a*f*(d*e + 5*c*f))*(x/(2*e*(e + f*x^2)) + ArcTan[(Sqrt[f]*x)/Sqrt[e]]/(2*
e^(3/2)*Sqrt[f])))/(4*e*f))/(6*e*f)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.8.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(5ac f^2+adef+bcef+bd e^2\right)x^5}{16e^3}+\frac{\left(5ac f^2+adef+bcef-bd e^2\right)x^3}{6e^2 f}+\frac{\left(11ac f^2-adef-bcef-bd e^2\right)x}{16e f^2}}{\left(f x^2+e\right)^3}+\frac{\left(5ac f^2+adef+bcef+bd e^2\right)\arctan\left(\right.}{16e^3 f^2\sqrt{ef}}$
risch	$\frac{\frac{\left(5ac f^2+adef+bcef+bd e^2\right)x^5}{16e^3}+\frac{\left(5ac f^2+adef+bcef-bd e^2\right)x^3}{6e^2 f}+\frac{\left(11ac f^2-adef-bcef-bd e^2\right)x}{16e f^2}}{\left(f x^2+e\right)^3}-\frac{5\ln\left(fx+\sqrt{-ef}\right)ac}{32\sqrt{-ef}\,e^3}-\frac{\ln\left(fx+\sqrt{-ef}\right)}{32\sqrt{-ef}\,f\,e}$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+a*d*e*f+b
*c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e*f-b*d*e^2)/e/f^2*
x)/(f*x^2+e)^3+1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3/f^2/(e*f)^(1/2
)*arctan(f*x/(e*f)^(1/2))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(155) = 310.

Time = 0.29 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$$
$$= \left[\frac{6\,(bde^3f^3+5\,acef^5+(bc+ad)e^2f^4)x^5-16\,(bde^4f^2-5\,ace^2f^4-(bc+ad)e^3f^3)x^3-3\,(bde^5+5\,ace^3f}{}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 16*(b*d*e
^4*f^2 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 - 3*(b*d*e^5 + 5*a*c*e^3
*f^2 + (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4
*f + 3*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*
f + 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sq
rt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)
*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3), 1/48
*(3*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 8*(b*d*e^4*f^2
 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 + 3*(b*d*e^5 + 5*a*c*e^3*f^2 +
 (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3
*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*
a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) -
3*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e
^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3)]
```

3.8.6 Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.83

$$
\begin{aligned}
&\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx \\
&= -\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32} \\
&\quad+\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32} \\
&\quad+\frac{x^5\cdot(15acf^4+3adef^3+3bcef^3+3bde^2f^2)+x^3\cdot(40acef^3+8ade^2f^2+8bce^2f^2-8bde^3f)+x(33ace^2}{48e^6f^2+144e^5f^3x^2+144e^4f^4x^4+48e^3f^5x^6}
\end{aligned}
$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
-sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(-e**
4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a
*d*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 +
(x**5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3
*(40*a*c*e*f**3 + 8*a*d*e**2*f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(3
3*a*c*e**2*f**2 - 3*a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2
 + 144*e**5*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

3.8.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(bde^2+bcef+adef+5\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\,\sqrt{ef}e^3f^2} + \frac{3\,bde^2f^2x^5+3\,bcef^3x^5+3\,adef^3x^5+15\,acf^4x^5-8\,bde^3fx^3+8\,bce^2f^2x^3+8\,ade^2f^2x^3+40\,acef^3x^3 -}{48\,(fx^2+e)^3e^3f^2}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(b*d*e^2 + b*c*e*f + a*d*e*f + 5*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt
(e*f)*e^3*f^2) + 1/48*(3*b*d*e^2*f^2*x^5 + 3*b*c*e*f^3*x^5 + 3*a*d*e*f^3*x
^5 + 15*a*c*f^4*x^5 - 8*b*d*e^3*f*x^3 + 8*b*c*e^2*f^2*x^3 + 8*a*d*e^2*f^2*
x^3 + 40*a*c*e*f^3*x^3 - 3*b*d*e^4*x - 3*b*c*e^3*f*x - 3*a*d*e^3*f*x + 33*
a*c*e^2*f^2*x)/((f*x^2 + e)^3*e^3*f^2)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$$
$$= \frac{\frac{x^5\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^3}-\frac{x\,(b\,d\,e^2-11\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e\,f^2}+\frac{x^3\,(5\,a\,c\,f^2-b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{6\,e^2\,f}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^{7/2}\,f^{5/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

output
```
((x^5*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e^3) - (x*(b*d*e^2 -
11*a*c*f^2 + a*d*e*f + b*c*e*f))/(16*e*f^2) + (x^3*(5*a*c*f^2 - b*d*e^2 +
a*d*e*f + b*c*e*f))/(6*e^2*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4)
 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(
16*e^(7/2)*f^(5/2))
```

3.9 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$

3.9.1 Optimal result

Integrand size = 26, antiderivative size = 226

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx &= ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3\\ &+\frac{1}{5}e\big(bce(2de+3cf)+a\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^5\\ &+\frac{1}{7}\big(af\big(3d^2e^2+6cdef+c^2f^2\big)\\ &\qquad+be\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^7\\ &+\frac{1}{9}f\big(adf(3de+2cf)+b\big(3d^2e^2+6cdef+c^2f^2\big)\big)x^9\\ &+\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}\end{aligned}$$

output
```
a*c^2*e^3*x+1/3*c*e^2*(3*a*c*f+2*a*d*e+b*c*e)*x^3+1/5*e*(b*c*e*(3*c*f+2*d*
e)+a*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5+1/7*(a*f*(c^2*f^2+6*c*d*e*f+3*d^2*
e^2)+b*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*f*(a*d*f*(2*c*f+3*d*e)+b*(
c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^9+1/11*d*f^2*(a*d*f+2*b*c*f+3*b*d*e)*x^11+
1/13*b*d^2*f^3*x^13
```

3.9.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3 + \frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5 + \frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right) + be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 + \frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9 + \frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
 + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 +
(d*f^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

3.9.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx$$

$$\Big\downarrow \; 396$$

$$\int \left(fx^8\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+x^6\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
 + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 +
(d*f^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

3.9.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05

method	result
default	$\frac{b\,d^2f^3x^{13}}{13}+\frac{\left(\left(a\,d^2+2bcd\right)f^3+3b\,d^2e\,f^2\right)x^{11}}{11}+\frac{\left(\left(2acd+b\,c^2\right)f^3+3\left(a\,d^2+2bcd\right)e\,f^2+3b\,d^2e^2f\right)x^9}{9}+\frac{\left(c^2a\,f^3+3\left(2acd+b\,c^2\right.\right.}{}$
norman	$\frac{b\,d^2f^3x^{13}}{13}+\left(\frac{1}{11}a\,d^2f^3+\frac{2}{11}bcd\,f^3+\frac{3}{11}b\,d^2e\,f^2\right)x^{11}+\left(\frac{2}{9}acd\,f^3+\frac{1}{3}a\,d^2e\,f^2+\frac{1}{9}b\,c^2f^3+\frac{2}{3}bcde\right.$
gosper	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$
risch	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$
parallelrisch	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11+1/9*((2*
a*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*x^9+1/7*(c^2*a*f^3
+3*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*f+b*d^2*e^3)*x^7+1/5*(3*c^2
*a*e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^2+2*b*c*d)*e^3)*x^5+1/3*(3*c^2*a*e^2
*f+(2*a*c*d+b*c^2)*e^3)*x^3+a*c^2*e^3*x
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int (a+bx^2)\,(c+dx^2)^2\,(e+fx^2)^3\,dx \\
&= \frac{1}{13}bd^2f^3x^{13} + \frac{1}{11}\left(3\,bd^2ef^2 + \left(2\,bcd + ad^2\right)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(3\,bd^2e^2f + 3\left(2\,bcd + ad^2\right)ef^2 + \left(bc^2 + 2\,acd\right)f^3\right)x^9 \\
&\quad + \frac{1}{7}\left(bd^2e^3 + ac^2f^3 + 3\left(2\,bcd + ad^2\right)e^2f + 3\left(bc^2 + 2\,acd\right)ef^2\right)x^7 \\
&\quad + ac^2e^3x + \frac{1}{5}\left(3\,ac^2ef^2 + \left(2\,bcd + ad^2\right)e^3 + 3\left(bc^2 + 2\,acd\right)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\,ac^2e^2f + \left(bc^2 + 2\,acd\right)e^3\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 +
1/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^
9 + 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*
a*c*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e
^3 + 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*
d)*e^3)*x^3
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx = ac^2e^3x + \frac{bd^2f^3x^{13}}{13} + x^{11}\left(\frac{ad^2f^3}{11} + \frac{2bcdf^3}{11} + \frac{3bd^2ef^2}{11}\right) + x^9 \cdot \left(\frac{2acdf^3}{9} + \frac{ad^2ef^2}{3} + \frac{bc^2f^3}{9} + \frac{2bcdef^2}{3} + \frac{bd^2e^2f}{3}\right) + x^7\left(\frac{ac^2f^3}{7} + \frac{6acdef^2}{7} + \frac{3ad^2e^2f}{7} + \frac{3bc^2ef^2}{7} + \frac{6bcde^2f}{7} + \frac{bd^2e^3}{7}\right) + x^5 \cdot \left(\frac{3ac^2ef^2}{5} + \frac{6acde^2f}{5} + \frac{ad^2e^3}{5} + \frac{3bc^2e^2f}{5} + \frac{2bcde^3}{5}\right) + x^3\left(ac^2e^2f + \frac{2acde^3}{3} + \frac{bc^2e^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

output
```
a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*b*c*d*f**
3/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d**2*e*f**2/3 + b*c*
*2*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3) + x**7*(a*c**2*f**3/7 + 6*
a*c*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7
+ b*d**2*e**3/7) + x**5*(3*a*c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**
3/5 + 3*b*c**2*e**2*f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*
e**3/3 + b*c**2*e**3/3)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx \\
&= \frac{1}{13}bd^2f^3x^{13} + \frac{1}{11}\left(3\,bd^2ef^2 + (2\,bcd+ad^2)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(3\,bd^2e^2f + 3\,(2\,bcd+ad^2)ef^2 + (bc^2+2\,acd)f^3\right)x^9 \\
&\quad + \frac{1}{7}\left(bd^2e^3 + ac^2f^3 + 3\,(2\,bcd+ad^2)e^2f + 3\,(bc^2+2\,acd)ef^2\right)x^7 \\
&\quad + ac^2e^3x + \frac{1}{5}\left(3\,ac^2ef^2 + (2\,bcd+ad^2)e^3 + 3\,(bc^2+2\,acd)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\,ac^2e^2f + (bc^2+2\,acd)e^3\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 +
1/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^
9 + 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*
a*c*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e
^3 + 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*
d)*e^3)*x^3
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\begin{aligned}
\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx &= \frac{1}{13}bd^2f^3x^{13} + \frac{3}{11}bd^2ef^2x^{11} + \frac{2}{11}bcdf^3x^{11} \\
&\quad + \frac{1}{11}ad^2f^3x^{11} + \frac{1}{3}bd^2e^2fx^9 + \frac{2}{3}bcdef^2x^9 \\
&\quad + \frac{1}{3}ad^2ef^2x^9 + \frac{1}{9}bc^2f^3x^9 + \frac{2}{9}acdf^3x^9 + \frac{1}{7}bd^2e^3x^7 \\
&\quad + \frac{6}{7}bcde^2fx^7 + \frac{3}{7}ad^2e^2fx^7 + \frac{3}{7}bc^2ef^2x^7 \\
&\quad + \frac{6}{7}acdef^2x^7 + \frac{1}{7}ac^2f^3x^7 + \frac{2}{5}bcde^3x^5 + \frac{1}{5}ad^2e^3x^5 \\
&\quad + \frac{3}{5}bc^2e^2fx^5 + \frac{6}{5}acde^2fx^5 + \frac{3}{5}ac^2ef^2x^5 \\
&\quad + \frac{1}{3}bc^2e^3x^3 + \frac{2}{3}acde^3x^3 + ac^2e^2fx^3 + ac^2e^3x
\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/13*b*d^2*f^3*x^13 + 3/11*b*d^2*e*f^2*x^11 + 2/11*b*c*d*f^3*x^11 + 1/11*a
*d^2*f^3*x^11 + 1/3*b*d^2*e^2*f*x^9 + 2/3*b*c*d*e*f^2*x^9 + 1/3*a*d^2*e*f^
2*x^9 + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 + 1/7*b*d^2*e^3*x^7 + 6/7*b*
c*d*e^2*f*x^7 + 3/7*a*d^2*e^2*f*x^7 + 3/7*b*c^2*e*f^2*x^7 + 6/7*a*c*d*e*f^
2*x^7 + 1/7*a*c^2*f^3*x^7 + 2/5*b*c*d*e^3*x^5 + 1/5*a*d^2*e^3*x^5 + 3/5*b*
c^2*e^2*f*x^5 + 6/5*a*c*d*e^2*f*x^5 + 3/5*a*c^2*e*f^2*x^5 + 1/3*b*c^2*e^3*
x^3 + 2/3*a*c*d*e^3*x^3 + a*c^2*e^2*f*x^3 + a*c^2*e^3*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int (a+bx^2)\,(c+dx^2)^2\,(e+fx^2)^3\,dx = x^5\left(\frac{3\,b\,c^2\,e^2\,f}{5}+\frac{3\,a\,c^2\,e\,f^2}{5}+\frac{2\,b\,c\,d\,e^3}{5}+\frac{6\,a\,c\,d\,e^2\,f}{5}+\frac{a\,d^2\,e^3}{5}\right)+x^9\left(\frac{b\,c^2\,f^3}{9}+\frac{2\,b\,c\,d\,e\,f^2}{3}+\frac{2\,a\,c\,d\,f^3}{9}+\frac{b\,d^2\,e^2\,f}{3}+\frac{a\,d^2\,e\,f^2}{3}\right)+x^7\left(\frac{3\,b\,c^2\,e\,f^2}{7}+\frac{a\,c^2\,f^3}{7}+\frac{6\,b\,c\,d\,e^2\,f}{7}+\frac{6\,a\,c\,d\,e\,f^2}{7}+\frac{b\,d^2\,e^3}{7}+\frac{3\,a\,d^2\,e^2\,f}{7}\right)+\frac{b\,d^2\,f^3\,x^{13}}{13}+\frac{c\,e^2\,x^3\,(3\,a\,c\,f+2\,a\,d\,e+b\,c\,e)}{3}+\frac{d\,f^2\,x^{11}\,(a\,d\,f+2\,b\,c\,f+3\,b\,d\,e)}{11}+a\,c^2\,e^3\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

output
```
x^5*((a*d^2*e^3)/5 + (2*b*c*d*e^3)/5 + (3*a*c^2*e*f^2)/5 + (3*b*c^2*e^2*f)
/5 + (6*a*c*d*e^2*f)/5) + x^9*((b*c^2*f^3)/9 + (2*a*c*d*f^3)/9 + (a*d^2*e*
f^2)/3 + (b*d^2*e^2*f)/3 + (2*b*c*d*e*f^2)/3) + x^7*((a*c^2*f^3)/7 + (b*d^
2*e^3)/7 + (3*a*d^2*e^2*f)/7 + (3*b*c^2*e*f^2)/7 + (6*a*c*d*e*f^2)/7 + (6*
b*c*d*e^2*f)/7) + (b*d^2*f^3*x^13)/13 + (c*e^2*x^3*(3*a*c*f + 2*a*d*e + b*
c*e))/3 + (d*f^2*x^11*(a*d*f + 2*b*c*f + 3*b*d*e))/11 + a*c^2*e^3*x
```

3.10 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx$

3.10.1 Optimal result

Integrand size = 26, antiderivative size = 158

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3 +\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 +\frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 +\frac{1}{9}df(adf+2b(de+cf))x^9+\frac{1}{11}bd^2f^2x^{11}$$

output
```
a*c^2*e^2*x+1/3*c*e*(b*c*e+2*a*(c*f+d*e))*x^3+1/5*(2*b*c*e*(c*f+d*e)+a*(c^
2*f^2+4*c*d*e*f+d^2*e^2))*x^5+1/7*(2*a*d*f*(c*f+d*e)+b*(c^2*f^2+4*c*d*e*f+
d^2*e^2))*x^7+1/9*d*f*(a*d*f+2*b*(c*f+d*e))*x^9+1/11*b*d^2*f^2*x^11
```

3.10.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3 +\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 +\frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 +\frac{1}{9}df(adf+2b(de+cf))x^9+\frac{1}{11}bd^2f^2x^{11}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f
) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*
(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x
^9)/9 + (b*d^2*f^2*x^11)/11
```

3.10.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \; 396$$

$$\int \left(x^6\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+x^4\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+dfx^8(adf+2b$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f
) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*
(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x
^9)/9 + (b*d^2*f^2*x^11)/11
```

3.10.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

method	result
default	$\frac{b\,d^{2} f^{2} x^{11}}{11}+\frac{\left(\left(a\,d^{2}+2 b c d \right) f^{2}+2 b\,d^{2} e f \right) x^{9}}{9}+\frac{\left(\left(2 a c d +b\,c^{2}\right) f^{2}+2 \left(a\,d^{2}+2 b c d \right) e f +b\,d^{2} e^{2}\right) x^{7}}{7}+\frac{(c^{2} a\,f^{2}+2 \left(2 a c d +b\,c^{2}\right) e f +(}{5}$
norman	$\frac{b\,d^{2} f^{2} x^{11}}{11}+\left(\frac{1}{9} a\,d^{2} f^{2}+\frac{2}{9} b c d\,f^{2}+\frac{2}{9} b\,d^{2} e f \right) x^{9}+\left(\frac{2}{7} a c d\,f^{2}+\frac{2}{7} a\,d^{2} e f +\frac{1}{7} b\,c^{2} f^{2}+\frac{4}{7} b c d e f +\frac{1}{7} b\right.$
gosper	$\frac{1}{11} b\,d^{2} f^{2} x^{11}+\frac{1}{9} x^{9} a\,d^{2} f^{2}+\frac{2}{9} x^{9} b c d\,f^{2}+\frac{2}{9} x^{9} b\,d^{2} e f +\frac{2}{7} x^{7} a c d\,f^{2}+\frac{2}{7} x^{7} a\,d^{2} e f +\frac{1}{7} x^{7} b\,c^{2} f^{2}+\frac{4}{7}$
risch	$\frac{1}{11} b\,d^{2} f^{2} x^{11}+\frac{1}{9} x^{9} a\,d^{2} f^{2}+\frac{2}{9} x^{9} b c d\,f^{2}+\frac{2}{9} x^{9} b\,d^{2} e f +\frac{2}{7} x^{7} a c d\,f^{2}+\frac{2}{7} x^{7} a\,d^{2} e f +\frac{1}{7} x^{7} b\,c^{2} f^{2}+\frac{4}{7}$
parallelrisch	$\frac{1}{11} b\,d^{2} f^{2} x^{11}+\frac{1}{9} x^{9} a\,d^{2} f^{2}+\frac{2}{9} x^{9} b c d\,f^{2}+\frac{2}{9} x^{9} b\,d^{2} e f +\frac{2}{7} x^{7} a c d\,f^{2}+\frac{2}{7} x^{7} a\,d^{2} e f +\frac{1}{7} x^{7} b\,c^{2} f^{2}+\frac{4}{7}$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*e*f)*x^9+1/7*((2*a*c*
d+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*e*f+b*d^2*e^2)*x^7+1/5*(c^2*a*f^2+2*(2*a*c*
d+b*c^2)*e*f+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*c^2*a*e*f+(2*a*c*d+b*c^2)*e^2
)*x^3+a*c^2*e^2*x
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{11}bd^2f^2x^{11} + \frac{1}{9}\left(2bd^2ef + (2bcd+ad^2)f^2\right)x^9 + \frac{1}{7}\left(bd^2e^2 + 2(2bcd+ad^2)ef + (bc^2+2acd)f^2\right)x^7 + ac^2e^2x + \frac{1}{5}\left(ac^2f^2 + (2bcd+ad^2)e^2 + 2(bc^2+2acd)ef\right)x^5 + \frac{1}{3}\left(2ac^2ef + (bc^2+2acd)e^2\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*
(b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*
e^2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*
x^5 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x + \frac{bd^2f^2x^{11}}{11} + x^9\left(\frac{ad^2f^2}{9} + \frac{2bcdf^2}{9} + \frac{2bd^2ef}{9}\right) + x^7\cdot\left(\frac{2acdf^2}{7} + \frac{2ad^2ef}{7} + \frac{bc^2f^2}{7} + \frac{4bcdef}{7} + \frac{bd^2e^2}{7}\right) + x^5\left(\frac{ac^2f^2}{5} + \frac{4acdef}{5} + \frac{ad^2e^2}{5} + \frac{2bc^2ef}{5} + \frac{2bcde^2}{5}\right) + x^3\cdot\left(\frac{2ac^2ef}{3} + \frac{2acde^2}{3} + \frac{bc^2e^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

output
```
a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*c*d*f**2/
9 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/
7 + 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 +
 a*d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 +
 2*a*c*d*e**2/3 + b*c**2*e**2/3)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{11}bd^2f^2x^{11} + \frac{1}{9}\left(2\,bd^2ef + \left(2\,bcd + ad^2\right)f^2\right)x^9 + \frac{1}{7}\left(bd^2e^2 + 2\left(2\,bcd + ad^2\right)ef + \left(bc^2 + 2\,acd\right)f^2\right)x^7 + ac^2e^2x + \frac{1}{5}\left(ac^2f^2 + \left(2\,bcd + ad^2\right)e^2 + 2\left(bc^2 + 2\,acd\right)ef\right)x^5 + \frac{1}{3}\left(2\,ac^2ef + \left(bc^2 + 2\,acd\right)e^2\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*
(b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*
e^2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*
x^5 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{11}bd^2f^2x^{11} + \frac{2}{9}bd^2efx^9 + \frac{2}{9}bcdf^2x^9 + \frac{1}{9}ad^2f^2x^9 + \frac{1}{7}bd^2e^2x^7 + \frac{4}{7}bcdefx^7 + \frac{2}{7}ad^2efx^7 + \frac{1}{7}bc^2f^2x^7 + \frac{2}{7}acdf^2x^7 + \frac{2}{5}bcde^2x^5 + \frac{1}{5}ad^2e^2x^5 + \frac{2}{5}bc^2efx^5 + \frac{4}{5}acdefx^5 + \frac{1}{5}ac^2f^2x^5 + \frac{1}{3}bc^2e^2x^3 + \frac{2}{3}acde^2x^3 + \frac{2}{3}ac^2efx^3 + ac^2e^2x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/11*b*d^2*f^2*x^11 + 2/9*b*d^2*e*f*x^9 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^
2*x^9 + 1/7*b*d^2*e^2*x^7 + 4/7*b*c*d*e*f*x^7 + 2/7*a*d^2*e*f*x^7 + 1/7*b*
c^2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 2/5*b*c*d*e^2*x^5 + 1/5*a*d^2*e^2*x^5 +
2/5*b*c^2*e*f*x^5 + 4/5*a*c*d*e*f*x^5 + 1/5*a*c^2*f^2*x^5 + 1/3*b*c^2*e^2*
x^3 + 2/3*a*c*d*e^2*x^3 + 2/3*a*c^2*e*f*x^3 + a*c^2*e^2*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\,(c+dx^2)^2\,(e+fx^2)^2\,dx = x^5\left(\frac{2\,b\,c^2\,e\,f}{5}+\frac{a\,c^2\,f^2}{5}+\frac{2\,b\,c\,d\,e^2}{5}+\frac{4\,a\,c\,d\,e\,f}{5}+\frac{a\,d^2\,e^2}{5}\right)+x^7\left(\frac{b\,c^2\,f^2}{7}+\frac{4\,b\,c\,d\,e\,f}{7}+\frac{2\,a\,c\,d\,f^2}{7}+\frac{b\,d^2\,e^2}{7}+\frac{2\,a\,d^2\,e\,f}{7}\right)+\frac{b\,d^2\,f^2\,x^{11}}{11}+a\,c^2\,e^2\,x+\frac{c\,e\,x^3\,(2\,a\,c\,f+2\,a\,d\,e+b\,c\,e)}{3}+\frac{d\,f\,x^9\,(a\,d\,f+2\,b\,c\,f+2\,b\,d\,e)}{9}$$

input
```
int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

output
```
x^5*((a*c^2*f^2)/5 + (a*d^2*e^2)/5 + (2*b*c*d*e^2)/5 + (2*b*c^2*e*f)/5 + (
4*a*c*d*e*f)/5) + x^7*((b*c^2*f^2)/7 + (b*d^2*e^2)/7 + (2*a*c*d*f^2)/7 + (
2*a*d^2*e*f)/7 + (4*b*c*d*e*f)/7) + (b*d^2*f^2*x^11)/11 + a*c^2*e^2*x + (c
*e*x^3*(2*a*c*f + 2*a*d*e + b*c*e))/3 + (d*f*x^9*(a*d*f + 2*b*c*f + 2*b*d*
e))/9
```

3.11 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx$

3.11.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}(bc(2de+cf)+ad(de+2cf))x^5+\frac{1}{7}d(bde+2bcf+adf)x^7+\frac{1}{9}bd^2fx^9$$

output
```
a*c^2*e*x+1/3*c*(a*c*f+2*a*d*e+b*c*e)*x^3+1/5*(b*c*(c*f+2*d*e)+a*d*(2*c*f+
d*e))*x^5+1/7*d*(a*d*f+2*b*c*f+b*d*e)*x^7+1/9*b*d^2*f*x^9
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}\left(2bcde+ad^2e+bc^2f+2acdf\right)x^5+\frac{1}{7}d(bde+2bcf+adf)x^7+\frac{1}{9}bd^2fx^9$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

output
```
a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((2*b*c*d*e + a*d^2*e +
b*c^2*f + 2*a*c*d*f)*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2
*f*x^9)/9
```

3.11.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(dx^6(adf+2bcf+bde)+x^4(ad(2cf+de)+bc(cf+2de))+cx^2(acf+2ade+bce)+ac^2e+bd^2fx^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

output
```
a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e + c*f) + a*
d*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f*x
^9)/9
```

3.11.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

method	result
norman	$\frac{b\,d^2 f\,x^9}{9}+\left(\frac{1}{7}a\,d^2 f+\frac{2}{7}bcfd+\frac{1}{7}b\,d^2 e\right)x^7+\left(\frac{2}{5}acdf+\frac{1}{5}ae\,d^2+\frac{1}{5}c^2 bf+\frac{2}{5}bcde\right)x^5+\left(\frac{1}{3}c^2 af+\right.$
default	$\frac{b\,d^2 f\,x^9}{9}+\frac{((a\,d^2+2bcd)f+b\,d^2 e)x^7}{7}+\frac{((2acd+b\,c^2)f+(a\,d^2+2bcd)e)x^5}{5}+\frac{(c^2 af+(2acd+b\,c^2)e)x^3}{3}+a\,c^2 ex$
gosper	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+$
risch	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+$
parallelrisch	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/9*b*d^2*f*x^9+(1/7*a*d^2*f+2/7*b*c*f*d+1/7*b*d^2*e)*x^7+(2/5*a*c*d*f+1/5
*a*e*d^2+1/5*c^2*b*f+2/5*b*c*d*e)*x^5+(1/3*c^2*a*f+2/3*a*c*d*e+1/3*b*c^2*e
)*x^3+a*c^2*e*x
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int (a+bx^2)\,(c+dx^2)^2\,(e+fx^2)\;dx=\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\,(bd^2e+(2\,bcd+ad^2)f)x^7+\frac{1}{5}\,((2\,bcd+ad^2)e+(bc^2+2\,acd)f)x^5+ac^2ex+\frac{1}{3}\,(ac^2f+(bc^2+2\,acd)e)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*((2*b*c*d
+ a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*(a*c^2*f + (b*c^2
+ 2*a*c*d)*e)*x^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = ac^2ex + \frac{bd^2fx^9}{9} + x^7\left(\frac{ad^2f}{7} + \frac{2bcdf}{7} + \frac{bd^2e}{7}\right) + x^5\cdot\left(\frac{2acdf}{5} + \frac{ad^2e}{5} + \frac{bc^2f}{5} + \frac{2bcde}{5}\right) + x^3\left(\frac{ac^2f}{3} + \frac{2acde}{3} + \frac{bc^2e}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

output
```
a*c**2*e*x + b*d**2*f*x**9/9 + x**7*(a*d**2*f/7 + 2*b*c*d*f/7 + b*d**2*e/7
) + x**5*(2*a*c*d*f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*d*e/5) + x**3*(a*c
**2*f/3 + 2*a*c*d*e/3 + b*c**2*e/3)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = \frac{1}{9}bd^2fx^9 + \frac{1}{7}\left(bd^2e + (2\,bcd + ad^2)f\right)x^7 + \frac{1}{5}\left((2\,bcd + ad^2)e + (bc^2 + 2\,acd)f\right)x^5 + ac^2ex + \frac{1}{3}\left(ac^2f + (bc^2 + 2\,acd)e\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*((2*b*c*d
+ a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*(a*c^2*f + (b*c^2
+ 2*a*c*d)*e)*x^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = \frac{1}{9}bd^2fx^9+\frac{1}{7}bd^2ex^7+\frac{2}{7}bcdfx^7+\frac{1}{7}ad^2fx^7 +\frac{2}{5}bcdex^5+\frac{1}{5}ad^2ex^5+\frac{1}{5}bc^2fx^5+\frac{2}{5}acdfx^5 +\frac{1}{3}bc^2ex^3+\frac{2}{3}acdex^3+\frac{1}{3}ac^2fx^3+ac^2ex$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="giac")
```

output
```
1/9*b*d^2*f*x^9 + 1/7*b*d^2*e*x^7 + 2/7*b*c*d*f*x^7 + 1/7*a*d^2*f*x^7 + 2/
5*b*c*d*e*x^5 + 1/5*a*d^2*e*x^5 + 1/5*b*c^2*f*x^5 + 2/5*a*c*d*f*x^5 + 1/3*
b*c^2*e*x^3 + 2/3*a*c*d*e*x^3 + 1/3*a*c^2*f*x^3 + a*c^2*e*x
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = x^5\left(\frac{a\,d^2\,e}{5}+\frac{b\,c^2\,f}{5}+\frac{2\,a\,c\,d\,f}{5}+\frac{2\,b\,c\,d\,e}{5}\right) +x^3\left(\frac{a\,c^2\,f}{3}+\frac{b\,c^2\,e}{3}+\frac{2\,a\,c\,d\,e}{3}\right) +x^7\left(\frac{a\,d^2\,f}{7}+\frac{b\,d^2\,e}{7}+\frac{2\,b\,c\,d\,f}{7}\right)+a\,c^2\,e\,x+\frac{b\,d^2\,f\,x^9}{9}$$

input
```
int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x)
```

output
```
x^5*((a*d^2*e)/5 + (b*c^2*f)/5 + (2*a*c*d*f)/5 + (2*b*c*d*e)/5) + x^3*((a*
c^2*f)/3 + (b*c^2*e)/3 + (2*a*c*d*e)/3) + x^7*((a*d^2*f)/7 + (b*d^2*e)/7 +
 (2*b*c*d*f)/7) + a*c^2*e*x + (b*d^2*f*x^9)/9
```

3.12 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$

3.12.1 Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3} - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} - \frac{(be-af)(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

output
```
-1/15*(5*a*d*f*(-5*c*f+3*d*e)-b*(8*c^2*f^2-25*c*d*e*f+15*d^2*e^2))*x/f^3-1
/15*(-5*a*d*f-4*b*c*f+5*b*d*e)*x*(d*x^2+c)/f^2+1/5*b*x*(d*x^2+c)^2/f-(-a*f
+b*e)*(-c*f+d*e)^2*arctan(x*f^(1/2)/e^(1/2))/f^(7/2)/e^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{(b(de-cf)^2+adf(-de+2cf))\,x}{f^3} + \frac{d(-bde+2bcf+adf)x^3}{3f^2} + \frac{bd^2x^5}{5f} - \frac{(be-af)(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e) + 2*b*c*
f + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*A
rcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

3.12.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {403, 25, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^2}{e+fx^2}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{\left(dx^2+c\right)\left((5bde-4bcf-5adf)x^2+c(be-5af)\right)}{fx^2+e}dx}{5f}+\frac{bx\left(c+dx^2\right)^2}{5f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left((5bde-4bcf-5adf)x^2+c(be-5af)\right)}{fx^2+e}dx}{5f}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{\int -\frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}+\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{3f}}{5f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{3f}-\frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}}{5f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{bx(c+dx^2)^2}{5f}-}{\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)(de-cf)^2\int\frac{1}{fx^2+e}dx}{f}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}}$$

$\downarrow$ 218

$$\frac{\frac{bx(c+dx^2)^2}{5f}-}{\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
(b*x*(c + d*x^2)^2)/(5*f) - (((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))
/(3*f) - (-(((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2
*f^2))*x)/f) - (15*(b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(3/2)))/(3*f))/(5*f)
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.12.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

method	result
default	$\frac{\frac{1}{5}b\,d^2x^5f^2+\frac{1}{3}a\,d^2f^2x^3+\frac{2}{3}bcd\,f^2x^3-\frac{1}{3}b\,d^2ef\,x^3+2acd\,f^2x-a\,d^2efx+b\,c^2f^2x-2bcdefx+b\,d^2e^2x}{f^3}+\frac{(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b}{}$
risch	$\frac{b\,d^2x^5}{5f}+\frac{a\,d^2x^3}{3f}+\frac{2bcd\,x^3}{3f}-\frac{b\,d^2e\,x^3}{3f^2}+\frac{2acdx}{f}-\frac{a\,d^2ex}{f^2}+\frac{b\,c^2x}{f}-\frac{2bcdex}{f^2}+\frac{b\,d^2e^2x}{f^3}-\frac{\ln(fx+\sqrt{-ef})c^2a}{2\sqrt{-ef}}+\frac{\ln(fx+}{f}$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+2/3*b*c*d*f^2*x^3-1/3*b*d^2*e*f
*x^3+2*a*c*d*f^2*x-a*d^2*e*f*x+b*c^2*f^2*x-2*b*c*d*e*f*x+b*d^2*e^2*x)+(a*c
^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^3/
(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$=\left[\frac{6\,bd^2ef^3x^5-10\,(bd^2e^2f^2-(2\,bcd+ad^2)ef^3)x^3+15\,(bd^2e^3-ac^2f^3-(2\,bcd+ad^2)e^2f+(bc^2+2\,acd)e}{30\,ef^4}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/30*(6*b*d^2*e*f^3*x^5 - 10*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^
3 + 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d
)*e*f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(b*
d^2*e^3*f - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4
), 1/15*(3*b*d^2*e*f^3*x^5 - 5*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x
^3 - 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*
d)*e*f^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(b*d^2*e^3*f - (2*b*c*d + a
*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4)]
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 0.52 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.44

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{bd^2x^5}{5f} + x^3\left(\frac{ad^2}{3f} + \frac{2bcd}{3f} - \frac{bd^2e}{3f^2}\right) + x\left(\frac{2acd}{f} - \frac{ad^2e}{f^2} + \frac{bc^2}{f} - \frac{2bcde}{f^2} + \frac{bd^2e^2}{f^3}\right) - \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
b*d**2*x**5/(5*f) + x**3*(a*d**2/(3*f) + 2*b*c*d/(3*f) - b*d**2*e/(3*f**2)
) + x*(2*a*c*d/f - a*d**2*e/f**2 + b*c**2/f - 2*b*c*d*e/f**2 + b*d**2*e**2
/f**3) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(-e*f**3*sqrt(-1/
(e*f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**
2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + sqrt(-1/
(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(a*f - b
*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*
e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= -\frac{(bd^2e^3-2\,bcde^2f-ad^2e^2f+bc^2ef^2+2\,acdef^2-ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^3}$$
$$+\frac{3\,bd^2f^4x^5-5\,bd^2ef^3x^3+10\,bcdf^4x^3+5\,ad^2f^4x^3+15\,bd^2e^2f^2x-30\,bcdef^3x-15\,ad^2ef^3x+15\,bc^2f^4x}{15\,f^5}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
-(b*d^2*e^3 - 2*b*c*d*e^2*f - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 -
a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^3) + 1/15*(3*b*d^2*f^4*x^5 -
 5*b*d^2*e*f^3*x^3 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 + 15*b*d^2*e^2*f^2
*x - 30*b*c*d*e*f^3*x - 15*a*d^2*e*f^3*x + 15*b*c^2*f^4*x + 30*a*c*d*f^4*x
)/f^5
```

3.12.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= x^3\left(\frac{a\,d^2+2\,b\,c\,d}{3\,f}-\frac{b\,d^2\,e}{3\,f^2}\right)+x\left(\frac{b\,c^2+2\,a\,d\,c}{f}-\frac{e\left(\frac{a\,d^2+2\,b\,c\,d}{f}-\frac{b\,d^2\,e}{f^2}\right)}{f}\right)+\frac{b\,d^2\,x^5}{5\,f}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}\right)(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,f^{7/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

output
```
x^3*((a*d^2 + 2*b*c*d)/(3*f) - (b*d^2*e)/(3*f^2)) + x*((b*c^2 + 2*a*c*d)/f
 - (e*((a*d^2 + 2*b*c*d)/f - (b*d^2*e)/f^2))/f) + (b*d^2*x^5)/(5*f) + (ata
n((f^(1/2)*x*(a*f - b*e)*(c*f - d*e)^2)/(e^(1/2)*(a*c^2*f^3 - b*d^2*e^3 +
a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f)))*(a*f - b*e)*(
c*f - d*e)^2)/(e^(1/2)*f^(7/2))
```

3.13 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx$

3.13.1 Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx = -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

output
```
-1/6*d*(b*e*(-13*c*f+15*d*e)-3*a*f*(-c*f+3*d*e))*x/e/f^3+1/6*d*(-3*a*f+5*b
*e)*x*(d*x^2+c)/e/f^2-1/2*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)+1/2*(-c*f
+d*e)*(b*e*(-c*f+5*d*e)-a*f*(c*f+3*d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)
/f^(7/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx = \frac{d(-2bde+2bcf+adf)x}{f^3} + \frac{bd^2x^3}{3f^2} - \frac{(be-af)(de-cf)^2x}{2ef^3(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
(d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((b*e - a*f
)*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f)
 - a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

3.13.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {401, 25, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}+\frac{dx(c+dx^2)(5be-3af)}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\int \frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3(de-cf)(be(5de-cf)-af(cf+3de))\int\frac{1}{fx^2+e}dx}{f}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
-1/2*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) + ((d*(5*b*e - 3*a*f)
*x*(c + d*x^2))/(3*f) - ((d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*
x)/f - (3*(d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(Sqrt
[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f)
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.13.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

method	result
default	$\frac{d\left(\frac{1}{3}bdf\,x^{3}+adfx+2bcfx-2bdex\right)}{f^{3}}+\frac{\frac{\left(c^{2}a\,f^{3}-2acde\,f^{2}+a\,d^{2}e^{2}f-b\,c^{2}e\,f^{2}+2bcd\,e^{2}f-b\,d^{2}e^{3}\right)x}{2e\left(f\,x^{2}+e\right)}+\frac{\left(c^{2}a\,f^{3}+2acde\,f^{2}-3a\,d^{2}e^{2}f+b\,c^{2}e\,f^{2}-6bc\right.}{2e\sqrt{ef}}}{f^{3}}$
risch	$\frac{d^{2}b\,x^{3}}{3f^{2}}+\frac{d^{2}ax}{f^{2}}+\frac{2dbcx}{f^{2}}-\frac{2d^{2}bex}{f^{3}}+\frac{(c^{2}a\,f^{3}-2acde\,f^{2}+a\,d^{2}e^{2}f-b\,c^{2}e\,f^{2}+2bcd\,e^{2}f-b\,d^{2}e^{3})x}{2e\,f^{3}(f\,x^{2}+e)}-\frac{\ln(fx+\sqrt{-ef})c^{2}a}{4\sqrt{-ef}\,e}-\ln(f$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
d/f^3*(1/3*b*d*f*x^3+a*d*f*x+2*b*c*f*x-2*b*d*e*x)+1/f^3*(1/2*(a*c^2*f^3-2*
a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/e*x/(f*x^2+e)
+1/2*(a*c^2*f^3+2*a*c*d*e*f^2-3*a*d^2*e^2*f+b*c^2*e*f^2-6*b*c*d*e^2*f+5*b*
d^2*e^3)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.37

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \left[\frac{4\,bd^2e^2f^3x^5 - 4\,(5\,bd^2e^3f^2 - 3\,(2\,bcd + ad^2)e^2f^3)x^3 - 3\,(5\,bd^2e^4 + ac^2ef^3 - 3\,(2\,bcd + ad^2)e^3f + (bc^2 + \right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b*d^2*e^2*f^3*x^5 - 4*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*
f^3)*x^3 - 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c
^2 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e
^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f
)*x - e)/(f*x^2 + e)) - 6*(5*b*d^2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^
2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4), 1/6*(2
*b*d^2*e^2*f^3*x^5 - 2*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f^3)*x^3
 + 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a
*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*f^2 +
 (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(5*b*d^
2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^
2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4)]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(151) = 302$.

Time = 1.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.95

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{bd^2x^3}{3f^2} + x\left(\frac{ad^2}{f^2} + \frac{2bcd}{f^2} - \frac{2bd^2e}{f^3}\right) + \frac{x(ac^2f^3 - 2acdef^2 + ad^2e^2f - bc^2ef^2 + 2bcde^2f - bd^2e^3)}{2e^2f^3 + 2ef^4x^2} - \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(-\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
b*d**2*x**3/(3*f**2) + x*(a*d**2/f**2 + 2*b*c*d/f**2 - 2*b*d**2*e/f**3) +
x*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*
e**2*f - b*d**2*e**3)/(2*e**2*f**3 + 2*e*f**4*x**2) - sqrt(-1/(e**3*f**7))
*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(-e**2*f**3*
sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e
**2)/(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b
*c*d*e**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*
c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f**3*sqrt(-1/(e**3*f**
7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/(a*c**2*f**3
 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b
*d**2*e**3) + x)/4
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{(5\,bd^2e^3-6\,bcde^2f-3\,ad^2e^2f+bc^2ef^2+2\,acdef^2+ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\sqrt{ef}ef^3} - \frac{bd^2e^3x-2\,bcde^2fx-ad^2e^2fx+bc^2ef^2x+2\,acdef^2x-ac^2f^3x}{2\,(fx^2+e)ef^3} + \frac{bd^2f^4x^3-6\,bd^2ef^3x+6\,bcdf^4x+3\,ad^2f^4x}{3\,f^6}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(5*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e
*f^2 + a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e*f^3) - 1/2*(b*d^2*e^3
*x - 2*b*c*d*e^2*f*x - a*d^2*e^2*f*x + b*c^2*e*f^2*x + 2*a*c*d*e*f^2*x - a
*c^2*f^3*x)/((f*x^2 + e)*e*f^3) + 1/3*(b*d^2*f^4*x^3 - 6*b*d^2*e*f^3*x + 6
*b*c*d*f^4*x + 3*a*d^2*f^4*x)/f^6
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = x\left(\frac{a\,d^2+2\,b\,c\,d}{f^2}-\frac{2\,b\,d^2\,e}{f^3}\right)+\frac{b\,d^2\,x^3}{3\,f^2} + \frac{x\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}{2\,e\,(f^4\,x^2+e\,f^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^2\,e\,f^2+a\,c^2\,f^3-6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3-3\,a\,d^2\,e^2\,f)}\right)\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e}{2\,e^{3/2}\,f^{7/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

output
```
x*((a*d^2 + 2*b*c*d)/f^2 - (2*b*d^2*e)/f^3) + (b*d^2*x^3)/(3*f^2) + (x*(a*
c^2*f^3 - b*d^2*e^3 + a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*
e^2*f))/(2*e*(e*f^3 + f^4*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)*(a*c*f^2 -
5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^2*f^3 + 5*b*d^2*e^3 - 3*a*
d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 - 6*b*c*d*e^2*f)))*(c*f - d*e)*(a*
c*f^2 - 5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(7/2))
```

3.13. $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$

3.14 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx$

3.14.1 Optimal result

Integrand size = 26, antiderivative size = 207

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx &= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} \\ &- \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} \\ &- \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}\end{aligned}$$

output
```
1/8*d*(b*e*(-c*f+15*d*e)-3*a*f*(c*f+d*e))*x/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^
2+c)^2/e/f/(f*x^2+e)^2-1/8*(b*e*(-c*f+5*d*e)-a*f*(3*c*f+d*e))*x*(d*x^2+c)/
e^2/f^2/(f*x^2+e)-1/8*(b*e*(-c^2*f^2-6*c*d*e*f+15*d^2*e^2)-a*f*(3*c^2*f^2+
2*c*d*e*f+3*d^2*e^2))*arctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(7/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{bd^2x}{f^3} - \frac{(be-af)(de-cf)^2x}{4ef^3(e+fx^2)^2} + \frac{(de-cf)(be(9de-cf)-af(5de+3cf))x}{8e^2f^3(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2)^2) + ((
d*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^3*(e + f*
x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*
e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

3.14.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {401, 25, 401, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(dx^2+c)(d(5be-af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx^2+c)(d(5be-af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \; 299$$

$$\frac{-\frac{\frac{\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{f}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
-1/4*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(5*d*
e - c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b*
e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/f) + ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)
/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f)
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.14. $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.14.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.11

method	result
default	$\frac{b\,d^2 x}{f^3}+\frac{\frac{\frac{f\left(3c^2 a\,f^3+2acde\,f^2-5a\,d^2e^2f+b\,c^2e\,f^2-10bcd\,e^2f+9b\,d^2e^3\right)x^3}{8e^2}+\frac{\left(5c^2a\,f^3-2acde\,f^2-3a\,d^2e^2f-b\,c^2e\,f^2-6bcd\,e^2f+7b\,d^2e^3\right)x}{8e}}{\left(f\,x^2+e\right)^2}+\frac{(3}{}}{f^3}$
risch	$\frac{b\,d^2 x}{f^3}+\frac{\frac{f\left(3c^2 a\,f^3+2acde\,f^2-5a\,d^2e^2f+b\,c^2e\,f^2-10bcd\,e^2f+9b\,d^2e^3\right)x^3}{8e^2}+\frac{\left(5c^2a\,f^3-2acde\,f^2-3a\,d^2e^2f-b\,c^2e\,f^2-6bcd\,e^2f+7b\,d^2e^3\right)x}{8e}}{f^3\left(f\,x^2+e\right)^2}-$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
b*d^2/f^3*x+1/f^3*((1/8*f*(3*a*c^2*f^3+2*a*c*d*e*f^2-5*a*d^2*e^2*f+b*c^2*e
*f^2-10*b*c*d*e^2*f+9*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-2*a*c*d*e*f^2-3*
a*d^2*e^2*f-b*c^2*e*f^2-6*b*c*d*e^2*f+7*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3
*a*c^2*f^3+2*a*c*d*e*f^2+3*a*d^2*e^2*f+b*c^2*e*f^2+6*b*c*d*e^2*f-15*b*d^2*
e^3)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$

$$= \left[\frac{16\,bd^2e^3f^3x^5 + 2\,(25\,bd^2e^4f^2 + 3\,ac^2ef^5 - 5\,(2\,bcd + ad^2)e^3f^3 + (bc^2 + 2\,acd)e^2f^4)x^3 + (15\,bd^2e^5 - 3\,ac\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*b*d^2*e^3*f^3*x^5 + 2*(25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 - 5*(2*b
*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 + (15*b*d^2*e^5 - 3
*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2 + (
15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*
a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2
)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt
(-e*f)*x - e)/(f*x^2 + e)) + 2*(15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*
c*d + a*d^2)*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*
f^5*x^2 + e^5*f^4), 1/8*(8*b*d^2*e^3*f^3*x^5 + (25*b*d^2*e^4*f^2 + 3*a*c^2
*e*f^5 - 5*(2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 - (1
5*b*d^2*e^5 - 3*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c
*d)*e^3*f^2 + (15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^
3 - (b*c^2 + 2*a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(
2*b*c*d + a*d^2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(e*f)*arcta
n(sqrt(e*f)*x/e) + (15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a*d^2)
*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2 + e^
5*f^4)]
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(199) = 398$.

Time = 6.09 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{x^3\cdot(3ac^2f^4+2acdef^3-5ad^2e^2f^2+bc^2ef^3-10bcde^2f^2+9bd^2e^3f)+x(5ac^2ef^3-2acde^2f^2-3ad^2e}{8e^4f^3+16e^3f^4x^2+8e^2f^5x^4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a
*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(-e**3*
f**3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 +
2*a*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d
**2*e**3)*log(e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**
4 + 2*a*c*d*e*f**3 - 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f*
*2 + 9*b*d**2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*
e**3*f - b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**4*f**3
+ 16*e**3*f**4*x**2 + 8*e**2*f**5*x**4)
```

3.14.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{bd^2x}{f^3} - \frac{(15\,bd^2e^3 - 6\,bcde^2f - 3\,ad^2e^2f - bc^2ef^2 - 2\,acdef^2 - 3\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\sqrt{ef}e^2f^3}$$
$$+ \frac{9\,bd^2e^3fx^3 - 10\,bcde^2f^2x^3 - 5\,ad^2e^2f^2x^3 + bc^2ef^3x^3 + 2\,acdef^3x^3 + 3\,ac^2f^4x^3 + 7\,bd^2e^4x - 6\,bcde^3fx}{8\,(fx^2+e)^2e^2f^3}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b*d^2*x/f^3 - 1/8*(15*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f - b*c^2*e*
f^2 - 2*a*c*d*e*f^2 - 3*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^
3) + 1/8*(9*b*d^2*e^3*f*x^3 - 10*b*c*d*e^2*f^2*x^3 - 5*a*d^2*e^2*f^2*x^3 +
 b*c^2*e*f^3*x^3 + 2*a*c*d*e*f^3*x^3 + 3*a*c^2*f^4*x^3 + 7*b*d^2*e^4*x - 6
*b*c*d*e^3*f*x - 3*a*d^2*e^3*f*x - b*c^2*e^2*f^2*x - 2*a*c*d*e^2*f^2*x + 5
*a*c^2*e*f^3*x)/((f*x^2 + e)^2*e^2*f^3)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(b\,c^2\,e\,f^2 + 3\,a\,c^2\,f^3 + 6\,b\,c\,d\,e^2\,f + 2\,a\,c\,d\,e\,f^2 - 15\,b\,d^2\,e^3 + 3\,a\,d^2\,e^2\,f\right)}{8\,e^{5/2}\,f^{7/2}}$$
$$- \frac{\frac{x\,(b\,c^2\,e\,f^2 - 5\,a\,c^2\,f^3 + 6\,b\,c\,d\,e^2\,f + 2\,a\,c\,d\,e\,f^2 - 7\,b\,d^2\,e^3 + 3\,a\,d^2\,e^2\,f)}{8\,e} - \frac{x^3\,(b\,c^2\,e\,f^3 + 3\,a\,c^2\,f^4 - 10\,b\,c\,d\,e^2\,f^2 + 2\,a\,c\,d\,e\,f^3 + 9\,b\,d^2\,e^3\,f - 5\,a\,d^2\,e}{8\,e^2}}{e^2\,f^3 + 2\,e\,f^4\,x^2 + f^5\,x^4}$$
$$+ \frac{b\,d^2\,x}{f^3}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

output
```
(atan((f^(1/2)*x)/e^(1/2))*(3*a*c^2*f^3 - 15*b*d^2*e^3 + 3*a*d^2*e^2*f + b
*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c*d*e^2*f))/(8*e^(5/2)*f^(7/2)) - ((x*(3*
a*d^2*e^2*f - 7*b*d^2*e^3 - 5*a*c^2*f^3 + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*
b*c*d*e^2*f))/(8*e) - (x^3*(3*a*c^2*f^4 - 5*a*d^2*e^2*f^2 + b*c^2*e*f^3 +
9*b*d^2*e^3*f - 10*b*c*d*e^2*f^2 + 2*a*c*d*e*f^3))/(8*e^2))/(e^2*f^3 + f^5
*x^4 + 2*e*f^4*x^2) + (b*d^2*x)/f^3
```

3.14. $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$

3.15 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$

3.15.1 Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3d^2e^2+4cdef-15c^2f^2)+be(15d^2e^2-4cdef-3c^2f^2))\,x}{48e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

output
```
-1/6*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)^3-1/24*(d*e*(a*f+5*b*e)-c*f*(5
*a*f+b*e))*x*(d*x^2+c)/e^2/f^2/(f*x^2+e)^2-1/48*(a*f*(-15*c^2*f^2+4*c*d*e*
f+3*d^2*e^2)+b*e*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2))*x/e^3/f^3/(f*x^2+e)+1/
16*(b*e*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+a*f*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*a
rctan(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(7/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)(de-cf)^2x}{6ef^3(e+fx^2)^3}+\frac{(de-cf)(be(13de-cf)-af(7de+5cf))x}{24e^2f^3(e+fx^2)^2}+\frac{(be(-11d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^3(e+fx^2)}+\frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
-1/6*((b*e - a*f)*(d*e - c*f)^2*x)/(e*f^3*(e + f*x^2)^3) + ((d*e - c*f)*(b
*e*(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e + f*x^2)^2) + (
(b*e*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^
2*f^2))*x)/(16*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(16*e^(7/2)*f^(7/2))
```

3.15.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {401, 25, 401, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

$$\Big\downarrow \; 401$$

$$-\frac{\int -\frac{(dx^2+c)(d(5be+af)x^2+c(be+5af))}{(fx^2+e)^3}dx}{6ef}-\frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx^2+c)\left(d(5be+af)x^2+c(be+5af)\right)}{(fx^2+e)^3}dx}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 401

$$\frac{-\frac{\int -\frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 298

$$\frac{\frac{\frac{3\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{2ef}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)(de(af+5b}{4ef(e+}}{6ef}$$
$$\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{2e^{3/2}f^{3/2}}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)(de(af-}{4ef(e}}{6ef}$$
$$\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

3.15. $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$

output
```
-1/6*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^3) + (-1/4*((d*e*(5*b*
e + a*f) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)^2) + (-1/2*(
(a*f*(3*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f -
3*c^2*f^2))*x)/(e*f*(e + f*x^2)) + (3*(b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^
2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(
2*e^(3/2)*f^(3/2)))/(4*e*f))/(6*e*f)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.15.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.20

method	result
default	$\frac{\frac{\left(5c^2 a f^3+2acde f^2+a d^2 e^2 f+b c^2 e f^2+2bcd e^2 f-11b d^2 e^3\right) x^5}{16e^3 f}+\frac{\left(5c^2 a f^3+2acde f^2-a d^2 e^2 f+b c^2 e f^2-2bcd e^2 f-5b d^2 e^3\right) x^3}{6e^2 f^2}+\frac{\left(11c^2 a f^3-2acd\right.}{}}{\left(f x^2+e\right)^3}$
risch	$\frac{\frac{\left(5c^2 a f^3+2acde f^2+a d^2 e^2 f+b c^2 e f^2+2bcd e^2 f-11b d^2 e^3\right) x^5}{16e^3 f}+\frac{\left(5c^2 a f^3+2acde f^2-a d^2 e^2 f+b c^2 e f^2-2bcd e^2 f-5b d^2 e^3\right) x^3}{6e^2 f^2}+\frac{\left(11c^2 a f^3-2acd\right.}{}}{\left(f x^2+e\right)^3}$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f-11*
b*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^2*e^2*f+b*c^2*e*f^
2-2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(11*a*c^2*f^3-2*a*c*d*e*f^2-
a*d^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+1/
16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f+5*b*d^
2*e^3)/e^3/f^3/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(224) = 448$.

Time = 0.27 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.27

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

$$= \left[-\frac{6\,(11\,bd^2e^4f^3 - 5\,ac^2ef^6 - (2\,bcd + ad^2)e^3f^4 - (bc^2 + 2\,acd)e^2f^5)x^5 + 16\,(5\,bd^2e^5f^2 - 5\,ac^2e^2f^5 + (2\,b\right.$$

$$-\frac{3\,(11\,bd^2e^4f^3 - 5\,ac^2ef^6 - (2\,bcd + ad^2)e^3f^4 - (bc^2 + 2\,acd)e^2f^5)x^5 + 8\,(5\,bd^2e^5f^2 - 5\,ac^2e^2f^5 + (2\,bc}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[-1/96*(6*(11*b*d^2*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 -
(b*c^2 + 2*a*c*d)*e^2*f^5)*x^5 + 16*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (
2*b*c*d + a*d^2)*e^4*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 + 3*(5*b*d^2*e^6
 + 5*a*c^2*e^3*f^3 + (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 +
 (5*b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a
*c*d)*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*
e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*
f^4 + (2*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(-e*
f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6*(5*b*d^2*e^6*f - 11*a
*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*f^3)*x)/(
e^4*f^7*x^6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4), -1/48*(3*(11*b*d^2
*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (b*c^2 + 2*a*c*d)*e
^2*f^5)*x^5 + 8*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*b*c*d + a*d^2)*e^4
*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 - 3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 +
 (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e^3*f^3 +
5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)*e*f^5)*x^6 + 3
*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2
*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (2*b*c*d + a*d
^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x
/e) + 3*(5*b*d^2*e^6*f - 11*a*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 +...
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

output
```
Timed out
```

3.15.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

$$= \frac{(5\,bd^2e^3+2\,bcde^2f+ad^2e^2f+bc^2ef^2+2\,acdef^2+5\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\,\sqrt{ef}e^3f^3} - \frac{33\,bd^2e^3f^2x^5-6\,bcde^2f^3x^5-3\,ad^2e^2f^3x^5-3\,bc^2ef^4x^5-6\,acdef^4x^5-15\,ac^2f^5x^5+40\,bd^2e^4fx^3+16}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(5*b*d^2*e^3 + 2*b*c*d*e^2*f + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*
f^2 + 5*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^3) - 1/48*(33*b*
d^2*e^3*f^2*x^5 - 6*b*c*d*e^2*f^3*x^5 - 3*a*d^2*e^2*f^3*x^5 - 3*b*c^2*e*f^
4*x^5 - 6*a*c*d*e*f^4*x^5 - 15*a*c^2*f^5*x^5 + 40*b*d^2*e^4*f*x^3 + 16*b*c
*d*e^3*f^2*x^3 + 8*a*d^2*e^3*f^2*x^3 - 8*b*c^2*e^2*f^3*x^3 - 16*a*c*d*e^2*
f^3*x^3 - 40*a*c^2*e*f^4*x^3 + 15*b*d^2*e^5*x + 6*b*c*d*e^4*f*x + 3*a*d^2*
e^4*f*x + 3*b*c^2*e^3*f^2*x + 6*a*c*d*e^3*f^2*x - 33*a*c^2*e^2*f^3*x)/((f*
x^2 + e)^3*e^3*f^3)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3-2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-5\,b\,d^2\,e^3-a\,d^2\,e^2\,f)}{6\,e^2\,f^2}-\frac{x\,(b\,c^2\,e\,f^2-11\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e\,f^3}+}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e^{7/2}\,f^{7/2}}$$

input

```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

output

```
((x^3*(5*a*c^2*f^3 - 5*b*d^2*e^3 - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f
^2 - 2*b*c*d*e^2*f))/(6*e^2*f^2) - (x*(5*b*d^2*e^3 - 11*a*c^2*f^3 + a*d^2*
e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e*f^3) + (x^5*(5
*a*c^2*f^3 - 11*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*
b*c*d*e^2*f))/(16*e^3*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) + (a
tan((f^(1/2)*x)/e^(1/2))*(5*a*c^2*f^3 + 5*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*
e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e^(7/2)*f^(7/2))
```

3.16 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$

3.16.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = {} & ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3\\ & +\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5\\ & +\frac{1}{7}\left(3bce\left(d^2e^2+3cdef+c^2f^2\right)\right.\\ & \left.+a\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7\\ & +\frac{1}{9}\left(3adf\left(d^2e^2+3cdef+c^2f^2\right)\right.\\ & \left.+b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9\\ & +\frac{3}{11}df\left(adf(de+cf)+b\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{11}\\ & +\frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13}+\frac{1}{15}bd^3f^3x^{15}\end{aligned}$$

output
```
a*c^3*e^3*x+1/3*c^2*e^2*(b*c*e+3*a*(c*f+d*e))*x^3+3/5*c*e*(b*c*e*(c*f+d*e)
+a*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(3*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^
2)+a*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/9*(3*a*d*f*(c^2*
f^2+3*c*d*e*f+d^2*e^2)+b*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^
9+3/11*d*f*(a*d*f*(c*f+d*e)+b*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^11+1/13*d^2*f
^2*(a*d*f+3*b*(c*f+d*e))*x^13+1/15*b*d^3*f^3*x^15
```

3.16.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3 +\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5 +\frac{1}{7}\left(3bce\left(d^2e^2+3cdef+c^2f^2\right) +a\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7 +\frac{1}{9}\left(3adf\left(d^2e^2+3cdef+c^2f^2\right) +b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9 +\frac{3}{11}df\left(adf(de+cf)+b\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{11} +\frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13}+\frac{1}{15}bd^3f^3x^{15}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d
*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2
 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3
*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f
) + b*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d
*e + c*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

3.16.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$$

$$\Big\downarrow \ 396$$

$$\int \left(3dfx^{10}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+3cex^4\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+c^2e^2x^2(3$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d
*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2
 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3
*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f
) + b*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d
*e + c*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

3.16.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09

method	result
default	$\frac{b d^3 f^3 x^{15}}{15}+\frac{\left(\left(a d^3+3 b c d^2\right) f^3+3 b d^3 e f^2\right) x^{13}}{13}+\frac{\left(\left(3 a c d^2+3 b c^2 d\right) f^3+3\left(a d^3+3 b c d^2\right) e f^2+3 b d^3 e^2 f\right) x^{11}}{11}+$ $((3a c^2 d+c^3 b$
norman	$a c^3 e^3 x+\left(c^3 a e^2 f+a c^2 d e^3+\frac{1}{3} b c^3 e^3\right) x^3+\left(\frac{3}{5} c^3 a e f^2+\frac{9}{5} a c^2 d e^2 f+\frac{3}{5} a c d^2 e^3+\frac{3}{5} b c^3 e^2 f+\right.$
gosper	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9$
risch	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9$
parallelrisch	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9$

input
```
int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^13+1/11*(
(3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*d^3*e^2*f)*x^11+1/
9*((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d)*e*f^2+3*(a*d^3+3*b*c*d^2)
*e^2*f+b*d^3*e^3)*x^9+1/7*(c^3*a*f^3+3*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^
2+3*b*c^2*d)*e^2*f+(a*d^3+3*b*c*d^2)*e^3)*x^7+1/5*(3*c^3*a*e*f^2+3*(3*a*c^
2*d+b*c^3)*e^2*f+(3*a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*c^3*a*e^2*f+(3*a*c^
2*d+b*c^3)*e^3)*x^3+a*c^3*e^3*x
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$
\begin{aligned}
&\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx \\
&= \frac{1}{15}\,bd^3f^3x^{15}+\frac{1}{13}\left(3\,bd^3ef^2+\left(3\,bcd^2+ad^3\right)f^3\right)x^{13} \\
&\quad+\frac{3}{11}\left(bd^3e^2f+\left(3\,bcd^2+ad^3\right)ef^2+\left(bc^2d+acd^2\right)f^3\right)x^{11} \\
&\quad+\frac{1}{9}\left(bd^3e^3+3\left(3\,bcd^2+ad^3\right)e^2f+9\left(bc^2d+acd^2\right)ef^2+\left(bc^3+3\,ac^2d\right)f^3\right)x^9+ac^3e^3x \\
&\quad+\frac{1}{7}\left(ac^3f^3+\left(3\,bcd^2+ad^3\right)e^3+9\left(bc^2d+acd^2\right)e^2f+3\left(bc^3+3\,ac^2d\right)ef^2\right)x^7 \\
&\quad+\frac{3}{5}\left(ac^3ef^2+\left(bc^2d+acd^2\right)e^3+\left(bc^3+3\,ac^2d\right)e^2f\right)x^5 \\
&\quad+\frac{1}{3}\left(3\,ac^3e^2f+\left(bc^3+3\,ac^2d\right)e^3\right)x^3
\end{aligned}
$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13
+ 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)
*x^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^
2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (
3*b*c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*
d)*e*f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*
c^2*d)*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.36

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = ac^3e^3x + \frac{bd^3f^3x^{15}}{15} + x^{13}\left(\frac{ad^3f^3}{13} + \frac{3bcd^2f^3}{13} + \frac{3bd^3ef^2}{13}\right) + x^{11}\cdot\left(\frac{3acd^2f^3}{11} + \frac{3ad^3ef^2}{11} + \frac{3bc^2df^3}{11} + \frac{9bcd^2ef^2}{11} + \frac{3bd^3e^2f}{11}\right) + x^9\left(\frac{ac^2df^3}{3} + acd^2ef^2 + \frac{ad^3e^2f}{3} + \frac{bc^3f^3}{9} + bc^2def^2 + bcd^2e^2f + \frac{bd^3e^3}{9}\right) + x^7\left(\frac{ac^3f^3}{7} + \frac{9ac^2def^2}{7} + \frac{9acd^2e^2f}{7} + \frac{ad^3e^3}{7} + \frac{3bc^3ef^2}{7} + \frac{9bc^2de^2f}{7} + \frac{3bcd^2e^3}{7}\right) + x^5\cdot\left(\frac{3ac^3ef^2}{5} + \frac{9ac^2de^2f}{5} + \frac{3acd^2e^3}{5} + \frac{3bc^3e^2f}{5} + \frac{3bc^2de^3}{5}\right) + x^3\left(ac^3e^2f + ac^2de^3 + \frac{bc^3e^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

output
```
a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*b*c*d**2*
f**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/11 + 3*a*d**3*e*f**
2/11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/11 + 3*b*d**3*e**2*f/11) + x
**9*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 +
 b*c**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7
+ 9*a*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f
**2/7 + 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5
 + 9*a*c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*b*c**2*
d*e**3/5) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e**3/3)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int (a+bx^2)\,(c+dx^2)^3\,(e+fx^2)^3\,dx \\
&= \frac{1}{15}bd^3f^3x^{15} + \frac{1}{13}\left(3\,bd^3ef^2 + \left(3\,bcd^2 + ad^3\right)f^3\right)x^{13} \\
&\quad + \frac{3}{11}\left(bd^3e^2f + \left(3\,bcd^2 + ad^3\right)ef^2 + \left(bc^2d + acd^2\right)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(bd^3e^3 + 3\left(3\,bcd^2 + ad^3\right)e^2f + 9\left(bc^2d + acd^2\right)ef^2 + \left(bc^3 + 3\,ac^2d\right)f^3\right)x^9 + ac^3e^3x \\
&\quad + \frac{1}{7}\left(ac^3f^3 + \left(3\,bcd^2 + ad^3\right)e^3 + 9\left(bc^2d + acd^2\right)e^2f + 3\left(bc^3 + 3\,ac^2d\right)ef^2\right)x^7 \\
&\quad + \frac{3}{5}\left(ac^3ef^2 + \left(bc^2d + acd^2\right)e^3 + \left(bc^3 + 3\,ac^2d\right)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\,ac^3e^2f + \left(bc^3 + 3\,ac^2d\right)e^3\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13
+ 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)
*x^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^
2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (
3*b*c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*
d)*e*f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*
c^2*d)*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.32

$$\int (a+bx^2)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx = \frac{1}{15}bd^3f^3x^{15}+\frac{3}{13}bd^3ef^2x^{13}+\frac{3}{13}bcd^2f^3x^{13}+\frac{1}{13}ad^3f^3x^{13}+\frac{3}{11}bd^3e^2fx^{11}+\frac{9}{11}bcd^2ef^2x^{11}+\frac{3}{11}ad^3ef^2x^{11}+\frac{3}{11}bc^2df^3x^{11}+\frac{3}{11}acd^2f^3x^{11}+\frac{1}{9}bd^3e^3x^9+bcd^2e^2fx^9+\frac{1}{3}ad^3e^2fx^9+bc^2def^2x^9+acd^2ef^2x^9+\frac{1}{9}bc^3f^3x^9+\frac{1}{3}ac^2df^3x^9+\frac{3}{7}bcd^2e^3x^7+\frac{1}{7}ad^3e^3x^7+\frac{9}{7}bc^2de^2fx^7+\frac{9}{7}acd^2e^2fx^7+\frac{3}{7}bc^3ef^2x^7+\frac{9}{7}ac^2def^2x^7+\frac{1}{7}ac^3f^3x^7+\frac{3}{5}bc^2de^3x^5+\frac{3}{5}acd^2e^3x^5+\frac{3}{5}bc^3e^2fx^5+\frac{9}{5}ac^2de^2fx^5+\frac{3}{5}ac^3ef^2x^5+\frac{1}{3}bc^3e^3x^3+ac^2de^3x^3+ac^3e^2fx^3+ac^3e^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/15*b*d^3*f^3*x^15 + 3/13*b*d^3*e*f^2*x^13 + 3/13*b*c*d^2*f^3*x^13 + 1/13
*a*d^3*f^3*x^13 + 3/11*b*d^3*e^2*f*x^11 + 9/11*b*c*d^2*e*f^2*x^11 + 3/11*a
*d^3*e*f^2*x^11 + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f^3*x^11 + 1/9*b*d^
3*e^3*x^9 + b*c*d^2*e^2*f*x^9 + 1/3*a*d^3*e^2*f*x^9 + b*c^2*d*e*f^2*x^9 +
a*c*d^2*e*f^2*x^9 + 1/9*b*c^3*f^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + 3/7*b*c*d^2*
e^3*x^7 + 1/7*a*d^3*e^3*x^7 + 9/7*b*c^2*d*e^2*f*x^7 + 9/7*a*c*d^2*e^2*f*x^
7 + 3/7*b*c^3*e*f^2*x^7 + 9/7*a*c^2*d*e*f^2*x^7 + 1/7*a*c^3*f^3*x^7 + 3/5*
b*c^2*d*e^3*x^5 + 3/5*a*c*d^2*e^3*x^5 + 3/5*b*c^3*e^2*f*x^5 + 9/5*a*c^2*d*
e^2*f*x^5 + 3/5*a*c^3*e*f^2*x^5 + 1/3*b*c^3*e^3*x^3 + a*c^2*d*e^3*x^3 + a*
c^3*e^2*f*x^3 + a*c^3*e^3*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.08

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = x^5\left(\frac{3\,b\,c^3\,e^2\,f}{5}+\frac{3\,a\,c^3\,e\,f^2}{5}+\frac{3\,b\,c^2\,d\,e^3}{5}+\frac{9\,a\,c^2\,d\,e^2\,f}{5}+\frac{3\,a\,c\,d^2\,e^3}{5}\right)+x^{11}\left(\frac{3\,b\,c^2\,d\,f^3}{11}+\frac{9\,b\,c\,d^2\,e\,f^2}{11}+\frac{3\,a\,c\,d^2\,f^3}{11}+\frac{3\,b\,d^3\,e^2\,f}{11}+\frac{3\,a\,d^3\,e\,f^2}{11}\right)+x^7\left(\frac{3\,b\,c^3\,e\,f^2}{7}+\frac{a\,c^3\,f^3}{7}+\frac{9\,b\,c^2\,d\,e^2\,f}{7}+\frac{9\,a\,c^2\,d\,e\,f^2}{7}+\frac{3\,b\,c\,d^2\,e^3}{7}+\frac{9\,a\,c\,d^2\,e^2\,f}{7}+\frac{a\,d^3\,e^3}{7}\right)+x^9\left(\frac{b\,c^3\,f^3}{9}+b\,c^2\,d\,e\,f^2+\frac{a\,c^2\,d\,f^3}{3}+b\,c\,d^2\,e^2\,f+a\,c\,d^2\,e\,f^2+\frac{b\,d^3\,e^3}{9}+\frac{a\,d^3\,e^2\,f}{3}\right)+\frac{b\,d^3\,f^3\,x^{15}}{15}+\frac{c^2\,e^2\,x^3\,(3\,a\,c\,f+3\,a\,d\,e+b\,c\,e)}{3}+\frac{d^2\,f^2\,x^{13}\,(a\,d\,f+3\,b\,c\,f+3\,b\,d\,e)}{13}+a\,c^3\,e^3\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

output
```
x^5*((3*a*c*d^2*e^3)/5 + (3*b*c^2*d*e^3)/5 + (3*a*c^3*e*f^2)/5 + (3*b*c^3*
e^2*f)/5 + (9*a*c^2*d*e^2*f)/5) + x^11*((3*a*c*d^2*f^3)/11 + (3*b*c^2*d*f^
3)/11 + (3*a*d^3*e*f^2)/11 + (3*b*d^3*e^2*f)/11 + (9*b*c*d^2*e*f^2)/11) +
x^7*((a*c^3*f^3)/7 + (a*d^3*e^3)/7 + (3*b*c*d^2*e^3)/7 + (3*b*c^3*e*f^2)/7
 + (9*a*c*d^2*e^2*f)/7 + (9*a*c^2*d*e*f^2)/7 + (9*b*c^2*d*e^2*f)/7) + x^9*
((b*c^3*f^3)/9 + (b*d^3*e^3)/9 + (a*c^2*d*f^3)/3 + (a*d^3*e^2*f)/3 + a*c*d
^2*e*f^2 + b*c*d^2*e^2*f + b*c^2*d*e*f^2) + (b*d^3*f^3*x^15)/15 + (c^2*e^2
*x^3*(3*a*c*f + 3*a*d*e + b*c*e))/3 + (d^2*f^2*x^13*(a*d*f + 3*b*c*f + 3*b
*d*e))/13 + a*c^3*e^3*x
```

3.17 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx$

3.17.1 Optimal result

Integrand size = 26, antiderivative size = 226

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx &= ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3\\&+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5\\&+\frac{1}{7}\left(bc\left(3d^2e^2+6cdef+c^2f^2\right)\right.\\&\qquad\left.+ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7\\&+\frac{1}{9}d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9\\&+\frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}\end{aligned}$$

output
```
a*c^3*e^2*x+1/3*c^2*e*(2*a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(b*c*e*(2*c*f+3*d*
e)+a*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^5+1/7*(b*c*(c^2*f^2+6*c*d*e*f+3*d^2*
e^2)+a*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*d*(a*d*f*(3*c*f+2*d*e)+b*(
3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^9+1/11*d^2*f*(a*d*f+3*b*c*f+2*b*d*e)*x^11+
1/13*b*d^3*f^2*x^13
```

3.17.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3 + \frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(bc\left(3d^2e^2+6cdef+c^2f^2\right) + ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 + \frac{1}{9}d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9 + \frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

output
```
a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
 + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(d*(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 +
(d^2*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

3.17.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(dx^8\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+x^6\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right)+\frac{1}{3}c^2ex^3(2acf+3ade+bce)+\frac{1}{11}d^2fx^{11}(adf+3bcf+2bde)+ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

output
```
a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
 + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(d*(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 +
(d^2*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

3.17.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

method	result
default	$\frac{b\,d^3f^2x^{13}}{13}+\frac{\left(\left(a\,d^3+3bc\,d^2\right)f^2+2b\,d^3ef\right)x^{11}}{11}+\frac{\left(\left(3ac\,d^2+3b\,c^2d\right)f^2+2\left(a\,d^3+3bc\,d^2\right)ef+b\,d^3e^2\right)x^9}{9}+\cdots$
norman	$\frac{b\,d^3f^2x^{13}}{13}+\left(\frac{1}{11}a\,d^3f^2+\frac{3}{11}bc\,d^2f^2+\frac{2}{11}b\,d^3ef\right)x^{11}+\left(\frac{1}{3}ac\,d^2f^2+\frac{2}{9}a\,d^3ef+\frac{1}{3}b\,c^2d\,f^2+\cdots\right.$
gosper	$\frac{1}{13}b\,d^3f^2x^{13}+\frac{1}{11}x^{11}a\,d^3f^2+\frac{3}{11}x^{11}bc\,d^2f^2+\frac{2}{11}x^{11}b\,d^3ef+\frac{1}{3}x^9ac\,d^2f^2+\frac{2}{9}x^9a\,d^3ef+\cdots$
risch	$\frac{1}{13}b\,d^3f^2x^{13}+\frac{1}{11}x^{11}a\,d^3f^2+\frac{3}{11}x^{11}bc\,d^2f^2+\frac{2}{11}x^{11}b\,d^3ef+\frac{1}{3}x^9ac\,d^2f^2+\frac{2}{9}x^9a\,d^3ef+\cdots$
parallelrisch	$\frac{1}{13}b\,d^3f^2x^{13}+\frac{1}{11}x^{11}a\,d^3f^2+\frac{3}{11}x^{11}bc\,d^2f^2+\frac{2}{11}x^{11}b\,d^3ef+\frac{1}{3}x^9ac\,d^2f^2+\frac{2}{9}x^9a\,d^3ef+\cdots$

input
```
int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^3*f^2*x^13+1/11*((a*d^3+3*b*c*d^2)*f^2+2*b*d^3*e*f)*x^11+1/9*((3*
a*c*d^2+3*b*c^2*d)*f^2+2*(a*d^3+3*b*c*d^2)*e*f+b*d^3*e^2)*x^9+1/7*((3*a*c^
2*d+b*c^3)*f^2+2*(3*a*c*d^2+3*b*c^2*d)*e*f+(a*d^3+3*b*c*d^2)*e^2)*x^7+1/5*
(c^3*a*f^2+2*(3*a*c^2*d+b*c^3)*e*f+(3*a*c*d^2+3*b*c^2*d)*e^2)*x^5+1/3*(2*c
^3*a*e*f+(3*a*c^2*d+b*c^3)*e^2)*x^3+a*c^3*e^2*x
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int (a+bx^2)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx \\
&= \frac{1}{13}bd^3f^2x^{13} + \frac{1}{11}\left(2\,bd^3ef + \left(3\,bcd^2 + ad^3\right)f^2\right)x^{11} \\
&\quad + \frac{1}{9}\left(bd^3e^2 + 2\left(3\,bcd^2 + ad^3\right)ef + 3\left(bc^2d + acd^2\right)f^2\right)x^9 \\
&\quad + \frac{1}{7}\left(\left(3\,bcd^2 + ad^3\right)e^2 + 6\left(bc^2d + acd^2\right)ef + \left(bc^3 + 3\,ac^2d\right)f^2\right)x^7 \\
&\quad + ac^3e^2x + \frac{1}{5}\left(ac^3f^2 + 3\left(bc^2d + acd^2\right)e^2 + 2\left(bc^3 + 3\,ac^2d\right)ef\right)x^5 \\
&\quad + \frac{1}{3}\left(2\,ac^3ef + \left(bc^3 + 3\,ac^2d\right)e^2\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^11 +
1/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^
9 + 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*
a*c^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e
^2 + 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*
d)*e^2)*x^3
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx = ac^3e^2x+\frac{bd^3f^2x^{13}}{13}+x^{11}\left(\frac{ad^3f^2}{11}+\frac{3bcd^2f^2}{11}+\frac{2bd^3ef}{11}\right)+x^9\left(\frac{acd^2f^2}{3}+\frac{2ad^3ef}{9}+\frac{bc^2df^2}{3}+\frac{2bcd^2ef}{3}+\frac{bd^3e^2}{9}\right)+x^7\cdot\left(\frac{3ac^2df^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}+\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3bcd^2e^2}{7}\right)+x^5\left(\frac{ac^3f^2}{5}+\frac{6ac^2def}{5}+\frac{3acd^2e^2}{5}+\frac{2bc^3ef}{5}+\frac{3bc^2de^2}{5}\right)+x^3\cdot\left(\frac{2ac^3ef}{3}+ac^2de^2+\frac{bc^3e^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

output
```
a*c**3*e**2*x + b*d**3*f**2*x**13/13 + x**11*(a*d**3*f**2/11 + 3*b*c*d**2*
f**2/11 + 2*b*d**3*e*f/11) + x**9*(a*c*d**2*f**2/3 + 2*a*d**3*e*f/9 + b*c*
*2*d*f**2/3 + 2*b*c*d**2*e*f/3 + b*d**3*e**2/9) + x**7*(3*a*c**2*d*f**2/7
+ 6*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b*c**3*f**2/7 + 6*b*c**2*d*e*f/7 + 3*
b*c*d**2*e**2/7) + x**5*(a*c**3*f**2/5 + 6*a*c**2*d*e*f/5 + 3*a*c*d**2*e**
2/5 + 2*b*c**3*e*f/5 + 3*b*c**2*d*e**2/5) + x**3*(2*a*c**3*e*f/3 + a*c**2*
d*e**2 + b*c**3*e**2/3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx &= \frac{1}{13}bd^3f^2x^{13}+\frac{1}{11}\left(2\,bd^3ef+\left(3\,bcd^2+ad^3\right)f^2\right)x^{11}\\&+\frac{1}{9}\left(bd^3e^2+2\left(3\,bcd^2+ad^3\right)ef+3\left(bc^2d+acd^2\right)f^2\right)x^9\\&+\frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e^2+6\left(bc^2d+acd^2\right)ef+\left(bc^3+3\,ac^2d\right)f^2\right)x^7\\&+ac^3e^2x+\frac{1}{5}\left(ac^3f^2+3\left(bc^2d+acd^2\right)e^2+2\left(bc^3+3\,ac^2d\right)ef\right)x^5\\&+\frac{1}{3}\left(2\,ac^3ef+\left(bc^3+3\,ac^2d\right)e^2\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^11 +
1/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^
9 + 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*
a*c^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e
^2 + 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*
d)*e^2)*x^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx = {} & \frac{1}{13}bd^3f^2x^{13}+\frac{2}{11}bd^3efx^{11}+\frac{3}{11}bcd^2f^2x^{11} \\ & +\frac{1}{11}ad^3f^2x^{11}+\frac{1}{9}bd^3e^2x^9+\frac{2}{3}bcd^2efx^9 \\ & +\frac{2}{9}ad^3efx^9+\frac{1}{3}bc^2df^2x^9+\frac{1}{3}acd^2f^2x^9 \\ & +\frac{3}{7}bcd^2e^2x^7+\frac{1}{7}ad^3e^2x^7+\frac{6}{7}bc^2defx^7 \\ & +\frac{6}{7}acd^2efx^7+\frac{1}{7}bc^3f^2x^7+\frac{3}{7}ac^2df^2x^7+\frac{3}{5}bc^2de^2x^5 \\ & +\frac{3}{5}acd^2e^2x^5+\frac{2}{5}bc^3efx^5+\frac{6}{5}ac^2defx^5+\frac{1}{5}ac^3f^2x^5 \\ & +\frac{1}{3}bc^3e^2x^3+ac^2de^2x^3+\frac{2}{3}ac^3efx^3+ac^3e^2x\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/13*b*d^3*f^2*x^13 + 2/11*b*d^3*e*f*x^11 + 3/11*b*c*d^2*f^2*x^11 + 1/11*a
*d^3*f^2*x^11 + 1/9*b*d^3*e^2*x^9 + 2/3*b*c*d^2*e*f*x^9 + 2/9*a*d^3*e*f*x^
9 + 1/3*b*c^2*d*f^2*x^9 + 1/3*a*c*d^2*f^2*x^9 + 3/7*b*c*d^2*e^2*x^7 + 1/7*
a*d^3*e^2*x^7 + 6/7*b*c^2*d*e*f*x^7 + 6/7*a*c*d^2*e*f*x^7 + 1/7*b*c^3*f^2*
x^7 + 3/7*a*c^2*d*f^2*x^7 + 3/5*b*c^2*d*e^2*x^5 + 3/5*a*c*d^2*e^2*x^5 + 2/
5*b*c^3*e*f*x^5 + 6/5*a*c^2*d*e*f*x^5 + 1/5*a*c^3*f^2*x^5 + 1/3*b*c^3*e^2*
x^3 + a*c^2*d*e^2*x^3 + 2/3*a*c^3*e*f*x^3 + a*c^3*e^2*x
```

3.17.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx = x^5\left(\frac{2\,b\,c^3\,e\,f}{5}+\frac{a\,c^3\,f^2}{5}+\frac{3\,b\,c^2\,d\,e^2}{5}+\frac{6\,a\,c^2\,d\,e\,f}{5}+\frac{3\,a\,c\,d^2\,e^2}{5}\right)+x^9\left(\frac{b\,c^2\,d\,f^2}{3}+\frac{2\,b\,c\,d^2\,e\,f}{3}+\frac{a\,c\,d^2\,f^2}{3}+\frac{b\,d^3\,e^2}{9}+\frac{2\,a\,d^3\,e\,f}{9}\right)+x^7\left(\frac{b\,c^3\,f^2}{7}+\frac{6\,b\,c^2\,d\,e\,f}{7}+\frac{3\,a\,c^2\,d\,f^2}{7}+\frac{3\,b\,c\,d^2\,e^2}{7}+\frac{6\,a\,c\,d^2\,e\,f}{7}+\frac{a\,d^3\,e^2}{7}\right)+\frac{b\,d^3\,f^2\,x^{13}}{13}+\frac{c^2\,e\,x^3\,(2\,a\,c\,f+3\,a\,d\,e+b\,c\,e)}{3}+\frac{d^2\,f\,x^{11}\,(a\,d\,f+3\,b\,c\,f+2\,b\,d\,e)}{11}+a\,c^3\,e^2\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x)
```

output
```
x^5*((a*c^3*f^2)/5 + (2*b*c^3*e*f)/5 + (3*a*c*d^2*e^2)/5 + (3*b*c^2*d*e^2)
/5 + (6*a*c^2*d*e*f)/5) + x^9*((b*d^3*e^2)/9 + (2*a*d^3*e*f)/9 + (a*c*d^2*
f^2)/3 + (b*c^2*d*f^2)/3 + (2*b*c*d^2*e*f)/3) + x^7*((a*d^3*e^2)/7 + (b*c^
3*f^2)/7 + (3*a*c^2*d*f^2)/7 + (3*b*c*d^2*e^2)/7 + (6*a*c*d^2*e*f)/7 + (6*
b*c^2*d*e*f)/7) + (b*d^3*f^2*x^13)/13 + (c^2*e*x^3*(2*a*c*f + 3*a*d*e + b*
c*e))/3 + (d^2*f*x^11*(a*d*f + 3*b*c*f + 2*b*d*e))/11 + a*c^3*e^2*x
```

3.18 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx$

3.18.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx &= ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3\\&+\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5\\&+\frac{1}{7}d(3bc(de+cf)+ad(de+3cf))x^7\\&+\frac{1}{9}d^2(bde+3bcf+adf)x^9+\frac{1}{11}bd^3fx^{11}\end{aligned}$$

output
```
a*c^3*e*x+1/3*c^2*(a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(3*a*d*(c*f+d*e)+b*c*(c*
f+3*d*e))*x^5+1/7*d*(3*b*c*(c*f+d*e)+a*d*(3*c*f+d*e))*x^7+1/9*d^2*(a*d*f+3
*b*c*f+b*d*e)*x^9+1/11*b*d^3*f*x^11
```

3.18.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx &= ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3\\&+\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5\\&+\frac{1}{7}d(3bc(de+cf)+ad(de+3cf))x^7\\&+\frac{1}{9}d^2(bde+3bcf+adf)x^9+\frac{1}{11}bd^3fx^{11}\end{aligned}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

output
```
a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f)
+ b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x
^7)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

3.18.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx$$

$$\Big\downarrow \text{396}$$

$$\int (c^2x^2(acf+3ade+bce)+d^2x^8(adf+3bcf+bde)+dx^6(ad(3cf+de)+3bc(cf+de))+cx^4(3ad(cf+de)+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de))+ac^3ex+\frac{1}{11}bd^3fx^{11}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

output
```
a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f)
+ b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x
^7)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

3.18.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
norman	$\frac{b\,d^3 f\,x^{11}}{11}+\left(\frac{1}{9}a\,d^3 f+\frac{1}{3}bc\,d^2 f+\frac{1}{9}b\,d^3 e\right)x^9+\left(\frac{3}{7}ac\,d^2 f+\frac{1}{7}a\,d^3 e+\frac{3}{7}b\,c^2 f d+\frac{3}{7}bce\,d^2\right)x^7+\left(\frac{3}{5}a\right.$
default	$\frac{b\,d^3 f\,x^{11}}{11}+\frac{\left(\left(a\,d^3+3bc\,d^2\right)f+b\,d^3 e\right)x^9}{9}+\frac{\left(\left(3ac\,d^2+3b\,c^2 d\right)f+\left(a\,d^3+3bc\,d^2\right)e\right)x^7}{7}+\frac{\left(\left(3a\,c^2 d+c^3 b\right)f+\left(3ac\,d^2+3b\,c^2 d\right)e\right)x}{5}$
gosper	$\frac{1}{11}b\,d^3 f\,x^{11}+\frac{1}{9}x^9 a\,d^3 f+\frac{1}{3}x^9 bc\,d^2 f+\frac{1}{9}x^9 b\,d^3 e+\frac{3}{7}x^7 ac\,d^2 f+\frac{1}{7}x^7 a\,d^3 e+\frac{3}{7}x^7 b\,c^2 f d+\frac{3}{7}x^7 bc$
risch	$\frac{1}{11}b\,d^3 f\,x^{11}+\frac{1}{9}x^9 a\,d^3 f+\frac{1}{3}x^9 bc\,d^2 f+\frac{1}{9}x^9 b\,d^3 e+\frac{3}{7}x^7 ac\,d^2 f+\frac{1}{7}x^7 a\,d^3 e+\frac{3}{7}x^7 b\,c^2 f d+\frac{3}{7}x^7 bc$
parallelrisch	$\frac{1}{11}b\,d^3 f\,x^{11}+\frac{1}{9}x^9 a\,d^3 f+\frac{1}{3}x^9 bc\,d^2 f+\frac{1}{9}x^9 b\,d^3 e+\frac{3}{7}x^7 ac\,d^2 f+\frac{1}{7}x^7 a\,d^3 e+\frac{3}{7}x^7 b\,c^2 f d+\frac{3}{7}x^7 bc$

input
```
int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d^3*f*x^11+(1/9*a*d^3*f+1/3*b*c*d^2*f+1/9*b*d^3*e)*x^9+(3/7*a*c*d^2
*f+1/7*a*d^3*e+3/7*b*c^2*f*d+3/7*b*c*e*d^2)*x^7+(3/5*a*c^2*d*f+3/5*a*c*d^2
*e+1/5*b*c^3*f+3/5*b*c^2*d*e)*x^5+(1/3*c^3*a*f+a*c^2*d*e+1/3*b*c^3*e)*x^3+
a*c^3*e*x
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int (a+bx^2) (c+dx^2)^3 (e+fx^2) \, dx = \frac{1}{11} bd^3 fx^{11} + \frac{1}{9} (bd^3 e + (3 bcd^2 + ad^3) f) x^9 + \frac{1}{7} ((3 bcd^2 + ad^3) e + 3 (bc^2 d + acd^2) f) x^7 + ac^3 ex + \frac{1}{5} (3 (bc^2 d + acd^2) e + (bc^3 + 3 ac^2 d) f) x^5 + \frac{1}{3} (ac^3 f + (bc^3 + 3 ac^2 d) e) x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*((3*b*
c*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*e*x + 1/5*(3*(b*c^
2*d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + 1/3*(a*c^3*f + (b*c^3 + 3*
a*c^2*d)*e)*x^3
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int (a+bx^2) (c+dx^2)^3 (e+fx^2) \, dx = ac^3 ex + \frac{bd^3 fx^{11}}{11} + x^9 \left(\frac{ad^3 f}{9} + \frac{bcd^2 f}{3} + \frac{bd^3 e}{9}\right) + x^7 \cdot \left(\frac{3acd^2 f}{7} + \frac{ad^3 e}{7} + \frac{3bc^2 df}{7} + \frac{3bcd^2 e}{7}\right) + x^5 \cdot \left(\frac{3ac^2 df}{5} + \frac{3acd^2 e}{5} + \frac{bc^3 f}{5} + \frac{3bc^2 de}{5}\right) + x^3 \left(\frac{ac^3 f}{3} + ac^2 de + \frac{bc^3 e}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

output
```
a*c**3*e*x + b*d**3*f*x**11/11 + x**9*(a*d**3*f/9 + b*c*d**2*f/3 + b*d**3*
e/9) + x**7*(3*a*c*d**2*f/7 + a*d**3*e/7 + 3*b*c**2*d*f/7 + 3*b*c*d**2*e/7
) + x**5*(3*a*c**2*d*f/5 + 3*a*c*d**2*e/5 + b*c**3*f/5 + 3*b*c**2*d*e/5) +
 x**3*(a*c**3*f/3 + a*c**2*d*e + b*c**3*e/3)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = \frac{1}{11}bd^3fx^{11} + \frac{1}{9}\left(bd^3e + (3\,bcd^2+ad^3)f\right)x^9 + \frac{1}{7}\left((3\,bcd^2+ad^3)e + 3\left(bc^2d+acd^2\right)f\right)x^7 + ac^3ex + \frac{1}{5}\left(3\left(bc^2d+acd^2\right)e + \left(bc^3+3\,ac^2d\right)f\right)x^5 + \frac{1}{3}\left(ac^3f + \left(bc^3+3\,ac^2d\right)e\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*((3*b*
c*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*e*x + 1/5*(3*(b*c^
2*d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + 1/3*(a*c^3*f + (b*c^3 + 3*
a*c^2*d)*e)*x^3
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = \frac{1}{11}bd^3fx^{11} + \frac{1}{9}bd^3ex^9 + \frac{1}{3}bcd^2fx^9 + \frac{1}{9}ad^3fx^9 + \frac{3}{7}bcd^2ex^7 + \frac{1}{7}ad^3ex^7 + \frac{3}{7}bc^2dfx^7 + \frac{3}{7}acd^2fx^7 + \frac{3}{5}bc^2dex^5 + \frac{3}{5}acd^2ex^5 + \frac{1}{5}bc^3fx^5 + \frac{3}{5}ac^2dfx^5 + \frac{1}{3}bc^3ex^3 + ac^2dex^3 + \frac{1}{3}ac^3fx^3 + ac^3ex$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="giac")
```

output
```
1/11*b*d^3*f*x^11 + 1/9*b*d^3*e*x^9 + 1/3*b*c*d^2*f*x^9 + 1/9*a*d^3*f*x^9
+ 3/7*b*c*d^2*e*x^7 + 1/7*a*d^3*e*x^7 + 3/7*b*c^2*d*f*x^7 + 3/7*a*c*d^2*f*
x^7 + 3/5*b*c^2*d*e*x^5 + 3/5*a*c*d^2*e*x^5 + 1/5*b*c^3*f*x^5 + 3/5*a*c^2*
d*f*x^5 + 1/3*b*c^3*e*x^3 + a*c^2*d*e*x^3 + 1/3*a*c^3*f*x^3 + a*c^3*e*x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = x^5\left(\frac{b\,c^3\,f}{5}+\frac{3\,a\,c\,d^2\,e}{5}+\frac{3\,a\,c^2\,d\,f}{5}+\frac{3\,b\,c^2\,d\,e}{5}\right) + x^7\left(\frac{a\,d^3\,e}{7}+\frac{3\,a\,c\,d^2\,f}{7}+\frac{3\,b\,c\,d^2\,e}{7}+\frac{3\,b\,c^2\,d\,f}{7}\right) + x^3\left(\frac{a\,c^3\,f}{3}+\frac{b\,c^3\,e}{3}+a\,c^2\,d\,e\right) + x^9\left(\frac{a\,d^3\,f}{9}+\frac{b\,d^3\,e}{9}+\frac{b\,c\,d^2\,f}{3}\right)+a\,c^3\,e\,x+\frac{b\,d^3\,f\,x^{11}}{11}$$

input
```
int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x)
```

output
```
x^5*((b*c^3*f)/5 + (3*a*c*d^2*e)/5 + (3*a*c^2*d*f)/5 + (3*b*c^2*d*e)/5) +
x^7*((a*d^3*e)/7 + (3*a*c*d^2*f)/7 + (3*b*c*d^2*e)/7 + (3*b*c^2*d*f)/7) +
x^3*((a*c^3*f)/3 + (b*c^3*e)/3 + a*c^2*d*e) + x^9*((a*d^3*f)/9 + (b*d^3*e)
/9 + (b*c*d^2*f)/3) + a*c^3*e*x + (b*d^3*f*x^11)/11
```

3.19 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$

3.19.1 Optimal result

Integrand size = 26, antiderivative size = 227

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx &= \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3))\,x}{105f^4} \\ &\quad - \frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))\,x(c+dx^2)}{105f^3} \\ &\quad - \frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2}+\frac{bx(c+dx^2)^3}{7f}+\frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}\end{aligned}$$

output
```
1/105*(7*a*d*f*(33*c^2*f^2-40*c*d*e*f+15*d^2*e^2)-b*(-48*c^3*f^3+231*c^2*d
*e*f^2-280*c*d^2*e^2*f+105*d^3*e^3))*x/f^4-1/105*(7*a*d*f*(-9*c*f+5*d*e)-b
*(24*c^2*f^2-63*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)/f^3-1/35*(-7*a*d*f-6*b*c*
f+7*b*d*e)*x*(d*x^2+c)^2/f^2+1/7*b*x*(d*x^2+c)^3/f+(-a*f+b*e)*(-c*f+d*e)^3
*arctan(x*f^(1/2)/e^(1/2))/f^(9/2)/e^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{(-b(de-cf)^3+adf(d^2e^2-3cdef+3c^2f^2))x}{f^4} + \frac{d(adf(-de+3cf)+b(d^2e^2-3cdef+3c^2f^2))x^3}{3f^3} + \frac{d^2(-bde+3bcf+adf)x^5}{5f^2} + \frac{bd^3x^7}{7f} + \frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^4 + (
d*(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f
^3) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5*f^2) + (b*d^3*x^7)/(7*f) +
 ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

3.19.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {403, 25, 403, 25, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$

$$\Big\downarrow \ 403$$

$$\frac{\int -\frac{(dx^2+c)^2((7bde-6bcf-7adf)x^2+c(be-7af))}{fx^2+e}dx}{7f}+\frac{bx(c+dx^2)^3}{7f}$$

$$\Big\downarrow \ 25$$

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\int \frac{(dx^2+c)^2((7bde-6bcf-7adf)x^2+c(be-7af))}{fx^2+e}dx}{7f}$$

$\downarrow$ 403

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{\int -\frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}+\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}}{7f}$$

$\downarrow$ 25

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}$$

$\downarrow$ 403

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\int \frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3\right)\right)x^2+c\left(7af\left(5d^2e^2-12cdfe+15c^2f^2\right)-\cdots\right.}{fx^2+e}}{3f}\cdots}{5f}}{7f}$$

$\downarrow$ 299

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)(de-cf)^3\int \frac{1}{fx^2+e}dx}{f}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3\right)\right)}{f}}{3f}\cdots}{5f}}{7f}$$

$\downarrow$ 218

$$\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3\cdots\right.\right.}{f}}{3f}\cdots}{5f}}{7f}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

3.19. $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$

output
```
(b*x*(c + d*x^2)^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^
2)/(5*f) - (-1/3*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f +
24*c^2*f^2))*x*(c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c
^2*f^2) - b*(105*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3)
)*x)/f + (105*(b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt
[e]*f^(3/2)))/(3*f))/(5*f))/(7*f)
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.19.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

method	result
default	$\frac{\frac{1}{7}b\,d^3x^7f^3+\frac{1}{5}a\,d^3f^3x^5+\frac{3}{5}bc\,d^2f^3x^5-\frac{1}{5}b\,d^3e\,f^2x^5+ac\,d^2f^3x^3-\frac{1}{3}a\,d^3e\,f^2x^3+b\,c^2d\,f^3x^3-bc\,d^2e\,f^2x^3+\frac{1}{3}b\,d^3e^2f\,x^3+3a\,c^2d\,f^3x-3ac}{f^4}$
risch	$\frac{b\,d^3x^7}{7f}+\frac{a\,d^3x^5}{5f}+\frac{b\,c^3x}{f}-\frac{\ln\left(fx+\sqrt{-ef}\right)a\,c^3}{2\sqrt{-ef}}+\frac{ac\,d^2x^3}{f}-\frac{a\,d^3e\,x^3}{3f^2}+\frac{b\,c^2d\,x^3}{f}+\frac{b\,d^3e^2x^3}{3f^3}-\frac{3\ln\left(-fx+\sqrt{-ef}\right)a\,c^2de}{2f\sqrt{-ef}}-$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/7*b*d^3*x^7*f^3+1/5*a*d^3*f^3*x^5+3/5*b*c*d^2*f^3*x^5-1/5*b*d^3*e
*f^2*x^5+a*c*d^2*f^3*x^3-1/3*a*d^3*e*f^2*x^3+b*c^2*d*f^3*x^3-b*c*d^2*e*f^2
*x^3+1/3*b*d^3*e^2*f*x^3+3*a*c^2*d*f^3*x-3*a*c*d^2*e*f^2*x+a*d^3*e^2*f*x+b
*c^3*f^3*x-3*b*c^2*d*e*f^2*x+3*b*c*d^2*e^2*f*x-b*d^3*e^3*x)+(a*c^3*f^4-3*a
*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-3
*b*c*d^2*e^3*f+b*d^3*e^4)/f^4/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$

$$= \left[\frac{30\,bd^3ef^4x^7 - 42\,(bd^3e^2f^3 - (3\,bcd^2 + ad^3)ef^4)x^5 + 70\,(bd^3e^3f^2 - (3\,bcd^2 + ad^3)e^2f^3 + 3\,(bc^2d + acd^2)e}{}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/210*(30*b*d^3*e*f^4*x^7 - 42*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*d^3)*e*f^4
)*x^5 + 70*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^2*d + a*c
*d^2)*e*f^4)*x^3 - 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3*f
+ 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(-e*f)*lo
g((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 210*(b*d^3*e^4*f - (3*b*c*d^
2 + a*d^3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e
*f^4)*x)/(e*f^5), 1/105*(15*b*d^3*e*f^4*x^7 - 21*(b*d^3*e^2*f^3 - (3*b*c*d
^2 + a*d^3)*e*f^4)*x^5 + 35*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 +
 3*(b*c^2*d + a*c*d^2)*e*f^4)*x^3 + 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^
2 + a*d^3)*e^3*f + 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f
^3)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 105*(b*d^3*e^4*f - (3*b*c*d^2 + a*d^
3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x)
/(e*f^5)]
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(228) = 456$.

Time = 0.85 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{bd^3x^7}{7f} + x^5\left(\frac{ad^3}{5f} + \frac{3bcd^2}{5f} - \frac{bd^3e}{5f^2}\right) + x^3\left(\frac{acd^2}{f} - \frac{ad^3e}{3f^2} + \frac{bc^2d}{f} - \frac{bcd^2e}{f^2} + \frac{bd^3e^2}{3f^3}\right)$$
$$+ x\left(\frac{3ac^2d}{f} - \frac{3acd^2e}{f^2} + \frac{ad^3e^2}{f^3} + \frac{bc^3}{f} - \frac{3bc^2de}{f^2} + \frac{3bcd^2e^2}{f^3} - \frac{bd^3e^3}{f^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4} + x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4} + x\right)}{2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
b*d**3*x**7/(7*f) + x**5*(a*d**3/(5*f) + 3*b*c*d**2/(5*f) - b*d**3*e/(5*f*
*2)) + x**3*(a*c*d**2/f - a*d**3*e/(3*f**2) + b*c**2*d/f - b*c*d**2*e/f**2
 + b*d**3*e**2/(3*f**3)) + x*(3*a*c**2*d/f - 3*a*c*d**2*e/f**2 + a*d**3*e*
*2/f**3 + b*c**3/f - 3*b*c**2*d*e/f**2 + 3*b*c*d**2*e**2/f**3 - b*d**3*e**
3/f**4) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log(-e*f**4*sqrt(-1
/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3
*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2
 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)
*(c*f - d*e)**3*log(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a
*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*
c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)
/2
```

3.19.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{(bd^3e^4 - 3\,bcd^2e^3f - ad^3e^3f + 3\,bc^2de^2f^2 + 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 + ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^4} + \frac{15\,bd^3f^6x^7 - 21\,bd^3ef^5x^5 + 63\,bcd^2f^6x^5 + 21\,ad^3f^6x^5 + 35\,bd^3e^2f^4x^3 - 105\,bcd^2ef^5x^3 - 35\,ad^3ef^5x^3 + \cdots}{\cdots}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
(b*d^3*e^4 - 3*b*c*d^2*e^3*f - a*d^3*e^3*f + 3*b*c^2*d*e^2*f^2 + 3*a*c*d^2
*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 + a*c^3*f^4)*arctan(f*x/sqrt(e*f)
)/(sqrt(e*f)*f^4) + 1/105*(15*b*d^3*f^6*x^7 - 21*b*d^3*e*f^5*x^5 + 63*b*c*
d^2*f^6*x^5 + 21*a*d^3*f^6*x^5 + 35*b*d^3*e^2*f^4*x^3 - 105*b*c*d^2*e*f^5*
x^3 - 35*a*d^3*e*f^5*x^3 + 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x^3 - 105
*b*d^3*e^3*f^3*x + 315*b*c*d^2*e^2*f^4*x + 105*a*d^3*e^2*f^4*x - 315*b*c^2
*d*e*f^5*x - 315*a*c*d^2*e*f^5*x + 105*b*c^3*f^6*x + 315*a*c^2*d*f^6*x)/f^
7
```

3.19.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = x\left(\frac{b\,c^3+3\,a\,d\,c^2}{f}+\frac{e\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{f}-\frac{3\,c\,d\,(a\,d+b\,c)}{f}\right)}{f}\right) + x^5\left(\frac{a\,d^3+3\,b\,c\,d^2}{5\,f}-\frac{b\,d^3\,e}{5\,f^2}\right) - x^3\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{3\,f}-\frac{c\,d\,(a\,d+b\,c)}{f}\right) + \frac{b\,d^3\,x^7}{7\,f} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}\right)(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
x*((b*c^3 + 3*a*c^2*d)/f + (e*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))
/f - (3*c*d*(a*d + b*c))/f))/f) + x^5*((a*d^3 + 3*b*c*d^2)/(5*f) - (b*d^3*
e)/(5*f^2)) - x^3*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/(3*f) - (c*
d*(a*d + b*c))/f) + (b*d^3*x^7)/(7*f) + (atan((f^(1/2)*x*(a*f - b*e)*(c*f
- d*e)^3)/(e^(1/2)*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b*c^3*e*f^3 - 3*
a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2)))
*(a*f - b*e)*(c*f - d*e)^3)/(e^(1/2)*f^(9/2))
```

3.20 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

3.20.1 Optimal result

Integrand size = 26, antiderivative size = 242

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx &= -\frac{d(5af(15d^2e^2-22cdef+3c^2f^2)-be(105d^2e^2-190cdef+81c^2f^2))x}{30ef^4}\\ &\quad -\frac{d(be(35de-33cf)-5af(5de-3cf))x(c+dx^2)}{30ef^3}+\frac{d(7be-5af)x(c+dx^2)^2}{10ef^2}\\ &\quad -\frac{(be-af)x(c+dx^2)^3}{2ef(e+fx^2)}-\frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}\end{aligned}$$

output
```
-1/30*d*(5*a*f*(3*c^2*f^2-22*c*d*e*f+15*d^2*e^2)-b*e*(81*c^2*f^2-190*c*d*e
*f+105*d^2*e^2))*x/e/f^4-1/30*d*(b*e*(-33*c*f+35*d*e)-5*a*f*(-3*c*f+5*d*e)
)*x*(d*x^2+c)/e/f^3+1/10*d*(-5*a*f+7*b*e)*x*(d*x^2+c)^2/e/f^2-1/2*(-a*f+b*
e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)-1/2*(-c*f+d*e)^2*(b*e*(-c*f+7*d*e)-a*f*(c*f
+5*d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(9/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{d(3b(de-cf)^2+adf(-2de+3cf))\,x}{f^4} + \frac{d^2(-2bde+3bcf+adf)x^3}{3f^3} + \frac{bd^3x^5}{5f^2} + \frac{(be-af)(de-cf)^3x}{2ef^4(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
(d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2*b*d*e +
3*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((b*e - a*f)*(d*e -
c*f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*
(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

3.20.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {401, 25, 403, 25, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef} - \frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef} - \frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{(dx^2+c)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}+\frac{dx(c+dx^2)^2(7be-5af)}{5f}}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\int \frac{(dx^2+c)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\int \frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\left(5d^2e^2-6cdfe-3c^2f^2\right)-be\left(35d^2e^2-54cdfe+15c^2f^2 \cdots\right.\right.}{fx^2+e}}{3f} \cdots}{5f}}{2ef} \cdots$$

$$-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15(de-cf)^2(be(7de-cf)-af(cf+5de))\int \frac{1}{fx^2+e}dx}{f}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+\frac{dx(c+ \cdots}{\cdots}}{5f}}{2ef} \cdots$$

$$-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+\frac{dx \cdots}{\cdots}}{5f}}{2ef} \cdots$$

$$-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

3.20. $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

output
```
-1/2*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*a*f)
*x*(c + d*x^2)^2)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e - 3*c*
f))*x*(c + d*x^2))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2
) - b*e*(105*d^2*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*f)^2
*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqr
t[e]*f^(3/2)))/(3*f))/(5*f))/(2*e*f)
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.20.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.27

method	result
default	$\frac{d\left(\frac{1}{5}b\,d^{2}x^{5}f^{2}+\frac{1}{3}a\,d^{2}f^{2}x^{3}+bcd\,f^{2}x^{3}-\frac{2}{3}b\,d^{2}ef\,x^{3}+3acd\,f^{2}x-2a\,d^{2}efx+3b\,c^{2}f^{2}x-6bcdefx+3b\,d^{2}e^{2}x\right)}{f^{4}}+\frac{\left(a\,c^{3}f^{4}-3a\,c^{2}de\,f^{3}+3ac\,d^{2}\right.}{}$
risch	$\frac{3d^{2}acx}{f^{2}}-\frac{2d^{3}aex}{f^{3}}+\frac{3db\,c^{2}x}{f^{2}}+\frac{3d^{3}b\,e^{2}x}{f^{4}}-\frac{\ln\left(fx+\sqrt{-ef}\right)a\,c^{3}}{4\sqrt{-ef}\,e}+\frac{9e\ln\left(fx+\sqrt{-ef}\right)ac\,d^{2}}{4f^{2}\sqrt{-ef}}+\frac{9e\ln\left(fx+\sqrt{-ef}\right)b\,c^{2}d}{4f^{2}\sqrt{-ef}}-\frac{15e^{2}}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
d/f^4*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+b*c*d*f^2*x^3-2/3*b*d^2*e*f*x^3
+3*a*c*d*f^2*x-2*a*d^2*e*f*x+3*b*c^2*f^2*x-6*b*c*d*e*f*x+3*b*d^2*e^2*x)+1/
f^4*(1/2*(a*c^3*f^4-3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*
f^3+3*b*c^2*d*e^2*f^2-3*b*c*d^2*e^3*f+b*d^3*e^4)/e*x/(f*x^2+e)+1/2*(a*c^3*
f^4+3*a*c^2*d*e*f^3-9*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3-9*b*c^2*d*
e^2*f^2+15*b*c*d^2*e^3*f-7*b*d^3*e^4)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)
))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.45

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^2}\,dx = \left[\frac{12\,bd^3e^2f^4x^7-4\,(7\,bd^3e^3f^3-5\,(3\,bcd^2+ad^3)e^2f^4)x^5+20\,(7\,bd^3e^4f^2-5\,(3\,bcd^2+ad^3)e^3f^3+9\,(bc^2d+}{}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b*d^3*e^2*f^4*x^7 - 4*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e
^2*f^4)*x^5 + 20*(7*b*d^3*e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c
^2*d + a*c*d^2)*e^2*f^4)*x^3 + 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^
2 + a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2
*f^3 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c
^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(-e*f)*log((
f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(7*b*d^3*e^5*f + a*c^3*e*f^5
 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 - (b*c^3
+ 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5), 1/30*(6*b*d^3*e^2*f^4*x^
7 - 2*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^2*f^4)*x^5 + 10*(7*b*d^3*
e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4)*x
^3 - 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4*f + 9*(b*c^
2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3 + (7*b*d^3*e^4*f - a*
c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d + a*c*d^2)*e^2*f^3 -
(b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(7*b*
d^3*e^5*f + a*c^3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c
*d^2)*e^3*f^3 - (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5)]
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(231) = 462$.

Time = 2.22 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$
$$= \frac{bd^3x^5}{5f^2} + x^3\left(\frac{ad^3}{3f^2} + \frac{bcd^2}{f^2} - \frac{2bd^3e}{3f^3}\right) + x\left(\frac{3acd^2}{f^2} - \frac{2ad^3e}{f^3} + \frac{3bc^2d}{f^2} - \frac{6bcd^2e}{f^3} + \frac{3bd^3e^2}{f^4}\right)$$
$$+ \frac{x(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4)}{2e^2f^4 + 2ef^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2 + 5adef + bcef - 7bde^2)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2-}\right.}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2 + 5adef + bcef - 7bde^2)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bd}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+1}\right.}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
b*d**3*x**5/(5*f**2) + x**3*(a*d**3/(3*f**2) + b*c*d**2/f**2 - 2*b*d**3*e/
(3*f**3)) + x*(3*a*c*d**2/f**2 - 2*a*d**3*e/f**3 + 3*b*c**2*d/f**2 - 6*b*c
*d**2*e/f**3 + 3*b*d**3*e**2/f**4) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 +
3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**
2 - 3*b*c*d**2*e**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(
-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**
2)*log(-e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*
f + b*c*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e*
*2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*
d**2*e**3*f - 7*b*d**3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*
(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*
f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**
3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f + b*c*
*3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**3*e**4) + x
)/4
```

3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = -\frac{(7\,bd^3e^4-15\,bcd^2e^3f-5\,ad^3e^3f+9\,bc^2de^2f^2+9\,acd^2e^2f^2-bc^3ef^3-3\,ac^2def^3-ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\sqrt{ef}ef^4} + \frac{bd^3e^4x-3\,bcd^2e^3fx-ad^3e^3fx+3\,bc^2de^2f^2x+3\,acd^2e^2f^2x-bc^3ef^3x-3\,ac^2def^3x+ac^3f^4x}{2\,(fx^2+e)ef^4} + \frac{3\,bd^3f^8x^5-10\,bd^3ef^7x^3+15\,bcd^2f^8x^3+5\,ad^3f^8x^3+45\,bd^3e^2f^6x-90\,bcd^2ef^7x-30\,ad^3ef^7x+45\,bc^2\cdots}{15\,f^{10}}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(7*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2 +
 9*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - a*c^3*f^4)*arctan(f*x
/sqrt(e*f))/(sqrt(e*f)*e*f^4) + 1/2*(b*d^3*e^4*x - 3*b*c*d^2*e^3*f*x - a*d
^3*e^3*f*x + 3*b*c^2*d*e^2*f^2*x + 3*a*c*d^2*e^2*f^2*x - b*c^3*e*f^3*x - 3
*a*c^2*d*e*f^3*x + a*c^3*f^4*x)/((f*x^2 + e)*e*f^4) + 1/15*(3*b*d^3*f^8*x^
5 - 10*b*d^3*e*f^7*x^3 + 15*b*c*d^2*f^8*x^3 + 5*a*d^3*f^8*x^3 + 45*b*d^3*e
^2*f^6*x - 90*b*c*d^2*e*f^7*x - 30*a*d^3*e*f^7*x + 45*b*c^2*d*f^8*x + 45*a
*c*d^2*f^8*x)/f^10
```

3.20.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = x^3\left(\frac{a\,d^3+3\,b\,c\,d^2}{3\,f^2}-\frac{2\,b\,d^3\,e}{3\,f^3}\right) - x\left(\frac{2\,e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^2}-\frac{2\,b\,d^3\,e}{f^3}\right)}{f}+\frac{b\,d^3\,e^2}{f^4}-\frac{3\,c\,d\,(a\,d+b\,c)}{f^2}\right)+\frac{b\,d^3\,x^5}{5\,f^2} + \frac{x\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}{2\,e\,(f^5\,x^2+e\,f^4)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)^2\,(a\,c\,f^2-7\,b\,d\,e^2+5\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+a\,c^3\,f^4-9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f-9\,a\,c\,d^2\,e^2\,f^2-7\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f)}\right)(c\,f-d\,e)^2\,(a\,c\,f^2-7\cdots}{2\,e^{3/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
x^3*((a*d^3 + 3*b*c*d^2)/(3*f^2) - (2*b*d^3*e)/(3*f^3)) - x*((2*e*((a*d^3
+ 3*b*c*d^2)/f^2 - (2*b*d^3*e)/f^3))/f + (b*d^3*e^2)/f^4 - (3*c*d*(a*d + b
*c))/f^2) + (b*d^3*x^5)/(5*f^2) + (x*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f
- b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*
b*c^2*d*e^2*f^2))/(2*e*(e*f^4 + f^5*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)^2
*(a*c*f^2 - 7*b*d*e^2 + 5*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^3*f^4 - 7*b*d^
3*e^4 + 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f -
 9*a*c*d^2*e^2*f^2 - 9*b*c^2*d*e^2*f^2)))*(c*f - d*e)^2*(a*c*f^2 - 7*b*d*e
^2 + 5*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(9/2))
```

3.20. $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

3.21 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3} dx$

3.21.1 Optimal result

Integrand size = 26, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx &= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))\,x}{24e^2f^4} \\ &+ \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} \\ &- \frac{(be-af)x(c+dx^2)^3}{4ef\,(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2\,(e+fx^2)} \\ &+ \frac{(de-cf)\,(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}\end{aligned}$$

output
```
1/24*d*(3*a*f*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2)-b*e*(3*c^2*f^2-100*c*d*e*f
+105*d^2*e^2))*x/e^2/f^4+1/24*d*(b*e*(-3*c*f+35*d*e)-3*a*f*(3*c*f+5*d*e))*
x*(d*x^2+c)/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^2-1/8*(b*e*
(-c*f+7*d*e)-3*a*f*(c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)+1/8*(-c*f+d*
e)*(b*e*(-c^2*f^2-10*c*d*e*f+35*d^2*e^2)-3*a*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^
2))*arctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(9/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{d^2(-3bde+3bcf+adf)x}{f^4}+\frac{bd^3x^3}{3f^3}+\frac{(be-af)(de-cf)^3x}{4ef^4(e+fx^2)^2}$$
$$-\frac{(de-cf)^2(be(13de-cf)-3af(3de+cf))x}{8e^2f^4(e+fx^2)}$$
$$+\frac{(de-cf)\left(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) + ((b*e - a
*f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c*f)^2*(b*e*(13*d*e
 - c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e + f*x^2)) + ((d*e - c*f)*(
b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c
^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

3.21.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {401, 25, 401, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$\Big\downarrow \text{401}$$
$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-3af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)^2(d(7be-3af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

$\Big\downarrow$ 401

$$-\frac{-\frac{\int \frac{(dx^2+c)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

$\Big\downarrow$ 403

$$-\frac{-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}-\frac{dx\left(c+dx^2\right)(be(35de-3cf)-3c}{3f}}{2ef}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

$\Big\downarrow$ 299

$$-\frac{-\frac{\frac{-\frac{3(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{f}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{f}}{3f}-dx}{2ef}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{f}}{3f}-}{2ef}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

input

```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

3.21. $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

output
```
-1/4*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*
e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(
d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-((d*
(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d*e
*f + 3*c^2*f^2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^
2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.21.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.18

method	result
default	$\frac{d^2\left(\frac{1}{3}bdf\,x^3+adfx+3bcfx-3bdex\right)}{f^4}+\frac{\frac{f\left(3a\,c^3f^4+3a\,c^2de\,f^3-15ac\,d^2e^2f^2+9a\,d^3e^3f+b\,c^3e\,f^3-15b\,c^2d\,e^2f^2+27bc\,d^2e^3f-13b\,d^3e^4\right)x^3}{8e^2}+\frac{(5}{}}{\left(f\,x^2+e\right)^2}$
risch	$\frac{d^3b\,x^3}{3f^3}+\frac{d^3ax}{f^3}+\frac{3d^2bcx}{f^3}-\frac{3d^3bex}{f^4}+\frac{\frac{f\left(3a\,c^3f^4+3a\,c^2de\,f^3-15ac\,d^2e^2f^2+9a\,d^3e^3f+b\,c^3e\,f^3-15b\,c^2d\,e^2f^2+27bc\,d^2e^3f-13b\,d^3e^4\right)x^3}{8e^2}}{f^4(f\,x^2}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
d^2/f^4*(1/3*b*d*f*x^3+a*d*f*x+3*b*c*f*x-3*b*d*e*x)+1/f^4*((1/8*f*(3*a*c^3
*f^4+3*a*c^2*d*e*f^3-15*a*c*d^2*e^2*f^2+9*a*d^3*e^3*f+b*c^3*e*f^3-15*b*c^2
*d*e^2*f^2+27*b*c*d^2*e^3*f-13*b*d^3*e^4)/e^2*x^3+1/8*(5*a*c^3*f^4-3*a*c^2
*d*e*f^3-9*a*c*d^2*e^2*f^2+7*a*d^3*e^3*f-b*c^3*e*f^3-9*b*c^2*d*e^2*f^2+21*
b*c*d^2*e^3*f-11*b*d^3*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a*c^3*f^4+3*a*c^2*d*e*
f^3+9*a*c*d^2*e^2*f^2-15*a*d^3*e^3*f+b*c^3*e*f^3+9*b*c^2*d*e^2*f^2-45*b*c*
d^2*e^3*f+35*b*d^3*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.79

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*b*d^3*e^3*f^4*x^7 - 16*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*
e^3*f^4)*x^5 - 2*(175*b*d^3*e^5*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^
3)*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^
5)*x^3 - 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f
+ 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*
e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*
d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^
3*e*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 +
 (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x
- e)/(f*x^2 + e)) - 6*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 +
a*d^3)*e^5*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f
^4)*x)/(e^3*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5), 1/24*(8*b*d^3*e^3*f^4*x^7
- 8*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e^3*f^4)*x^5 - (175*b*d^3*e^5
*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)*e^4*f^3 + 45*(b*c^2*d + a*c*
d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^3 + 3*(35*b*d^3*e^6 + 3*a*
c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2
 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b
*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2
*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5 - 15*(3*b*c*d^2 + a*d^3
)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4...
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(291) = 582$.

Time = 58.36 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.97

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{bd^3x^3}{3f^3} + x\left(\frac{ad^3}{f^3} + \frac{3bcd^2}{f^3} - \frac{3bd^3e}{f^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)\left(3ac^2f^3 + 6acdef^2 + 15ad^2e^2f + bc^2ef^2 + 10bcde^2f - 35bd^2e^3\right)\log\left(-\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf}{3ac^3f^4+3ac^2def^3}\right.}{16}$$
$$+ \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)\left(3ac^2f^3 + 6acdef^2 + 15ad^2e^2f + bc^2ef^2 + 10bcde^2f - 35bd^2e^3\right)\log\left(\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf-}{3ac^3f^4+3ac^2def^3+9}\right.}{16}$$
$$+ \frac{x^3\cdot(3ac^3f^5 + 3ac^2def^4 - 15acd^2e^2f^3 + 9ad^3e^3f^2 + bc^3ef^4 - 15bc^2de^2f^3 + 27bcd^2e^3f^2 - 13bd^3e^4f) +}{8e^4f^4 + 16e^3f^5x^2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
b*d**3*x**3/(3*f**3) + x*(a*d**3/f**3 + 3*b*c*d**2/f**3 - 3*b*d**3*e/f**4)
 - sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(-e**3
*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 1
5*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c
**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b
*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4)
 + x)/16 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**
2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*l
og(e**3*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f
**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)
/(3*a*c**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**
3*f + b*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**
3*e**4) + x)/16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c*d**2*e
**2*f**3 + 9*a*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*d*e**2*f**3 + 27
*b*c*d**2*e**3*f**2 - 13*b*d**3*e**4*f) + x*(5*a*c**3*e*f**4 - 3*a*c**2*d*
e**2*f**3 - 9*a*c*d**2*e**3*f**2 + 7*a*d**3*e**4*f - b*c**3*e**2*f**3 - 9*
b*c**2*d*e**3*f**2 + 21*b*c*d**2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4 +
16*e**3*f**5*x**2 + 8*e**2*f**6*x**4)
```

3.21.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{(35\,bd^3e^4 - 45\,bcd^2e^3f - 15\,ad^3e^3f + 9\,bc^2de^2f^2 + 9\,acd^2e^2f^2 + bc^3ef^3 + 3\,ac^2def^3 + 3\,ac^3f^4)\arctan\left(\frac{f\cdots}{\sqrt{e\cdots}}\right.}{8\sqrt{ef}e^2f^4}$$
$$- \frac{13\,bd^3e^4fx^3 - 27\,bcd^2e^3f^2x^3 - 9\,ad^3e^3f^2x^3 + 15\,bc^2de^2f^3x^3 + 15\,acd^2e^2f^3x^3 - bc^3ef^4x^3 - 3\,ac^2def^4x^3 \cdots}{\cdots}$$
$$+ \frac{bd^3f^6x^3 - 9\,bd^3ef^5x + 9\,bcd^2f^6x + 3\,ad^3f^6x}{3\,f^9}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(35*b*d^3*e^4 - 45*b*c*d^2*e^3*f - 15*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2
+ 9*a*c*d^2*e^2*f^2 + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 3*a*c^3*f^4)*arctan(
f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^4) - 1/8*(13*b*d^3*e^4*f*x^3 - 27*b*c*d^2*
e^3*f^2*x^3 - 9*a*d^3*e^3*f^2*x^3 + 15*b*c^2*d*e^2*f^3*x^3 + 15*a*c*d^2*e^
2*f^3*x^3 - b*c^3*e*f^4*x^3 - 3*a*c^2*d*e*f^4*x^3 - 3*a*c^3*f^5*x^3 + 11*b
*d^3*e^5*x - 21*b*c*d^2*e^4*f*x - 7*a*d^3*e^4*f*x + 9*b*c^2*d*e^3*f^2*x +
9*a*c*d^2*e^3*f^2*x + b*c^3*e^2*f^3*x + 3*a*c^2*d*e^2*f^3*x - 5*a*c^3*e*f^
4*x)/((f*x^2 + e)^2*e^2*f^4) + 1/3*(b*d^3*f^6*x^3 - 9*b*d^3*e*f^5*x + 9*b*
c*d^2*f^6*x + 3*a*d^3*f^6*x)/f^9
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^3\,e\,f^4+3\,a\,c^3\,f^5-15\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4+27\,b\,c\,d^2\,e^3\,f^2-15\,a\,c\,d^2\,e^2\,f^3-13\,b\,d^3\,e^4\,f+9\,a\,d^3\,e^3\,f^2)}{8\,e^2} - \frac{x\,(b\,c^3\,e\,f^3-5\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^{\cdots}}{\cdots}}{e^2\,f^4+2\,e\,f^5\,x^2+f^6\,x^4}$$
$$+ x\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^3} - \frac{3\,b\,d^3\,e}{f^4}\right) + \frac{b\,d^3\,x^3}{3\,f^3}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+10\,b\,c\,d\,e^2\,f+6\,a\,c\,d\,e\,f^2-35\,b\,d^2\,e^3+15\,a\,d^2\,e^2\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+3\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3-45\,b\,c\,d^2\,e^3\,f+9\,a\,c\,d^2\,e^2\,f^2+35\,b\,d^3\,e^4-15\,a\,d^3\,e^3\,f)}\right)(c\,f-d\,e)\,(b\,c^2\,e\,f^{\cdots}}{8\,e^{5/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
((x^3*(3*a*c^3*f^5 + 9*a*d^3*e^3*f^2 + b*c^3*e*f^4 - 13*b*d^3*e^4*f + 3*a*
c^2*d*e*f^4 - 15*a*c*d^2*e^2*f^3 + 27*b*c*d^2*e^3*f^2 - 15*b*c^2*d*e^2*f^3
))/(8*e^2) - (x*(11*b*d^3*e^4 - 5*a*c^3*f^4 - 7*a*d^3*e^3*f + b*c^3*e*f^3
+ 3*a*c^2*d*e*f^3 - 21*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e^2*f
^2))/(8*e))/(e^2*f^4 + f^6*x^4 + 2*e*f^5*x^2) + x*((a*d^3 + 3*b*c*d^2)/f^3
 - (3*b*d^3*e)/f^4) + (b*d^3*x^3)/(3*f^3) + (atan((f^(1/2)*x*(c*f - d*e)*(
3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*e*f^2 + 6*a*c*d*e*f^2
+ 10*b*c*d*e^2*f))/(e^(1/2)*(3*a*c^3*f^4 + 35*b*d^3*e^4 - 15*a*d^3*e^3*f +
 b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 - 45*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*
b*c^2*d*e^2*f^2)))*(c*f - d*e)*(3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*
f + b*c^2*e*f^2 + 6*a*c*d*e*f^2 + 10*b*c*d*e^2*f))/(8*e^(5/2)*f^(9/2))
```

3.21. $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

3.22 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$

3.22.1 Optimal result

Integrand size = 26, antiderivative size = 348

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx &= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))\,x}{48e^3f^4} \\ &\quad - \frac{(be-af)x(c+dx^2)^3}{6ef\,(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2\,(e+fx^2)^2} \\ &\quad - \frac{(be(35d^2e^2-8cdef-3c^2f^2)-af(5d^2e^2+4cdef+15c^2f^2))\,x(c+dx^2)}{48e^3f^3\,(e+fx^2)} \\ &\quad - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}\end{aligned}$$

output
```
1/48*d*(b*e*(-3*c^2*f^2-10*c*d*e*f+105*d^2*e^2)-a*f*(15*c^2*f^2+14*c*d*e*f
+15*d^2*e^2))*x/e^3/f^4-1/6*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^3-1/24*
(b*e*(-c*f+7*d*e)-a*f*(5*c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)^2-1/48*
(b*e*(-3*c^2*f^2-8*c*d*e*f+35*d^2*e^2)-a*f*(15*c^2*f^2+4*c*d*e*f+5*d^2*e^2
))*x*(d*x^2+c)/e^3/f^3/(f*x^2+e)-1/16*(b*e*(-c^3*f^3-3*c^2*d*e*f^2-15*c*d^
2*e^2*f+35*d^3*e^3)-a*f*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))
*arctan(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(9/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{bd^3x}{f^4} + \frac{(be-af)(de-cf)^3x}{6ef^4(e+fx^2)^3} - \frac{(de-cf)^2(be(19de-cf)-af(13de+5cf))x}{24e^2f^4(e+fx^2)^2}$$
$$+ \frac{(de-cf)(be(29d^2e^2-4cdef-c^2f^2)-af(11d^2e^2+8cdef+5c^2f^2))x}{16e^3f^4(e+fx^2)}$$
$$- \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2)^3) - ((
d*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))*x)/(24*e^2*f^4*(e
 + f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 - 4*c*d*e*f - c^2*f^2) - a*f*
(11*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*
(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 +
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(16*e^(7/2)*f^(9/2))
```

3.22.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {401, 25, 401, 25, 401, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$\Big\downarrow \text{ 401}$$
$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-af)x^2+c(be+5af))}{(fx^2+e)^3}dx}{6ef} - \frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{(dx^2+c)^2(d(7be-af)x^2+c(be+5af))}{(fx^2+e)^3}dx}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 401$$

$$\frac{-\frac{\int -\frac{(dx^2+c)(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af)))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{(dx^2+c)(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af)))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 401$$

$$\frac{\frac{-\frac{\int \frac{c(af(5d^2e^2+6cdfe-15c^2f^2)-be(35d^2e^2+6cdfe+3c^2f^2))-d(be(105d^2e^2-10cdfe-3c^2f^2)-af(15d^2e^2+14cdfe+15c^2f^2))x^2}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)(be(-3c^2f^2-}{}}{4ef}}{6ef}$$

$$-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 299$$

$$\frac{\frac{-\frac{\frac{3(be(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3)-af(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3))\int\frac{1}{fx^2+e}dx}{f}-\frac{dx(be(-3c^2f^2-10cdef+105d^2e^2)-af(15c^2f^2+14cdef+1}{f}}{2ef}}{4ef}}{6ef}$$

$$-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 218$$

3.22. $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$

$$-\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-af\left(15c^2f^2+14cdef\right.\right.}{f}}{2ef}}{4ef}$$

$$6ef$$

$$\frac{x\left(c+dx^2\right)^3(be-af)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
-1/6*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^3) + (-1/4*((b*e*(7*d*
e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2
*((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f*(5*d^2*e^2 + 4*c*d*e*f +
 15*c^2*f^2))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b*e*(105*d^2*e^2 -
 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))*x)/
f) + (3*(b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f
*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*
x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f))/(6*e*f)
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.22.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.20

method	result
default	$\frac{b d^3 x}{f^4} + \frac{\frac{f^2\left(5a\,c^3 f^4+3a\,c^2 de\,f^3+3ac\,d^2 e^2 f^2-11a\,d^3 e^3 f+b\,c^3 e\,f^3+3b\,c^2 d\,e^2 f^2-33bc\,d^2 e^3 f+29b\,d^3 e^4\right) x^5}{16e^3} + \frac{f\left(5a\,c^3 f^4+3a\,c^2 de\,f^3-3ac\,d^2 e^2 f^2 \ldots\right.}{\ldots}}{\ldots}$
risch	$\frac{b\,d^3 x}{f^4} + \frac{\frac{f^2\left(5a\,c^3 f^4+3a\,c^2 de\,f^3+3ac\,d^2 e^2 f^2-11a\,d^3 e^3 f+b\,c^3 e\,f^3+3b\,c^2 d\,e^2 f^2-33bc\,d^2 e^3 f+29b\,d^3 e^4\right) x^5}{16e^3} + \frac{f\left(5a\,c^3 f^4+3a\,c^2 de\,f^3-3ac\,d^2 e^2 f^2 \ldots\right.}{\ldots}}{\ldots}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
b*d^3/f^4*x+1/f^4*((1/16*f^2*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^
2-11*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-33*b*c*d^2*e^3*f+29*b*d^3*e
^4)/e^3*x^5+1/6*f*(5*a*c^3*f^4+3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e
^3*f+b*c^3*e*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+17*b*d^3*e^4)/e^2*x^3+
1/16*(11*a*c^3*f^4-3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*f-b*c^3*e
*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+19*b*d^3*e^4)/e*x)/(f*x^2+e)^3+1/1
6*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3
+3*b*c^2*d*e^2*f^2+15*b*c*d^2*e^3*f-35*b*d^3*e^4)/e^3/(e*f)^(1/2)*arctan(f
*x/(e*f)^(1/2)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. 2(330) = 660.

Time = 0.29 (sec) , antiderivative size = 1422, normalized size of antiderivative = 4.09

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(96*b*d^3*e^4*f^4*x^7 + 6*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*
b*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^
2*d)*e^2*f^6)*x^5 + 16*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2
+ a*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3
*f^5)*x^3 + 3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*
f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^
3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c
*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a
*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4
 - (b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5
- 5*(3*b*c*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 +
 3*a*c^2*d)*e^3*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x
^2 + e)) + 6*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^
6*f^2 - 3*(b*c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e
^4*f^8*x^6 + 3*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5), 1/48*(48*b*d^3*e^4*
f^4*x^7 + 3*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b*c*d^2 + a*d^3)*e^4
*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*d)*e^2*f^6)*x^5 +
8*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^3 - 3*
(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5)*x^3 - 3*(35*b*d
^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3*(b*c^2*d + a...
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Timed out
```

3.22.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \frac{bd^3x}{f^4} - \frac{(35\,bd^3e^4 - 15\,bcd^2e^3f - 5\,ad^3e^3f - 3\,bc^2de^2f^2 - 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 - 5\,ac^3f^4)\arctan\left(\ldots\right.}{16\,\sqrt{ef}e^3f^4} + \frac{87\,bd^3e^4f^2x^5 - 99\,bcd^2e^3f^3x^5 - 33\,ad^3e^3f^3x^5 + 9\,bc^2de^2f^4x^5 + 9\,acd^2e^2f^4x^5 + 3\,bc^3ef^5x^5 + 9\,ac^2def^5x\ldots}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
b*d^3*x/f^4 - 1/16*(35*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f - 3*b*
c^2*d*e^2*f^2 - 3*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 5*a*c^
3*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^4) + 1/48*(87*b*d^3*e^4*f^2*
x^5 - 99*b*c*d^2*e^3*f^3*x^5 - 33*a*d^3*e^3*f^3*x^5 + 9*b*c^2*d*e^2*f^4*x^
5 + 9*a*c*d^2*e^2*f^4*x^5 + 3*b*c^3*e*f^5*x^5 + 9*a*c^2*d*e*f^5*x^5 + 15*a
*c^3*f^6*x^5 + 136*b*d^3*e^5*f*x^3 - 120*b*c*d^2*e^4*f^2*x^3 - 40*a*d^3*e^
4*f^2*x^3 - 24*b*c^2*d*e^3*f^3*x^3 - 24*a*c*d^2*e^3*f^3*x^3 + 8*b*c^3*e^2*
f^4*x^3 + 24*a*c^2*d*e^2*f^4*x^3 + 40*a*c^3*e*f^5*x^3 + 57*b*d^3*e^6*x - 4
5*b*c*d^2*e^5*f*x - 15*a*d^3*e^5*f*x - 9*b*c^2*d*e^4*f^2*x - 9*a*c*d^2*e^4
*f^2*x - 3*b*c^3*e^3*f^3*x - 9*a*c^2*d*e^3*f^3*x + 33*a*c^3*e^2*f^4*x)/((f
*x^2 + e)^3*e^3*f^4)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$

$$= \frac{\frac{x^3\,(b\,c^3\,e\,f^4+5\,a\,c^3\,f^5-3\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4-15\,b\,c\,d^2\,e^3\,f^2-3\,a\,c\,d^2\,e^2\,f^3+17\,b\,d^3\,e^4\,f-5\,a\,d^3\,e^3\,f^2)}{6\,e^2}+\frac{x^5\,(b\,c^3\,e\,f^5+5\,a\,c^3\,f^6+3\,b\,c^2\,d\,e^2\,\dots}{\dots}}{e^3\,f^4\,\dots}$$

$$+\frac{b\,d^3\,x}{f^4}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^3\,e\,f^3+5\,a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2-35\,b\,d^3\,e^4\,\dots}{16\,e^{7/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
((x^3*(5*a*c^3*f^5 - 5*a*d^3*e^3*f^2 + b*c^3*e*f^4 + 17*b*d^3*e^4*f + 3*a*
c^2*d*e*f^4 - 3*a*c*d^2*e^2*f^3 - 15*b*c*d^2*e^3*f^2 - 3*b*c^2*d*e^2*f^3))
/(6*e^2) + (x^5*(5*a*c^3*f^6 - 11*a*d^3*e^3*f^3 + 29*b*d^3*e^4*f^2 + b*c^3
*e*f^5 + 3*a*c^2*d*e*f^5 + 3*a*c*d^2*e^2*f^4 - 33*b*c*d^2*e^3*f^3 + 3*b*c^
2*d*e^2*f^4))/(16*e^3) - (x*(5*a*d^3*e^3*f - 19*b*d^3*e^4 - 11*a*c^3*f^4 +
 b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*
b*c^2*d*e^2*f^2))/(16*e))/(e^3*f^4 + f^7*x^6 + 3*e*f^6*x^4 + 3*e^2*f^5*x^2
) + (b*d^3*x)/f^4 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c^3*f^4 - 35*b*d^3*e^4
 + 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*
c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2))/(16*e^(7/2)*f^(9/2))
```

3.23 $\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx$

3.23.1 Optimal result

Integrand size = 30, antiderivative size = 544

$$\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx = -\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))x\sqrt{c+dx^2}}{105d^2f^2\sqrt{e+fx^2}} + \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} + \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} + \frac{\sqrt{e}(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{105d^2f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{e^{3/2}(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2))\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-b*(-6*c^3*f^3+9*c^2*d*e*f
^2-19*c*d^2*e^2*f+8*d^3*e^3))*x*(d*x^2+c)^(1/2)/d^2/f^2/(f*x^2+e)^(1/2)-1/
105*e^(3/2)*(7*a*d*f*(-9*c*f+d*e)-b*(-3*c^2*f^2-9*c*d*e*f+4*d^2*e^2))*(1/(
1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e
)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2
+e))^(1/2)/(f*x^2+e)^(1/2)+1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)
-b*(-6*c^3*f^3+9*c^2*d*e*f^2-19*c*d^2*e^2*f+8*d^3*e^3))*(1/(1+f*x^2/e))^(1
/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e
/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)/(f*x^2+e)^(1/2)+1/35*(7*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(3/2)*(f*x
^2+e)^(1/2)/d/f+1/7*b*x*(d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/d+1/105*(7*a*d*f*(
3*c*f+d*e)-b*(6*c^2*f^2-6*c*d*e*f+4*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/d/f^2
```

3.23.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.69

$$\int (a+bx^2)\,(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)\,(e+fx^2)\,(7adf(6cf+d(e+3fx^2))+b(3c^2f^2+3cdf(3e+8fx^2)}{}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(6*c*f + d*(e + 3*f*x^2))
+ b*(3*c^2*f^2 + 3*c*d*f*(3*e + 8*f*x^2) + d^2*(-4*e^2 + 3*e*f*x^2 + 15*f^
2*x^4))) + I*e*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + b*(-8*d^3*e^
3 + 19*c*d^2*e^2*f - 9*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e
) + c*f)*(-14*a*d*f*(d*e - 3*c*f) + b*(8*d^2*e^2 - 15*c*d*e*f + 3*c^2*f^2)
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)])/(105*d*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.23.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {403, 25, 403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int -\frac{\left(dx^2+c\right)^{3/2}\left((bc-7ad)e-(bde-2bcf+7adf)x^2\right)}{\sqrt{fx^2+e}}dx}{7d}+\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}-\frac{\int \frac{\left(dx^2+c\right)^{3/2}\left((bc-7ad)e-(bde-2bcf+7adf)x^2\right)}{\sqrt{fx^2+e}}dx}{7d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}-\frac{\frac{\int \frac{\sqrt{dx^2+c}\left(ce(bde+3bcf-28adf)-\left(7adf(de+3cf)-b\left(4d^2e^2-6cdfe+6c^2f^2\right)\right)x^2\right)}{\sqrt{fx^2+e}}dx}{5f}-\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-2bcf+bde)}{5f}}{7d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}-\frac{\frac{\frac{\int \frac{\left(7adf\left(2d^2e^2-7cdfe-3c^2f^2\right)-b\left(8d^3e^3-19cd^2fe^2+9c^2df^2e-6c^3f^3\right)\right)x^2+ce\left(7adf(de-9cf)-b\left(4d^2e^2-9cdfe-3c^2f^2\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(7adf(3cf+de)\cdots\right.}{3f}}{5f}}{7d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}-\frac{\frac{\frac{ce\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx\cdots}}}{3f}}{5f}}{7d}$$

$$\Big\downarrow \; 320$$

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} - \frac{\frac{\frac{\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)\text{Elli}}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3f}}{5f}}{7d}$$

$\downarrow$ 388

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} - \frac{\frac{\frac{\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef-\right.\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e}{c}}}}{3f}}{5f}}{7d}$$

$\downarrow$ 313

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} - \frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8\right.\right.}{3f}}{5f}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

output
```
(b*x*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(7*d) - (-1/5*((b*d*e - 2*b*c*f +
7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/f + (-1/3*((7*a*d*f*(d*e + 3
*c*f) - b*(4*d^2*e^2 - 6*c*d*e*f + 6*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2])/f + ((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 -
 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*((x*Sqrt[c + d*x^2])/(d*Sqrt
[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])) + (e^(3/2)*(7*a*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*d*e
*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2]))/(3*f))/(5*f))/(7*d)
```

3.23. $\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx$

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.23.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b d x^5 \sqrt{d f x^4+c f x^2+d e x^2+c e}}{7}+\frac{\left(a d^2 f+2 b c f d+b d^2 e-\frac{b d(6 c f+6 d e)}{7}\right) x^3 \sqrt{d f x^4+c f x^2+d e x^2+c e}}{5 d f}+\frac{\left(2 a c d f+a e d^2+c^2 b\right.}{}\right.}{}$
risch	$\frac{x(15 b x^4 d^2 f^2+21 a d^2 f^2 x^2+24 b c d f^2 x^2+3 b d^2 e f x^2+42 a c d f^2+7 a d^2 e f+3 b c^2 f^2+9 b c d e f-4 b d^2 e^2)\sqrt{d x^2+c}\sqrt{f x^2+e}}{105 d f^2}+\frac{\left(-\frac{3 b c^3}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*d*x^5*(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*
(6*c*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(2*a*c*d*f+
a*e*d^2+c^2*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+
6*d*e))/d/f*(4*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^2*
a*e-1/3*(2*a*c*d*f+a*e*d^2+c^2*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^
2*e-1/7*b*d*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(c^2*a*f+2*a*c*d*e+b*c^2*e-3/5*
(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+6*d*e))/d/f*c*e-1/3*(2*a*c*d*f+a
*e*d^2+c^2*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+6
*d*e))/d/f*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(
-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx =$$

$$-\frac{(8\,bd^3e^4-(19\,bcd^2+14\,ad^3)e^3f+(9\,bc^2d+49\,acd^2)e^2f^2-3\,(2\,bc^3-7\,ac^2d)ef^3)\sqrt{df}\,x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{}}{}\right.}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*((8*b*d^3*e^4 - (19*b*c*d^2 + 14*a*d^3)*e^3*f + (9*b*c^2*d + 49*a*c
*d^2)*e^2*f^2 - 3*(2*b*c^3 - 7*a*c^2*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elli
ptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (8*b*d^3*e^4 - (19*b*c*d^2 + 14*
a*d^3)*e^3*f + (9*b*c^2*d + (49*a + 4*b)*c*d^2)*e^2*f^2 - (6*b*c^3 - 3*(7*
a - 3*b)*c^2*d + 7*a*c*d^2)*e*f^3 - 3*(b*c^3 - 21*a*c^2*d)*f^4)*sqrt(d*f)*
x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (15*b*d^3*f^4*x
^6 + 8*b*d^3*e^3*f - (19*b*c*d^2 + 14*a*d^3)*e^2*f^2 + (9*b*c^2*d + 49*a*c
*d^2)*e*f^3 - 3*(2*b*c^3 - 7*a*c^2*d)*f^4 + 3*(b*d^3*e*f^3 + (8*b*c*d^2 +
7*a*d^3)*f^4)*x^4 - (4*b*d^3*e^2*f^2 - (9*b*c*d^2 + 7*a*d^3)*e*f^3 - 3*(b*
c^2*d + 14*a*c*d^2)*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^2*f^4*x)
```

3.23.6 Sympy [F]

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx = \int \left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

3.23.7 Maxima [F]

$$\int (a+bx^2)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}\,dx = \int (bx^2+a)\,(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

3.23.8 Giac [F]

$$\int (a+bx^2)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}\,dx = \int (bx^2+a)\,(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}\,dx = \int (b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2), x)
```

3.24 $\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

3.24.1 Optimal result

Integrand size = 30, antiderivative size = 381

$$\begin{aligned}
&\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx \\
&= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} \\
&\quad + \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} \\
&\quad - \frac{\sqrt{e}(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{e^{3/2}(bde+bcf-10adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*x*(d*x^2+c)^(1/2)/d
^2/f/(f*x^2+e)^(1/2)-1/15*e^(3/2)*(-10*a*d*f+b*c*f+b*d*e)*(1/(1+f*x^2/e))^
(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d
*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(
f*x^2+e)^(1/2)-1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*(1/(
1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e
)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(3/2)/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/
2)/d+1/15*(5*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$
$$= \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(bcf+5adf+bd(e+3fx^2))+ie(-5adf(de+cf)+2b(d^2e^2-cdef+c^2f^2))\sqrt{}}{}$$

input
```
Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(b*c*f + 5*a*d*f + b*d*(e + 3*f*x^2
)) + I*e*(-5*a*d*f*(d*e + c*f) + 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[1
 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*e*(-(d*e) + c*f)*(-2*b*d*e + b*c*f + 5*a*d*f)*Sqrt[1 + (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15
*d*Sqrt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.24.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{\sqrt{dx^2+c}((bc-5ad)e-(bde-2bcf+5adf)x^2)}{\sqrt{fx^2+e}}dx}{5d}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\int \frac{\sqrt{dx^2+c}((bc-5ad)e-(bde-2bcf+5adf)x^2)}{\sqrt{fx^2+e}}dx}{5d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\frac{\int \frac{ce(bde+bcf-10adf)-\left(5adf(de+cf)-2b\left(d^2e^2-cdfe+c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-2bcf+bde)}{3f}}{5d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\frac{ce(-10adf+bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-\left(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-2bcf+bde}{3f}}{5d}$$

$$\Big\downarrow \; 320$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\left(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\cdots}{3f}}{5d}$$

$$\Big\downarrow \; 388$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\left(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)}{3f}-\frac{x\sqrt{c+\cdots}}{\cdots}}{5d}$$

$$\Big\downarrow \; 313$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\left(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\cdots\right.}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\cdots\right.}{3f}}{5d}$$

input
```
Int[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

3.24. $\int \left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

output
```
(b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*d) - (-1/3*((b*d*e - 2*b*c*f +
5*a*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/f + (-((5*a*d*f*(d*e + c*f) -
2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]
) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2]))) + (e^(3/2)*(b*d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f))/(5*d)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.24.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b x^3\sqrt{df x^4+cf x^2+de x^2+ce}}{5}+\frac{\left(adf+bcf+bde-\frac{b(4cf+4de)}{5}\right)x\sqrt{df x^4+cf x^2+de x^2+ce}}{3df}+\frac{\left(ace-\frac{\left(adf+bcf+bde-\frac{b(4cf}{}\right.}{3df}\right.}{}\right.}{}$
risch	$\frac{x(3bdf x^2+5adf+bcf+bde)\sqrt{d x^2+c}\sqrt{f x^2+e}}{15df}+\frac{\left(-\frac{b c^2 ef\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf x^2+de x^2+ce}}-\frac{bcd e^2\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F(}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf}}\right.}{}$
default	$\frac{\sqrt{d x^2+c}\sqrt{f x^2+e}\left(3\sqrt{-\frac{d}{c}}\,b d^2 f^3 x^7+5\sqrt{-\frac{d}{c}}\,a d^2 f^3 x^5+4\sqrt{-\frac{d}{c}}\,bcd f^3 x^5+4\sqrt{-\frac{d}{c}}\,b d^2 e f^2 x^5+5\sqrt{-\frac{d}{c}}\,acd f^3 x^3+5\sqrt{-\frac{d}{c}}\,a d^2 e f^2 x^3-\right.}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*x^3*(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*d*f+b*c*f+b*d*e-1/5*b*(4*c*f+4*d*e
))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c*e-1/3*(a*d*f+b*c*f+b*d*e
-1/5*b*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*
f+d*e)/e/d)^(1/2))-(a*c*f+a*d*e+2/5*b*c*e-1/3*(a*d*f+b*c*f+b*d*e-1/5*b*(4*
c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))
))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.80

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$= \frac{(2\,bd^2e^3-(2\,bcd+5\,ad^2)e^2f+(2\,bc^2-5\,acd)ef^2)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(2\,bd^2e^3-(2\,bcd}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
1/15*((2*b*d^2*e^3 - (2*b*c*d + 5*a*d^2)*e^2*f + (2*b*c^2 - 5*a*c*d)*e*f^2
)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (2*
b*d^2*e^3 - (2*b*c*d + 5*a*d^2)*e^2*f + (2*b*c^2 - (5*a - b)*c*d)*e*f^2 +
(b*c^2 - 10*a*c*d)*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f
)/x), c*f/(d*e)) + (3*b*d^2*f^3*x^4 - 2*b*d^2*e^2*f + (2*b*c*d + 5*a*d^2)*
e*f^2 - (2*b*c^2 - 5*a*c*d)*f^3 + (b*d^2*e*f^2 + (b*c*d + 5*a*d^2)*f^3)*x^
2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^2*f^3*x)
```

3.24.6 Sympy [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

3.24.7 Maxima [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

3.24.8 Giac [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int (bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int (b\,x^2+a)\ \sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

3.25 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

3.25.1 Optimal result

Integrand size = 30, antiderivative size = 283

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} - \frac{\sqrt{e}(bde-2bcf+3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(bc-3ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(3*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/d^2/(f*x^2+e)^(1/2)-1/3*(-3*
a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/
2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2
)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(3*a*d*f-2*b*c*f+b*d
*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1
+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(1/2)/(e*
(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)/d
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)+ie(-bde+2bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ibe(-}{3d\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) + I*e*(-(b*d*e) + 2*b*c*f - 3*a*d
*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*d*Sqrt[d/c]*f*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

3.25.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(bc-3ad)e-(bde-2bcf+3adf)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{\int \frac{(bc-3ad)e-(bde-2bcf+3adf)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}$$

$$\Big\downarrow \text{406}$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{e(bc-3ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-(3adf-2bcf+bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}$$

$\Big\downarrow$ 320

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-(3adf-2bcf+bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}$$

$\Big\downarrow$ 388

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-(3adf-2bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)}{3d}$$

$\Big\downarrow$ 313

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-(3adf-2bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{3d}$$

input
```
Int[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (-((b*d*e - 2*b*c*f + 3*a*d*
f)*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Ell
ipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))) + ((b*c - 3*a*d)*e^(3/2)*S
qrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c
*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*d)
```

3.25. $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.25. $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

3.25.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{bx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3d}+\frac{\left(ae-\frac{ceb}{3d}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{\left(af+be-\frac{b(2cf+2de)}{3d}\right)e\sqrt{1+\cdots}}{\cdots}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$
risch	$\frac{bx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3d}+\frac{\left(\frac{3ade\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{bce\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(3adf-\cdots}{\cdots}\right.}{3d\sqrt{d\,x^2+c}\cdots}$
default	$\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{d}{c}}\,bd\,f^2x^5+\sqrt{-\frac{d}{c}}\,bc\,f^2x^3+\sqrt{-\frac{d}{c}}\,bdef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bcef-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*b/d*x*(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*e-1/3*c/d*e*b)/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(a*f+b*e-1/3*b/d*(2*c*f+2*d*e))*e/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{(bde^2-(2\,bc-3\,ad)ef)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(bde^2-(2\,bc-3\,ad)ef-(bc-3\,ad)f^2)\sqrt{\cdots}}{3\,d^2f^2x}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*d*e^2 - (2*b*c - 3*a*d)*e*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(ar
csin(sqrt(-e/f)/x), c*f/(d*e)) - (b*d*e^2 - (2*b*c - 3*a*d)*e*f - (b*c - 3
*a*d)*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*
e)) - (b*d*f^2*x^2 + b*d*e*f - (2*b*c - 3*a*d)*f^2)*sqrt(d*x^2 + c)*sqrt(f
*x^2 + e))/(d^2*f^2*x)
```

3.25.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

3.25.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

3.25.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.26 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

3.26.1 Optimal result

Integrand size = 30, antiderivative size = 271

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
(-a*d+2*b*c)*f*x*(d*x^2+c)^(1/2)/c/d^2/(f*x^2+e)^(1/2)+b*e^(3/2)*(1/(1+f*x
^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/
2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e)
)^(1/2)/(f*x^2+e)^(1/2)-(-a*d+2*b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/
2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2
)*f^(1/2)*(d*x^2+c)^(1/2)/c/d^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^
(1/2)-(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(1/2)
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{-i(2bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-(bc-ad)\left(\sqrt{\frac{d}{c}}x(e\right.}{c^2\left(\frac{d}{c}\right)^{3/2}\sqrt{c+dx^2}\sqrt{}}$$

input
```
Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
((-I)*(2*b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (b*c - a*d)*(Sqrt[d/c]*x*(e + f*x^2)
- I*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)]))/(c^2*(d/c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.26.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(2bc-ad)fx^2+bce}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(2bc-ad)fx^2+bce}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{f(2bc-ad)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+bce\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 320

$$\frac{f(2bc-ad)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 388

$$\frac{f(2bc-ad)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 313

$$\frac{f(2bc-ad)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a*d)*
f*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (b*e^(3/2)*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(c*d)
```

3.26. $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.26. $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

3.26.4 Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{d}{c}}\,adf\,x^3-\sqrt{-\frac{d}{c}}\,bcf\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)b\right.}{d(df\,x^4+cf\,x^2+de\,x^2+c}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+de\right)(ad-bc)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{(ad-bc)(cf-de)}{d^2c}-\frac{e(ad-bc)}{dc}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+d}{ed}}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

input
```
int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*a*d*f*x^3-(-d/c)^(1/2)*b*c*f
*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e+(-d/c)^(1/2
)*a*d*e*x-(-d/c)^(1/2)*b*c*e*x)/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/c/(-d/c)^(
1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,bcd-ad^2)ex^3+(2\,bc^2-acd)ex)\sqrt{df}\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-((bcdf+(2\,bcd-ad^2)e)x^3+(bc^2}{cd^3fx^3+c}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-(((2*b*c*d - a*d^2)*e*x^3 + (2*b*c^2 - a*c*d)*e*x)*sqrt(d*f)*sqrt(-e/f)*e
lliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((b*c*d*f + (2*b*c*d - a*d^2)
*e)*x^3 + (b*c^2*f + (2*b*c^2 - a*c*d)*e)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic
_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (b*c*d*f*x^2 + (2*b*c^2 - a*c*d)*f)*
sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c*d^3*f*x^3 + c^2*d^2*f*x)
```

3.26.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

3.26.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

3.26.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

3.27 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

3.27.1 Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}} + \frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1
/2)/c^2/d/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(
-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(3/2)+1/3*(d*(2*a*d+b*c)*e-c*(a*
d+2*b*c)*f)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/d^(3/2)
/(-c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(e+fx^2)\left(ad(-3cde+2c^2f-2d^2ex^2+cdfx^2)+bc(c^2f-d^2ex^2+2cdfx\right.}{\ldots}$$

input
```
Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[d/c]*x*(e + f*x^2)*(a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^
2) + b*c*(c^2*f - d^2*e*x^2 + 2*c*d*f*x^2)) + I*e*(a*d*(-2*d*e + c*f) + b*
c*(-(d*e) + 2*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c + 2*a*d)*e*(-(d*e) +
 c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*c^3*(d/c)^(3/2)*(-(d*e) + c*f)*(c + d*x
^2)^(3/2)*Sqrt[e + f*x^2])
```

3.27.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{(2bc+ad)fx^2+(bc+2ad)e}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(2bc+ad)fx^2+(bc+2ad)e}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{ef(bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{(de(2ad+bc)-cf(ad+2bc))\int \frac{\sqrt{fx^2+e}}{(dx^2+c)^{3/2}}dx}{de-cf}}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{ef(bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

input
```
Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(3/2)) + (((d*(b*c +
 2*a*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]
*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(
d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c*d)
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.27.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{(ad-bc)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3c\,d^3\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df\,x^2+de\right)x\left(acdf-2ae\,d^2+2c^2bf-bcde\right)}{3d^2c^2(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\left(\frac{bf}{d^2}+\frac{(ad-bc)f}{3d^2c}-\frac{acdf-2ae\,d^2+2c^2bf}{3d^2c^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*(a*d-b*c)
/c/d^3*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)
/d^2/c^2/(c*f-d*e)*x*(a*c*d*f-2*a*d^2*e+2*b*c^2*f-b*c*d*e)/((x^2+c/d)*(d*f
*x^2+d*e))^(1/2)+(b*f/d^2+1/3*(a*d-b*c)/d^2*f/c-1/3/d^2*(a*c*d*f-2*a*d^2*e
+2*b*c^2*f-b*c*d*e)/c^2-1/3/d*e/c^2/(c*f-d*e)*(a*c*d*f-2*a*d^2*e+2*b*c^2*f
-b*c*d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1
/3/d*(a*c*d*f-2*a*d^2*e+2*b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)*e/(-d/c)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(Ell
ipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(
-1+(c*f+d*e)/e/d)^(1/2))))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{(((bcd^4+2\,ad^5)e-(2\,bc^2d^3+acd^4)f)x^4+2\,((bc^2d^3+2\,acd^4)e-(2\,bc^3d^2+ac^2d^3)f)x^2+(bc^3d^2+2\,ac^2d^{\cdots}}{\cdots}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((((b*c*d^4 + 2*a*d^5)*e - (2*b*c^2*d^3 + a*c*d^4)*f)*x^4 + 2*((b*c^2
*d^3 + 2*a*c*d^4)*e - (2*b*c^3*d^2 + a*c^2*d^3)*f)*x^2 + (b*c^3*d^2 + 2*a*
c^2*d^3)*e - (2*b*c^4*d + a*c^3*d^2)*f)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(ar
csin(x*sqrt(-d/c)), c*f/(d*e)) - (((b*c*d^4 + 2*a*d^5)*e - (b*c^3*d^2 - (a
 - 2*b)*c^2*d^3 + a*c*d^4)*f)*x^4 + 2*((b*c^2*d^3 + 2*a*c*d^4)*e - (b*c^4*
d - (a - 2*b)*c^3*d^2 + a*c^2*d^3)*f)*x^2 + (b*c^3*d^2 + 2*a*c^2*d^3)*e -
(b*c^5 - (a - 2*b)*c^4*d + a*c^3*d^2)*f)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(a
rcsin(x*sqrt(-d/c)), c*f/(d*e)) - (((b*c^2*d^3 + 2*a*c*d^4)*e - (2*b*c^3*d
^2 + a*c^2*d^3)*f)*x^3 + (3*a*c^2*d^3*e - (b*c^4*d + 2*a*c^3*d^2)*f)*x)*sq
rt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^5*d^3*e - c^6*d^2*f + (c^3*d^5*e - c^4*d
^4*f)*x^4 + 2*(c^4*d^4*e - c^5*d^3*f)*x^2)
```

3.27.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

3.27.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

3.27.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

3.28 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

3.28.1 Optimal result

Integrand size = 30, antiderivative size = 385

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}}$$
$$+\frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}}$$
$$+\frac{(2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2e^2-13cdef+3c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{e^{3/2}\sqrt{f}(2ad(2de-3cf)+bc(de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/15*e^(3/2)*(2*a*d*(-3*c*f+2*d*e)+b*c*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(
1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)
^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/d/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e
))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(5
/2)+1/15*(a*d*(-3*c*f+4*d*e)+b*c*(-2*c*f+d*e))*x*(f*x^2+e)^(1/2)/c^2/d/(-c
*f+d*e)/(d*x^2+c)^(3/2)+1/15*(2*b*c*(c^2*f^2-c*d*e*f+d^2*e^2)+a*d*(3*c^2*f
^2-13*c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
E(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c
^(5/2)/d^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2
)
```

3.28.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(ad(4de-3cf)+bc(de\right.}{}$$

input
```
Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(d*e - c*f
)*(a*d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*(c + d*x^2) - (2*b*c*(d^2*e^2
- c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*(c + d*x^
2)^2)) + I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((2*b*c
*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))
*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(b*c*(-2*
d*e + c*f) + a*d*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)]))/(15*c^4*(d/c)^(3/2)*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x
^2])
```

3.28.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {401, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{(2bc+3ad)fx^2+(bc+4ad)e}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(2bc+3ad)fx^2+(bc+4ad)e}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(3ad+2bc))}{3c(c+dx^2)^{3/2}(de-cf)}-\frac{\int -\frac{f(ad(4de-3cf)+bc(de-2cf))x^2+e(ad(8de-9cf)+bc(2de-cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{f(ad(4de-3cf)+bc(de-2cf))x^2+e(ad(8de-9cf)+bc(2de-cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(3ad+2bc))}{3c(c+dx^2)^{3/2}(de-cf)}}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{ef(2ad(2de-3cf)+bc(cf+de))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(3ad+2bc))}{3c(c+dx^2)^{3/2}(de-cf)}}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{ef(2ad(2de-3cf)+bc(cf+de))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(3ad+2bc))}{3c(c+dx^2)^{3/2}(de-cf)}}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 320$$

3.28. $\int \frac{\left(a+bx^2\right)\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}dx$

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2de-3cf)+bc(cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\right.}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}}{5cd}$$

$$\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

input
```
Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
-1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(5/2)) + (((d*(b*c +
 4*a*d)*e - c*(2*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c +
d*x^2)^(3/2)) + (((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[
(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c*f
) + b*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2]))/(3*c*(d*e - c*f)))/(5*c*d)
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.28.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.96

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{(ad-bc)x\sqrt{df x^4+cf x^2+de x^2+ce}}{5c d^4\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(3acdf-4ae d^2+2c^2bf-bcde\right)x\sqrt{df x^4+cf x^2+de x^2+ce}}{15d^3c^2(cf-de)\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df x^2+de\right)x\left(3a c^2 d f^2-\right.}{15d^2c^3}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*(a*d-b*c)
/c/d^4*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3+1/15*(3*a*c*d*f-4
*a*d^2*e+2*b*c^2*f-b*c*d*e)/d^3/c^2/(c*f-d*e)*x*(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)/(x^2+c/d)^2+1/15*(d*f*x^2+d*e)/d^2/c^3/(c*f-d*e)^2*x*(3*a*c^2*d*
f^2-13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^2*e^2)/((
x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(1/15*f*(3*a*c*d*f-4*a*d^2*e+2*b*c^2*f-b*c*d
*e)/c^2/(c*f-d*e)/d^2-1/15/d^2/(c*f-d*e)*(3*a*c^2*d*f^2-13*a*c*d^2*e*f+8*a
*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^2*e^2)/c^3-1/15/d*e/c^3/(c*f-d*
e)^2*(3*a*c^2*d*f^2-13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2
*b*c*d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))+1/15/d*(3*a*c^2*d*f^2-13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*
e*f+2*b*c*d^2*e^2)/c^3/(c*f-d*e)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(367) = 734.

Time = 0.13 (sec) , antiderivative size = 1105, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^2 - (2*b*c^2*d^5 + 13*a*c*d^6)*e*f + (2*b
*c^3*d^4 + 3*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 - (2*b
*c^3*d^4 + 13*a*c^2*d^5)*e*f + (2*b*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^4 + 2*(b
*c^4*d^3 + 4*a*c^3*d^4)*e^2 - (2*b*c^5*d^2 + 13*a*c^4*d^3)*e*f + (2*b*c^6*
d + 3*a*c^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 - (2*b*c^4*d^3 +
 13*a*c^3*d^4)*e*f + (2*b*c^5*d^2 + 3*a*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(
-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c*d^6 + 4*a*d^7
)*e^2 + (b*c^3*d^4 + 2*(2*a - b)*c^2*d^5 - 13*a*c*d^6)*e*f + (b*c^4*d^3 -
2*(3*a - b)*c^3*d^4 + 3*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)
*e^2 + (b*c^4*d^3 + 2*(2*a - b)*c^3*d^4 - 13*a*c^2*d^5)*e*f + (b*c^5*d^2 -
 2*(3*a - b)*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)
*e^2 + (b*c^6*d + 2*(2*a - b)*c^5*d^2 - 13*a*c^4*d^3)*e*f + (b*c^7 - 2*(3*
a - b)*c^6*d + 3*a*c^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + (b*
c^5*d^2 + 2*(2*a - b)*c^4*d^3 - 13*a*c^3*d^4)*e*f + (b*c^6*d - 2*(3*a - b)
*c^5*d^2 + 3*a*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x
*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 - (2*b*c^3*d^4
+ 13*a*c^2*d^5)*e*f + (2*b*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^5 + (5*(b*c^3*d^4
 + 4*a*c^2*d^5)*e^2 - (7*b*c^4*d^3 + 33*a*c^3*d^4)*e*f + 3*(2*b*c^5*d^2 +
3*a*c^4*d^3)*f^2)*x^3 + (15*a*c^3*d^4*e^2 + (b*c^5*d^2 - 26*a*c^4*d^3)*e*f
 + (b*c^6*d + 9*a*c^5*d^2)*f^2)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^...
```

3.28.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(7/2), x)
```

3.28.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

3.28.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

3.29 $\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx$

3.29.1 Optimal result

Integrand size = 30, antiderivative size = 543

$$
\begin{aligned}
&\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\,x\sqrt{c+dx^2}}{105d^3f\sqrt{e+fx^2}} \\
&+\frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} \\
&+\frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2}+\frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} \\
&-\frac{\sqrt{e}(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{105d^3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+\frac{e^{3/2}(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

output
```
1/7*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/d+1/105*(7*a*d*f*(-2*c^2*f^2+7*c*d
*e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c*d^2*e^2*f+6*d^3*e^3))*x*(
d*x^2+c)^(1/2)/d^3/f/(f*x^2+e)^(1/2)+1/105*e^(3/2)*(7*a*d*f*(-c*f+9*d*e)-b
*(-4*c^2*f^2+9*c*d*e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)
*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c
)^(1/2)/d^2/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/105*
(7*a*d*f*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c
*d^2*e^2*f+6*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1
/2)/d^3/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(7*a*
d*f-4*b*c*f+3*b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d^2+1/105*(14*a*d*f
*(-c*f+3*d*e)+b*(8*c^2*f^2-15*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2
+e)^(1/2)/d^2/f
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.69

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{-\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(4bc^2f^2-3bcdf(3e+fx^2)-7adf(6de+cf+3dfx^2)-3bd^2(e^2}{}$$

input
```
Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
(-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c^2*f^2 - 3*b*c*d*f*(3*e + f
*x^2) - 7*a*d*f*(6*d*e + c*f + 3*d*f*x^2) - 3*b*d^2*(e^2 + 8*e*f*x^2 + 5*f
^2*x^4))) - I*e*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) + b*(-6*d^3*e
^3 + 9*c*d^2*e^2*f - 19*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*
e) + c*f)*(-7*a*d*f*(3*d*e + c*f) + b*(6*d^2*e^2 - 6*c*d*e*f + 4*c^2*f^2))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)])/(105*c^2*(d/c)^(5/2)*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.29.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {403, 25, 403, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \ 403$$

$$\frac{\int -\sqrt{dx^2+c}\sqrt{fx^2+e}\left((bc-7ad)e-(3bde-4bcf+7adf)x^2\right)dx}{7d}+\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{\int \sqrt{dx^2+c}\sqrt{fx^2+e}\left((bc-7ad)e-(3bde-4bcf+7adf)x^2\right)dx}{7d}$$

$$\Big\downarrow \ 403$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{\frac{\int \frac{\sqrt{dx^2+c}\left(e(4bc(2de-cf)-7ad(5de-cf))-\left(14adf(3de-cf)+b\left(3d^2e^2-15cdfe+8c^2f^2\right)\right)x^2\right)}{\sqrt{fx^2+e}}dx}{5d}-\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-4bcf+3bde)}{5d}}{7d}$$

$$\Big\downarrow \ 403$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{\frac{\frac{\int -\frac{\left(7adf\left(3d^2e^2+7cdfe-2c^2f^2\right)-b\left(6d^3e^3-9cd^2fe^2+19c^2df^2e-8c^3f^3\right)\right)x^2+ce\left(7adf(9de-cf)-b\left(3d^2e^2+9cdfe-4c^2f^2\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14adf(3de-c\right.}{3}}{5d}}{7d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{\frac{-\frac{\int \frac{\left(7adf\left(3d^2e^2+7cdfe-2c^2f^2\right)-b\left(6d^3e^3-9cd^2fe^2+19c^2df^2e-8c^3f^3\right)\right)x^2+ce\left(7adf(9de-cf)-b\left(3d^2e^2+9cdfe-4c^2f^2\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14adf(3de-\right.}{3}}{5d}}{7d}$$

$$\Big\downarrow \ 406$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{-\frac{\frac{ce\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{\ldots}}}{3f}}{5d}}{7d}$$

$\downarrow$ 320

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{-\frac{\frac{\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)\text{E}\ldots}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}}{5d}}{7d}$$

$\downarrow$ 388

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{-\frac{\frac{\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cde\ldots\right.\right.}{\sqrt{f}\sqrt{e+fx^2}\ldots}}{3f}}{5d}}{7\ldots}$$

$\downarrow$ 313

$$\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}-\frac{-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+\ldots\right.\right.}{3f}}{5d}}{\ldots}$$

input
```
Int[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

3.29. $\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

output
```
(b*x*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(7*d) - (-1/5*((3*b*d*e - 4*b*c*
f + 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/d + (-1/3*((14*a*d*f*(3*
d*e - c*f) + b*(3*d^2*e^2 - 15*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])/f - ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3
*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*((x*Sqrt[c + d*x^2])/(
d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)
/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])) + (e^(3/2)*(7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9
*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2]))/(3*f))/(5*d))/(7*d)
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.29.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bfx^5\sqrt{dfx^4+cfx^2+dex^2+ce}}{7}+\frac{\left(adf^2+bcf^2+2ebdf-\frac{bf(6cf+6de)}{7}\right)x^3\sqrt{dfx^4+cfx^2+dex^2+ce}}{5df}+\frac{\left(acf^2+2adef+\frac{9bc}{7}\right.}{}\right.}{}$
risch	$\frac{x(15bx^4d^2f^2+21ad^2f^2x^2+3bcdf^2x^2+24bd^2efx^2+7acdf^2+42ad^2ef-4bc^2f^2+9bcdef+3bd^2e^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}{105fd^2}-\frac{\left(-\frac{4bc^3}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*f*x^5*(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*
(6*c*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*c*f^2+2*
a*d*e*f+9/7*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+
6*d*e))/d/f*(4*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c*
e^2-1/3*(a*c*f^2+2*a*d*e*f+9/7*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*
d*f-1/7*b*f*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(2*a*c*e*f+a*d*e^2+b*c*e^2-3/5*
(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+6*d*e))/d/f*c*e-1/3*(a*c*f^2+2*a
*d*e*f+9/7*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+6
*d*e))/d/f*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(
-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx = \frac{\left(6\,bd^3e^4-3\,(3\,bcd^2+7\,ad^3)e^3f+(19\,bc^2d-49\,acd^2)e^2f^2-2\,(4\,bc^3-7\,ac^2d)ef^3\right)\sqrt{df}\,x\sqrt{-\cdots}}{\cdots}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
1/105*((6*b*d^3*e^4 - 3*(3*b*c*d^2 + 7*a*d^3)*e^3*f + (19*b*c^2*d - 49*a*c
*d^2)*e^2*f^2 - 2*(4*b*c^3 - 7*a*c^2*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elli
ptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (6*b*d^3*e^4 - 3*(3*b*c*d^2 + 7*
a*d^3)*e^3*f + (19*b*c^2*d - (49*a - 3*b)*c*d^2)*e^2*f^2 - (8*b*c^3 - (14*
a + 9*b)*c^2*d + 63*a*c*d^2)*e*f^3 - (4*b*c^3 - 7*a*c^2*d)*f^4)*sqrt(d*f)*
x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) + (15*b*d^3*f^4*x
^6 - 6*b*d^3*e^3*f + 3*(3*b*c*d^2 + 7*a*d^3)*e^2*f^2 - (19*b*c^2*d - 49*a*
c*d^2)*e*f^3 + 2*(4*b*c^3 - 7*a*c^2*d)*f^4 + 3*(8*b*d^3*e*f^3 + (b*c*d^2 +
 7*a*d^3)*f^4)*x^4 + (3*b*d^3*e^2*f^2 + 3*(3*b*c*d^2 + 14*a*d^3)*e*f^3 - (
4*b*c^2*d - 7*a*c*d^2)*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^3*f^3
*x)
```

3.29.6 Sympy [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

3.29.7 Maxima [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

3.29.8 Giac [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

3.30 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.30.1 Optimal result

Integrand size = 30, antiderivative size = 400

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}}$$
$$+\frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2}+\frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d}$$
$$-\frac{\sqrt{e}(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{e^{3/2}(5ad(3de-cf)-b(6cde-4c^2f))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*b*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/d+1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8
*c^2*f^2-13*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)/d^3/(f*x^2+e)^(1/2)+1/15
*e^(3/2)*(5*a*d*(-c*f+3*d*e)-b*(-4*c^2*f+6*c*d*e))*(1/(1+f*x^2/e))^(1/2)*(
1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)
^(1/2))*(d*x^2+c)^(1/2)/c/d^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x
^2+e)^(1/2)-1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8*c^2*f^2-13*c*d*e*f+3*d^2*e^2)
)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f
*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^3/f^(1/2)/(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*(5*a*d*f-4*b*c*f+3*b*d*e)*
x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2
```

3.30.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(4bcf-5adf-3bd(2e+fx^2))-ie(10adf(2de-}{}$$

input
```
Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c*f - 5*a*d*f - 3*b*d*(2*e +
 f*x^2))) - I*e*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^
2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) + c*f)*(-3*b*d*e + 4*b*c*f - 5*a*d*f)*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(15*c^2*(d/c)^(5/2)*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.30.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int -\frac{\sqrt{fx^2+e}\,((bc-5ad)e-(3bde-4bcf+5adf)x^2)}{\sqrt{dx^2+c}}dx}{5d}+\frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d}-\frac{\int \frac{\sqrt{fx^2+e}\,((bc-5ad)e-(3bde-4bcf+5adf)x^2)}{\sqrt{dx^2+c}}dx}{5d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

3.30. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

3.30. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

3.30. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}dx$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \text{ 25}$$

3.30. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}dx$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{\frac{\int -\frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2+e\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -\frac{e(2bc(3de-2cf)-5ad(3de-cf))-\left(10adf(2de-cf)+b\left(3d^2e^2-13cdfe+8c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{3d}}{5d}$$

input
```
Int[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.30.4 Maple [A] (verified)

Time = 6.17 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b f x^3\sqrt{d f x^4+c f x^2+d e x^2+c e}}{5d}+\frac{\left(a f^2+2bfe-\frac{bf(4cf+4de)}{5d}\right)x\sqrt{d f x^4+c f x^2+d e x^2+c e}}{3df}+\frac{\left(e^2a-\frac{\left(a f^2+2bfe-\frac{bf(4cf+}{5d}\right.}{3df}\right.}{\sqrt{}}\right.}{}$
risch	$\frac{x(3bdf x^2+5adf-4bcf+6bde)\sqrt{d x^2+c}\sqrt{f x^2+e}}{15d^2}-\frac{\left(-\frac{15a d^2e^2\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{d f x^4+c f x^2+d e x^2+c e}}-\frac{4b c^2ef\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f}{}}}{\sqrt{-\frac{d}{c}}\sqrt{df x}}\right.}{}$
default	$\frac{\sqrt{f x^2+e}\sqrt{d x^2+c}\left(3\sqrt{-\frac{d}{c}}b d^2f^3x^7+5\sqrt{-\frac{d}{c}}a d^2f^3x^5-\sqrt{-\frac{d}{c}}bcd f^3x^5+9\sqrt{-\frac{d}{c}}b d^2e f^2x^5+5\sqrt{-\frac{d}{c}}acd f^3x^3+5\sqrt{-\frac{d}{c}}a d^2e f^2x^3-\right.}{}$

input
```
int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*f/d*x^3
*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*f^2+2*b*f*e-1/5*b*f/d*(4*c*f+4
*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(e^2*a-1/3*(a*f^2+2*b*f*e
-1/5*b*f/d*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1
+(c*f+d*e)/e/d)^(1/2))-(2*a*e*f+e^2*b-3/5*b*f/d*c*e-1/3*(a*f^2+2*b*f*e-1/5
*b*f/d*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*
(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e
/d)^(1/2))))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(3\,bd^2e^3-(13\,bcd-20\,ad^2)e^2f+2\,(4\,bc^2-5\,acd)ef^2)\sqrt{df}\,x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(3\,bd^2e^3-(13}{}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((3*b*d^2*e^3 - (13*b*c*d - 20*a*d^2)*e^2*f + 2*(4*b*c^2 - 5*a*c*d)*
e*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e))
- (3*b*d^2*e^3 - (13*b*c*d - 20*a*d^2)*e^2*f + (8*b*c^2 - 2*(5*a + 3*b)*c*
d + 15*a*d^2)*e*f^2 + (4*b*c^2 - 5*a*c*d)*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elli
ptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^2*f^3*x^4 + 3*b*d^2*e^2*f
 - (13*b*c*d - 20*a*d^2)*e*f^2 + 2*(4*b*c^2 - 5*a*c*d)*f^3 + (6*b*d^2*e*f^
2 - (4*b*c*d - 5*a*d^2)*f^3)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^3*f^
2*x)
```

3.30.6 Sympy [F]

$$\int \frac{(a + bx^2)(e + fx^2)^{3/2}}{\sqrt{c + dx^2}}\,dx = \int \frac{(a + bx^2)(e + fx^2)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.30.7 Maxima [F]

$$\int \frac{(a + bx^2)(e + fx^2)^{3/2}}{\sqrt{c + dx^2}}\,dx = \int \frac{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}{\sqrt{dx^2 + c}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

3.30.8 Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.31 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

3.31.1 Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{f}(bc(7de-8cf)-3ad(de-2cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(3bde-4bcf+3adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(1/2)+1/3*f*(b*c*(-8*c*f+7*d*e
)-3*a*d*(-2*c*f+d*e))*x*(d*x^2+c)^(1/2)/c/d^3/(f*x^2+e)^(1/2)+1/3*e^(3/2)*
(3*a*d*f-4*b*c*f+3*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Elliptic
F(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c
/d^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(b*c*(-8*
c*f+7*d*e)-3*a*d*(-2*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ell
ipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1
/2)*(d*x^2+c)^(1/2)/c/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+
1/3*(-3*a*d+4*b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/d^2
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}x(e+fx^2)(3ad(de-cf)+bc(-3de+4cf+dfx^2))+ie(3ad(de-\right.}{}$$

input
```
Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[d/c]*(Sqrt[d/c]*x*(e + f*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*d*e + 4*c
*f + d*f*x^2)) + I*e*(3*a*d*(d*e - 2*c*f) + b*c*(-7*d*e + 8*c*f))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)] - I*(4*b*c - 3*a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^3*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

3.31.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {401, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{\sqrt{fx^2+e}((4bc-3ad)fx^2+bce)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{fx^2+e}((4bc-3ad)fx^2+bce)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{f(bc(7de-8cf)-3ad(de-2cf))x^2+ce(3bde-4bcf+3adf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3d}}{cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{ce(3adf-4bcf+3bde)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+f(bc(7de-8cf)-3ad(de-2cf))\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3d}}{cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{f(bc(7de-8cf)-3ad(de-2cf))\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3d}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3\cdots}{3d}}{cd}\cdots$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{f(bc(7de-8cf)-3ad(de-2cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3d}+\frac{fx\sqrt{c+dx^2}\sqrt{e-\cdots}}{3\cdots}}{cd}\cdots$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+f(bc(7de-8cf)-3ad(de-2cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3d}+\cdots}{cd}\cdots$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

3.31. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$

output
```
-(((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2])) + (((4*b*c - 3*
a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) + (f*(b*c*(7*d*e - 8*c*f)
- 3*a*d*(d*e - 2*c*f))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]
*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3
/2)*(3*b*d*e - 4*b*c*f + 3*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f
]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]))/(3*d))/(c*d)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.31. $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.31.4 Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)} \left(-\frac{\left(df x^2+de\right)\left(acdf-ae d^2-c^2 bf+bcde\right)x}{d^3 c\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{bfx\sqrt{df x^4+cf x^2+de x^2+ce}}{3d^2}+\frac{\left(-\frac{acd f^2-2a d^2 ef-b c^2 f^2+2bcdef-b d^2 e^2}{d^3}\right.}{}\right.}{}$
default	$-\frac{\sqrt{f x^2+e}\sqrt{d x^2+c}\left(-\sqrt{-\frac{d}{c}}\, bcd f^2 x^5+3\sqrt{-\frac{d}{c}}\, acd f^2 x^3-3\sqrt{-\frac{d}{c}}\, a d^2 ef x^3-4\sqrt{-\frac{d}{c}}\, b c^2 f^2 x^3+2\sqrt{-\frac{d}{c}}\, bcdef x^3+3\sqrt{\frac{d x^2+c}{c}}\sqrt{\frac{f x^2}{e}}\right.}{}$
risch	$\frac{bx\sqrt{d x^2+c}\sqrt{f x^2+e}\, f}{3d^2}+\frac{\left(-\frac{(3adf-5bcf+4bde)e\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right)}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf x^2+de x^2+ce}}-\frac{\left(3acd f^2\right.}{}\right.}{}$

input
```
int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-(d*f*x^2+d*e
)*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3/c*x/((x^2+c/d)*(d*f*x^2+d*e))^(1/2
)+1/3*b*f/d^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(-(a*c*d*f^2-2*a*d^2*e
*f-b*c^2*f^2+2*b*c*d*e*f-b*d^2*e^2)/d^3+(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/
d^3*(c*f-d*e)/c+1/d^2*e*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c-1/3*b*f/d^2*c*
e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(1/d^2*f*
(a*d*f-b*c*f+2*b*d*e)+(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^2*f/c-1/3*b*f/d^
2*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/
d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx =$$

$$-\frac{(((7\,bcd^2-3\,ad^3)e^2-2\,(4\,bc^2d-3\,acd^2)ef)x^3+((7\,bc^2d-3\,acd^2)e^2-2\,(4\,bc^3-3\,ac^2d)ef)x)\sqrt{df}\sqrt{-\frac{e}{f}}E}{}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((((7*b*c*d^2 - 3*a*d^3)*e^2 - 2*(4*b*c^2*d - 3*a*c*d^2)*e*f)*x^3 + (
(7*b*c^2*d - 3*a*c*d^2)*e^2 - 2*(4*b*c^3 - 3*a*c^2*d)*e*f)*x)*sqrt(d*f)*sq
rt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (((7*b*c*d^2 - 3*a*
d^3)*e^2 - (8*b*c^2*d - 3*(2*a + b)*c*d^2)*e*f - (4*b*c^2*d - 3*a*c*d^2)*f
^2)*x^3 + ((7*b*c^2*d - 3*a*c*d^2)*e^2 - (8*b*c^3 - 3*(2*a + b)*c^2*d)*e*f
 - (4*b*c^3 - 3*a*c^2*d)*f^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sq
rt(-e/f)/x), c*f/(d*e)) - (b*c*d^2*f^2*x^4 + (7*b*c^2*d - 3*a*c*d^2)*e*f -
 2*(4*b*c^3 - 3*a*c^2*d)*f^2 + (4*b*c*d^2*e*f - (4*b*c^2*d - 3*a*c*d^2)*f^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c*d^4*f*x^3 + c^2*d^3*f*x)
```

3.31.6 Sympy [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

3.31.7 Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.31.8 Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

3.32 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

3.32.1 Optimal result

Integrand size = 30, antiderivative size = 373

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} + \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} + \frac{\sqrt{e}\sqrt{f}(bc(de-8cf)+2ad(de+cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(4bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(3/2)-1/3*f*(b*c*(-8*c*f+d
*e)+2*a*d*(c*f+d*e))*x*(d*x^2+c)^(1/2)/c^2/d^3/(f*x^2+e)^(1/2)+1/3*(-a*d+4
*b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/
e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/d
^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-8*c*f+d*e)+2
*a*d*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2
)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(
1/2)/c^2/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-4*
c*f+d*e)+a*d*(c*f+2*d*e))*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(1/2)
```

3.32.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{\left(\frac{d}{c}\right)^{3/2}\left(\sqrt{\frac{d}{c}}x(e+fx^2)(bc(-4c^2f+d^2ex^2-5cdfx^2)+ad(c^2f+2d^2ex^2+c\right.}{}$$

input
```
Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
((d/c)^(3/2)*(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(-4*c^2*f + d^2*e*x^2 - 5*c*d*f
*x^2) + a*d*(c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 2*f*x^2))) - I*e*(-2*a*d*(d*
e + c*f) + b*c*(-(d*e) + 8*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(a*d*(2*
d*e + c*f)) + b*c*(-(d*e) + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^4*(c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

3.32.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {401, 25, 401, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{\sqrt{fx^2+e}((4bc-ad)fx^2+(bc+2ad)e)}{(dx^2+c)^{3/2}}dx}{3cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{fx^2+e}((4bc-ad)fx^2+(bc+2ad)e)}{(dx^2+c)^{3/2}}dx}{3cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

$\Big\downarrow$ 401

$$\frac{\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{\int -\frac{f\left(c(4bc-ad)e-(bc(de-8cf)+2ad(de+cf))x^2\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}}{3cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{f\left(c(4bc-ad)e-(bc(de-8cf)+2ad(de+cf))x^2\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{f\int \frac{c(4bc-ad)e-(bc(de-8cf)+2ad(de+cf))x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{cd}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 406

$$\frac{\frac{f\left(ce(4bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-(2ad(cf+de)+bc(de-8cf))\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{cd}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 320

$$\frac{\frac{f\left(\frac{e^{3/2}\sqrt{c+dx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-(2ad(cf+de)+bc(de-8cf))\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{cd}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4c}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 388

3.32. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}}dx$

$$\frac{\frac{f\left(\frac{e^{3/2}\sqrt{c+dx^2}(4bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-(2ad(cf+de)+bc(de-8cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)\right)}{cd}+\frac{x\sqrt{e+fx^2}(ad(cf+2de}{cd\sqrt{c+d}}}{3cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 313$$

$$\frac{\frac{f\left(\frac{e^{3/2}\sqrt{c+dx^2}(4bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-(2ad(cf+de)+bc(de-8cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{cd}+\frac{x\sqrt{e-}}{}}{3cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

input
```
Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*(c + d*x^2)^(3/2)) + (((b*c*(d
*e - 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2])
+ (f*(-((b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*((x*Sqrt[c + d*x^2])/(d*Sq
rt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*
Sqrt[e + f*x^2]))) + ((4*b*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(c*d))/(3*c*d)
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.32.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.50

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{\left(acdf-ae d^2-c^2bf+bcde\right)x\sqrt{df x^4+cf x^2+de x^2+ce}}{3c d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df x^2+de\right)\left(2acdf+2ae d^2-5c^2bf+bcde\right)x}{3c^2d^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{\left(\frac{f(adf-2bcf+2b}{d^3}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^4*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2
+c/d)^2+1/3*(d*f*x^2+d*e)*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/c^2/d^3*
x/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(f*(a*d*f-2*b*c*f+2*b*d*e)/d^3-1/3*(a*c*
d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3*f/c-1/3*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*
c*d*e)/d^3*(c*f-d*e)/c^2-1/3/d^2*e*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)
/c^2)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*
e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(b*f^2
/d^2-1/3*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/d^2*f/c^2)*e/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f
*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1
/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{(((bcd^3+2\,ad^4)e^2-2\,(4\,bc^2d^2-acd^3)ef)x^5+2\,((bc^2d^2+2\,acd^3)e^2-2\,(4\,bc}{}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((((b*c*d^3 + 2*a*d^4)*e^2 - 2*(4*b*c^2*d^2 - a*c*d^3)*e*f)*x^5 + 2*((
b*c^2*d^2 + 2*a*c*d^3)*e^2 - 2*(4*b*c^3*d - a*c^2*d^2)*e*f)*x^3 + ((b*c^3*
d + 2*a*c^2*d^2)*e^2 - 2*(4*b*c^4 - a*c^3*d)*e*f)*x)*sqrt(d*f)*sqrt(-e/f)*
elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (((b*c*d^3 + 2*a*d^4)*e^2 -
2*(4*b*c^2*d^2 - a*c*d^3)*e*f - (4*b*c^2*d^2 - a*c*d^3)*f^2)*x^5 + 2*((b*c
^2*d^2 + 2*a*c*d^3)*e^2 - 2*(4*b*c^3*d - a*c^2*d^2)*e*f - (4*b*c^3*d - a*c
^2*d^2)*f^2)*x^3 + ((b*c^3*d + 2*a*c^2*d^2)*e^2 - 2*(4*b*c^4 - a*c^3*d)*e*
f - (4*b*c^4 - a*c^3*d)*f^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqr
t(-e/f)/x), c*f/(d*e)) + (3*b*c^2*d^2*f^2*x^4 - (b*c^3*d + 2*a*c^2*d^2)*e*
f + 2*(4*b*c^4 - a*c^3*d)*f^2 - ((2*b*c^2*d^2 + a*c*d^3)*e*f - 3*(4*b*c^3*
d - a*c^2*d^2)*f^2)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^2*d^5*f*x^5 +
 2*c^3*d^4*f*x^3 + c^4*d^3*f*x)
```

3.32.6 Sympy [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(5/2), x)
```

3.32.7 Maxima [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.32.8 Giac [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

3.33 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$

3.33.1 Optimal result

Integrand size = 30, antiderivative size = 376

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd(c+dx^2)^{5/2}} + \frac{(bc(2d^2e^2+3cdef-8c^2f^2)+ad(8d^2e^2-3cdef-2c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{e^{3/2}\sqrt{f}(bc(de-4cf)+ad(4de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(5/2)-1/15*e^(3/2)*(b*c*(-
4*c*f+d*e)+a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellip
ticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2
+c)^(1/2)/c^3/d^2/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/
2)+1/15*(d*(4*a*d+b*c)*e-c*(a*d+4*b*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2
+c)^(3/2)+1/15*(b*c*(-8*c^2*f^2+3*c*d*e*f+2*d^2*e^2)+a*d*(-2*c^2*f^2-3*c*d
*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/
2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/d^
(5/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.33.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.00 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(bc(de-7cf)+\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
(Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c
*(d*e - c*f)*(b*c*(d*e - 7*c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2) + (a*d*
(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2
*f^2))*(c + d*x^2)^2)) - I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*((a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*
c*d*e*f + 8*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*
e - c*f)*(a*d*(8*d*e + c*f) + 2*b*c*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)])))/(15*c^2*d^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[
e + f*x^2])
```

3.33.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {401, 25, 401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 401$$

$$-\frac{\int -\frac{\sqrt{fx^2+e}\,((4bc+ad)fx^2+(bc+4ad)e)}{(dx^2+c)^{5/2}}dx}{5cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{5cd(c+dx^2)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{fx^2+e}((4bc+ad)fx^2+(bc+4ad)e)}{(dx^2+c)^{5/2}}dx}{5cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 401

$$\frac{\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{3cd(c+dx^2)^{3/2}}-\frac{\int -\frac{f(2ad(2de+cf)+bc(de+8cf))x^2+e(ad(8de+cf)+2bc(de+2cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3cd}}{5cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{f(2ad(2de+cf)+bc(de+8cf))x^2+e(ad(8de+cf)+2bc(de+2cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3cd}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{3cd(c+dx^2)^{3/2}}}{5cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 400

$$\frac{\frac{\frac{\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{ef(ad(4de-cf)+bc(de-4cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3cd}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{3cd(c+dx^2)^{3/2}}}{5cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{ef(ad(4de-cf)+bc(de-4cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3cd}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{3cd(c+dx^2)^{3/2}}}{5cd}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 320

3.33. $\int \frac{(a+bx^2)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}dx$

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4de-cf)+bc(de-4cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\ldots}{3cd}\ldots}{5cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

input
```
Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
-1/5*((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*(c + d*x^2)^(5/2)) + (((d*(b*c
 + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(3*c*d*(c + d*x^2)^(3/
2)) + (((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2) + a*d*(8*d^2*e^2 - 3*c*d*
e*f - 2*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e +
 f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*
d*e - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]))/(3*c*d))/(5*c*d)
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

3.33.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{\left(acdf-ae\,d^2-c^2bf+bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5c\,d^5\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(2acdf+4ae\,d^2-7c^2bf+bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{15c^2d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{(df\,x\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^5*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2
+c/d)^3+1/15*(2*a*c*d*f+4*a*d^2*e-7*b*c^2*f+b*c*d*e)/c^2/d^4*x*(d*f*x^4+c*
f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/15*(d*f*x^2+d*e)/d^3/c^3/(c*f-d*e)*
x*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c
*d^2*e^2)/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(b*f^2/d^3+1/15*f*(2*a*c*d*f+4*a
*d^2*e-7*b*c^2*f+b*c*d*e)/c^2/d^3-1/15/d^3*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*
a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c*d^2*e^2)/c^3-1/15/d^2*e/c^3/(c*f
-d*e)*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2
*b*c*d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))+1/15/d^2*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d
*e*f-2*b*c*d^2*e^2)/c^3/(c*f-d*e)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(
-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)
)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(358) = 716$.

Time = 0.14 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.77

$$\int \frac{(a+bx^2)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx =$$

$$-\frac{((2\,(bcd^6+4\,ad^7)e^2+3\,(bc^2d^5-acd^6)ef-2\,(4\,bc^3d^4+ac^2d^5)f^2)x^6+3\,(2\,(bc^2d^5+4\,acd^6)e^2+3\,(bc^3d^4-}{}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^2 + 3*(b*c^2*d^5 - a*c*d^6)*e*f - 2*(4*b*
c^3*d^4 + a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + 3*(b*c^
3*d^4 - a*c^2*d^5)*e*f - 2*(4*b*c^4*d^3 + a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d
^3 + 4*a*c^3*d^4)*e^2 + 3*(b*c^5*d^2 - a*c^4*d^3)*e*f - 2*(4*b*c^6*d + a*c
^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + 3*(b*c^4*d^3 - a*c^3*d^
4)*e*f - 2*(4*b*c^5*d^2 + a*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*ellipt
ic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c*d^6 + 4*a*d^7)*e^2 + (b*c
^3*d^4 + (4*a + 3*b)*c^2*d^5 - 3*a*c*d^6)*e*f - (4*b*c^4*d^3 + (a + 8*b)*c
^3*d^4 + 2*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + (b*c^4
*d^3 + (4*a + 3*b)*c^3*d^4 - 3*a*c^2*d^5)*e*f - (4*b*c^5*d^2 + (a + 8*b)*c
^4*d^3 + 2*a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e^2 + (b*c^6*
d + (4*a + 3*b)*c^5*d^2 - 3*a*c^4*d^3)*e*f - (4*b*c^7 + (a + 8*b)*c^6*d +
2*a*c^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + (b*c^5*d^2 + (4*a
+ 3*b)*c^4*d^3 - 3*a*c^3*d^4)*e*f - (4*b*c^6*d + (a + 8*b)*c^5*d^2 + 2*a*c
^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*
f/(d*e)) - ((2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + 3*(b*c^3*d^4 - a*c^2*d^5)*e*f
 - 2*(4*b*c^4*d^3 + a*c^3*d^4)*f^2)*x^5 + (5*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2
 - 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e*f - 3*(3*b*c^5*d^2 + 2*a*c^4*d^3)*f^2)*x^
3 + (15*a*c^3*d^4*e^2 + (b*c^5*d^2 - 11*a*c^4*d^3)*e*f - (4*b*c^6*d + a*c^
5*d^2)*f^2)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^7*d^4*e - c^8*d^3*f ...
```

3.33.6 Sympy [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(7/2), x)
```

3.33.7 Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

3.33.8 Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

3.34 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$

3.34.1 Optimal result

Integrand size = 30, antiderivative size = 531

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2(c+dx^2)^{5/2}}$$
$$+\frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))\,x\sqrt{e+fx^2}}{105c^3d^2(de-cf)(c+dx^2)^{3/2}}$$
$$-\frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd(c+dx^2)^{7/2}}$$
$$+\frac{(6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))\sqrt{e+fx^2}\,E\Big(\arctan}{105c^{7/2}d^{5/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{e^{3/2}\sqrt{f}(3ad(8d^2e^2-11cdef+c^2f^2)+2bc(2d^2e^2-cdef+2c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\right.}{105c^4d^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/7*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(7/2)-1/105*e^(3/2)*(3*a*d
*(c^2*f^2-11*c*d*e*f+8*d^2*e^2)+2*b*c*(2*c^2*f^2-c*d*e*f+2*d^2*e^2))*(1/(1
+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)
^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^4/d^2/(-c*f+d*e)^2/(e*
(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(d*(6*a*d+b*c)*e-c*(3*a*
d+4*b*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(5/2)+1/105*(b*c*(-8*c^2*f
^2+c*d*e*f+4*d^2*e^2)+3*a*d*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2))*x*(f*x^2+e)^
(1/2)/c^3/d^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/105*(6*a*d*(c^3*f^3+2*c^2*d*e*f
^2-12*c*d^2*e^2*f+8*d^3*e^3)+b*c*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*
d^3*e^3))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(7/2)/d^(5/2)/(
-c*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}x(e+fx^2)\left(15c^3(bc-ad)(de-cf)^3-3c^2(de-cf)^2(bc(de-9c\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
(Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(15*c^3*(b*c - a*d)*(d*e - c*f)^3 -
3*c^2*(d*e - c*f)^2*(b*c*(d*e - 9*c*f) + 2*a*d*(3*d*e + c*f))*(c + d*x^2)
- c*(d*e - c*f)*(b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*e^2
- 5*c*d*e*f - 2*c^2*f^2))*(c + d*x^2)^2 - (6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2
*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e
*f^2 + 8*c^3*f^3))*(c + d*x^2)^3)) + I*e*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 +
 c^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*E
llipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(3*a*d*(-16
*d^2*e^2 + 16*c*d*e*f + c^2*f^2) + b*c*(-8*d^2*e^2 + c*d*e*f + 4*c^2*f^2))
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(105*c^3*d^3*(d*e - c*f
)^2*(c + d*x^2)^(7/2)*Sqrt[e + f*x^2])
```

3.34.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 401, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{\sqrt{fx^2+e}((4bc+3ad)fx^2+(bc+6ad)e)}{(dx^2+c)^{7/2}}dx}{7cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{fx^2+e}((4bc+3ad)fx^2+(bc+6ad)e)}{(dx^2+c)^{7/2}}dx}{7cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x\sqrt{e+fx^2}(de(6ad+bc)-cf(3ad+4bc))}{5cd(c+dx^2)^{5/2}}-\frac{\int -\frac{f(6ad(3de+cf)+bc(3de+8cf))x^2+e(4bc(de+cf)+3ad(8de+cf))}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5cd}}{7cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{f(6ad(3de+cf)+bc(3de+8cf))x^2+e(4bc(de+cf)+3ad(8de+cf))}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5cd}+\frac{x\sqrt{e+fx^2}(de(6ad+bc)-cf(3ad+4bc))}{5cd(c+dx^2)^{5/2}}}{7cd}-\frac{x(e+fx^2)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{x\sqrt{e+fx^2}\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+cdef+4d^2e^2\right)\right)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}-\frac{\int -\frac{f\left(bc\left(4d^2e^2+cdfe-8c^2f^2\right)+3ad\left(8d^2e^2-5cdfe-2c^2f^2\right)\right)x^2+e\left(bc\left(8d^2e^2-cdfe-4c^2\cdots\right.\right.}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}\cdots}{3c(de-cf)}}{5cd}\cdots}{7cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{f\left(bc\left(4d^2e^2+cdfe-8c^2f^2\right)+3ad\left(8d^2e^2-5cdfe-2c^2f^2\right)\right)x^2+e\left(bc\left(8d^2e^2-cdfe-4c^2f^2\right)+3ad\left(16d^2e^2-16cdfe-c^2f^2\right)\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}\left(3ad\left(-2c^2f^2-5cdef+8\cdots\right.\right.}{3c\left(c+dx^2\right)\cdots}}{5cd}\cdots}{7cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{\frac{\left(6ad\left(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3\right)+bc\left(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3\right)\right)\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{ef\left(3ad\left(c^2f^2-11cdef+8d^2e^2\right)+2bc\left(2c^2f^2-cdef+2d^2e^2\right)\cdots\right.}{de-cf}}{3c(de-cf)}\cdots}{5cd}\cdots}{7cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{\frac{\sqrt{e+fx^2}\left(6ad\left(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3\right)+bc\left(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{ef\left(3ad\left(c^2f^2-11cdef+8d^2e^2\right)+2bc\left(2c^2\cdots\right.\right.}{de-\cdots}}{3c(de-cf)}\cdots}{5cd}\cdots}{7\cdots}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \; 320$$

3.34. $\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}dx$

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(6ad\left(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3\right)+bc\left(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\left(3ad\left(c^2f^2-11cdef+8d^2\cdots\right.\right.}{c\sqrt{\cdots}}}{3c(de-cf)}}{5cd}$$

$$\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

input
```
Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
-1/7*((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*(c + d*x^2)^(7/2)) + (((d*(b*c
 + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(5*c*d*(c + d*x^2)^(
5/2)) + (((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*e^2 - 5*c*
d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2))
 + (((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(
8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d
*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2
)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*b*c*(2*d^2*e^2 - c
*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2]))/(3*c*(d*e - c*f)))/(5*c*d))/(7*c*d)
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.34.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.93

method	result	size
elliptic	Expression too large to display	1023
default	Expression too large to display	5113

input
```
int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/7*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^6*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2
+c/d)^4+1/35*(2*a*c*d*f+6*a*d^2*e-9*b*c^2*f+b*c*d*e)/c^2/d^5*x*(d*f*x^4+c*
f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3+1/105*(6*a*c^2*d*f^2+15*a*c*d^2*e*f-2
4*a*d^3*e^2+8*b*c^3*f^2-b*c^2*d*e*f-4*b*c*d^2*e^2)/d^4/c^3/(c*f-d*e)*x*(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/105*(d*f*x^2+d*e)/d^3/c^4/(
c*f-d*e)^2*x*(6*a*c^3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e
^3+8*b*c^4*f^3-5*b*c^3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3)/((x^2+c/d)
*(d*f*x^2+d*e))^(1/2)+(1/105*f*(6*a*c^2*d*f^2+15*a*c*d^2*e*f-24*a*d^3*e^2+
8*b*c^3*f^2-b*c^2*d*e*f-4*b*c*d^2*e^2)/d^3/c^3/(c*f-d*e)-1/105/d^3/(c*f-d*
e)*(6*a*c^3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*c^4
*f^3-5*b*c^3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3)/c^4-1/105/d^2*e/c^4/
(c*f-d*e)^2*(6*a*c^3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^
3+8*b*c^4*f^3-5*b*c^3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3))/(-d/c)^(1/
2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/105/d^2*(6*a*c^3*d*f
^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*c^4*f^3-5*b*c^3*d*
e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3)/c^4/(c*f-d*e)^2*e/(-d/c)^(1/2)*(1+d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(Ellip
ticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),...
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(507) = 1014$.

Time = 0.17 (sec) , antiderivative size = 1847, normalized size of antiderivative = 3.48

$$\int \frac{(a + bx^2)(e + fx^2)^{3/2}}{(c + dx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(((8*(b*c*d^8 + 6*a*d^9)*e^3 - (5*b*c^2*d^7 + 72*a*c*d^8)*e^2*f - (
5*b*c^3*d^6 - 12*a*c^2*d^7)*e*f^2 + 2*(4*b*c^4*d^5 + 3*a*c^3*d^6)*f^3)*x^8
 + 4*(8*(b*c^2*d^7 + 6*a*c*d^8)*e^3 - (5*b*c^3*d^6 + 72*a*c^2*d^7)*e^2*f -
 (5*b*c^4*d^5 - 12*a*c^3*d^6)*e*f^2 + 2*(4*b*c^5*d^4 + 3*a*c^4*d^5)*f^3)*x
^6 + 6*(8*(b*c^3*d^6 + 6*a*c^2*d^7)*e^3 - (5*b*c^4*d^5 + 72*a*c^3*d^6)*e^2
*f - (5*b*c^5*d^4 - 12*a*c^4*d^5)*e*f^2 + 2*(4*b*c^6*d^3 + 3*a*c^5*d^4)*f^
3)*x^4 + 8*(b*c^5*d^4 + 6*a*c^4*d^5)*e^3 - (5*b*c^6*d^3 + 72*a*c^5*d^4)*e^
2*f - (5*b*c^7*d^2 - 12*a*c^6*d^3)*e*f^2 + 2*(4*b*c^8*d + 3*a*c^7*d^2)*f^3
 + 4*(8*(b*c^4*d^5 + 6*a*c^3*d^6)*e^3 - (5*b*c^5*d^4 + 72*a*c^4*d^5)*e^2*f
 - (5*b*c^6*d^3 - 12*a*c^5*d^4)*e*f^2 + 2*(4*b*c^7*d^2 + 3*a*c^6*d^3)*f^3)
*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - (
(8*(b*c*d^8 + 6*a*d^9)*e^3 + (4*b*c^3*d^6 + (24*a - 5*b)*c^2*d^7 - 72*a*c*
d^8)*e^2*f - (2*b*c^4*d^5 + (33*a + 5*b)*c^3*d^6 - 12*a*c^2*d^7)*e*f^2 + (
4*b*c^5*d^4 + (3*a + 8*b)*c^4*d^5 + 6*a*c^3*d^6)*f^3)*x^8 + 4*(8*(b*c^2*d^
7 + 6*a*c*d^8)*e^3 + (4*b*c^4*d^5 + (24*a - 5*b)*c^3*d^6 - 72*a*c^2*d^7)*e
^2*f - (2*b*c^5*d^4 + (33*a + 5*b)*c^4*d^5 - 12*a*c^3*d^6)*e*f^2 + (4*b*c^
6*d^3 + (3*a + 8*b)*c^5*d^4 + 6*a*c^4*d^5)*f^3)*x^6 + 6*(8*(b*c^3*d^6 + 6*
a*c^2*d^7)*e^3 + (4*b*c^5*d^4 + (24*a - 5*b)*c^4*d^5 - 72*a*c^3*d^6)*e^2*f
 - (2*b*c^6*d^3 + (33*a + 5*b)*c^5*d^4 - 12*a*c^4*d^5)*e*f^2 + (4*b*c^7*d^
2 + (3*a + 8*b)*c^6*d^3 + 6*a*c^5*d^4)*f^3)*x^4 + 8*(b*c^5*d^4 + 6*a*c^...
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

3.34.7 Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

3.34.8 Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

3.35 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx$

3.35.1 Optimal result

Integrand size = 30, antiderivative size = 551

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c}{105df^3\sqrt{e+fx^2}}$$

$$-\frac{(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3}$$

$$-\frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

$$-\frac{\sqrt{e}(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3))\sqrt{c+dx^2}E\Big(\arctan}{105df^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}(7af(4d^2e^2-11cdef+15c^2f^2)-be(24d^2e^2-61cdef+45c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1\right.}{105f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
1/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d*
e*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*x*(d*x^2+c)^(1/2)/d/f^3/(f*x^2+e)^(1/2)
-1/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d
*e*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2
)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)
*(d*x^2+c)^(1/2)/d/f^(7/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
+1/105*(7*a*f*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2)-b*e*(45*c^2*f^2-61*c*d*e*f
+24*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/
e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/
2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/35*(-7*a*d*f-5*b*c*f+
6*b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/f^2+1/7*b*x*(d*x^2+c)^(5/2)*(f*
x^2+e)^(1/2)/f-1/105*(28*a*d*f*(-2*c*f+d*e)-b*(15*c^2*f^2-43*c*d*e*f+24*d^
2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f^3
```

3.35.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)\left(7adf(-4de+11cf+3dfx^2)+b(45c^2f^2+cdf(-6\right.}{}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(-4*d*e + 11*c*f + 3*d*f*x
^2) + b*(45*c^2*f^2 + c*d*f*(-61*e + 45*f*x^2) + 3*d^2*(8*e^2 - 6*e*f*x^2
+ 5*f^2*x^4))) - I*e*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) + b*(-
48*d^3*e^3 + 128*c*d^2*e^2*f - 103*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 + I*(-(d*e) + c*f)*(4*b*e*(12*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 7*a*f*
(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(105*Sqrt[d/c]*f^4*Sq
rt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.35.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {403, 25, 403, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(dx^2+c)^{3/2}((6bde-5bcf-7adf)x^2+c(be-7af))}{\sqrt{fx^2+e}}dx}{7f}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}-\frac{\int \frac{(dx^2+c)^{3/2}((6bde-5bcf-7adf)x^2+c(be-7af))}{\sqrt{fx^2+e}}dx}{7f}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}-\frac{\frac{\int \frac{\sqrt{dx^2+c}\left(\left(28adf(de-2cf)-b\left(24d^2e^2-43cdfe+15c^2f^2\right)\right)x^2+c(7af(de-5cf)-2be(3de-5cf))\right)}{\sqrt{fx^2+e}}dx}{5f}+\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}(-7adf-5bcf+6bde)}{5f}}{7f}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}-$$

$$\frac{\frac{\frac{\int -\frac{\left(7adf\left(8d^2e^2-23cdfe+23c^2f^2\right)-b\left(48d^3e^3-128cd^2fe^2+103c^2df^2e-15c^3f^3\right)\right)x^2+c\left(7af\left(4d^2e^2-11cdfe+15c^2f^2\right)-be\left(24d^2e^2-61cdfe+45c^2f^2\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{x\sqrt{c+}}{}}{5f}}{7f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}-$$

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{3f}-\frac{\int \frac{\left(7adf\left(8d^2e^2-23cdfe+23c^2f^2\right)-b\left(48d^3e^3-128cd^2fe^2+103c^2df^2e-15c^3f^3\right)\right)x^2+c\left(7af\left(4\right.\right.}{\sqrt{dx^2+c}\sqrt{fx^2+e}}}{3f}}{5f}}{7f}$$

$\downarrow$ 406

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} - \frac{\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{3f} - \frac{c\left(7af\left(15c^2f^2-11cdef+4d^2e^2\right)-be\left(45c^2f^2-61cdef+24d^2e^2\right)\right)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\left(7a\right.}{5f}}{7f}$$

$\downarrow$ 320

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} - \frac{\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{3f} - \frac{\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)\int\frac{x^2}{\sqrt{dx^2+c}}}{5f}}{}$$

$\downarrow$ 388

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} - \frac{\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{3f} - \frac{\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}\right.}{5f}}{}$$

$\downarrow$ 313

$$\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} - \frac{\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{3f} - \frac{\frac{\sqrt{e}\sqrt{c+dx^2}\left(7af\left(15c^2f^2-11cdef+4d^2e^2\right)-be\left(45c^2f^2-61cdef+24d^2e^2\right)\right)\text{EllipticF}\left(\arctan\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{5f}}{}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

3.35. $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx$

output
```
(b*x*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(7*f) - (((6*b*d*e - 5*b*c*f - 7*a
*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*f) + (((28*a*d*f*(d*e - 2*c*
f) - b*(24*d^2*e^2 - 43*c*d*e*f + 15*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2])/(3*f) - ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^
3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*((x*Sqrt[c + d*x^
2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[
f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f + 15*
c^2*f^2) - b*e*(24*d^2*e^2 - 61*c*d*e*f + 45*c^2*f^2))*Sqrt[c + d*x^2]*Ell
ipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f))/(5*f))/(7*f)
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.35.4 Maple [A] (verified)

Time = 7.48 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b d^2 x^5\sqrt{df x^4+cf x^2+de x^2+ce}}{7f}+\frac{\left(a d^3+3bc d^2-\frac{b d^2(6cf+6de)}{7f}\right)x^3\sqrt{df x^4+cf x^2+de x^2+ce}}{5df}+\frac{\left(3ac d^2+3b c^2 d-\frac{5b d^2 c\cdots}{7f}\right.}{}\right.}{}$
risch	$\frac{x(15b x^4 d^2 f^2+21a d^2 f^2 x^2+45bcd f^2 x^2-18b d^2 ef x^2+77acd f^2-28a d^2 ef+45b c^2 f^2-61bcdef+24b d^2 e^2)\sqrt{d x^2+c}\sqrt{f x^2+e}}{105 f^3}+\cdots$
default	Expression too large to display

input
```
int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*d^2/f*x
^5*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6
*c*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(3*a*c*d^2+3*
b*c^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6*d*e))/d/
f*(4*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^3*a-1/3*(3*a
*c*d^2+3*b*c^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6
*d*e))/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1
+(c*f+d*e)/e/d)^(1/2))-(3*a*c^2*d+c^3*b-3/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(
6*c*f+6*d*e))/d/f*c*e-1/3*(3*a*c*d^2+3*b*c^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+
3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))
*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{(48\,bd^3e^5 - 8\,(16\,bcd^2 + 7\,ad^3)e^4f + (103\,bc^2d + 161\,acd^2)e^3f^2 - (15\,bc^3 + 16}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
1/105*((48*b*d^3*e^5 - 8*(16*b*c*d^2 + 7*a*d^3)*e^4*f + (103*b*c^2*d + 161
*a*c*d^2)*e^3*f^2 - (15*b*c^3 + 161*a*c^2*d)*e^2*f^3)*sqrt(d*f)*x*sqrt(-e/
f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (48*b*d^3*e^5 - 105*a*c^3
*f^5 - 8*(16*b*c*d^2 + 7*a*d^3)*e^4*f + (103*b*c^2*d + (161*a + 24*b)*c*d^
2)*e^3*f^2 - (15*b*c^3 + (161*a + 61*b)*c^2*d + 28*a*c*d^2)*e^2*f^3 + (45*
b*c^3 + 77*a*c^2*d)*e*f^4)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-
e/f)/x), c*f/(d*e)) + (15*b*d^3*e*f^4*x^6 - 48*b*d^3*e^4*f + 8*(16*b*c*d^2
 + 7*a*d^3)*e^3*f^2 - (103*b*c^2*d + 161*a*c*d^2)*e^2*f^3 + (15*b*c^3 + 16
1*a*c^2*d)*e*f^4 - 3*(6*b*d^3*e^2*f^3 - (15*b*c*d^2 + 7*a*d^3)*e*f^4)*x^4
+ (24*b*d^3*e^3*f^2 - (61*b*c*d^2 + 28*a*d^3)*e^2*f^3 + (45*b*c^2*d + 77*a
*c*d^2)*e*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^5*x)
```

3.35.6 Sympy [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)*(c + d*x**2)**(5/2)/sqrt(e + f*x**2), x)
```

3.35.7 Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

3.35.8 Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2), x)
```

3.36 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$

3.36.1 Optimal result

Integrand size = 30, antiderivative size = 396

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx &= -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} \\ &- \frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} \\ &+ \frac{\sqrt{e}(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &- \frac{\sqrt{e}(5af(de-3cf)-b(4de^2-6cef))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*x*(d*x^2+
c)^(1/2)/d/f^2/(f*x^2+e)^(1/2)+1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13
*c*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f
^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2
)/d/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/15*(5*a*f*(-
3*c*f+d*e)-b*(-6*c*e*f+4*d*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*E
llipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d
*x^2+c)^(1/2)/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*
b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/f-1/15*(-5*a*d*f-3*b*c*f+4*b*d*e)*x*(d
*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f^2
```

3.36.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(5adf+b(-4de+6cf+3dfx^2)) - ie(-10adf(de\cdots}{}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(5*a*d*f + b*(-4*d*e + 6*c*f + 3*d*
f*x^2)) - I*e*(-10*a*d*f*(d*e - 2*c*f) + b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2
*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(5*a*f*(2*d*e - 3*c*f) + b*e*(-8*d*
e + 9*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)])/(15*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
])
```

3.36.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{\sqrt{dx^2+c}\left((4bde-3bcf-5adf)x^2+c(be-5af)\right)}{\sqrt{fx^2+e}}dx}{5f}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\int \frac{\sqrt{dx^2+c}\left((4bde-3bcf-5adf)x^2+c(be-5af)\right)}{\sqrt{fx^2+e}}dx}{5f}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\frac{\int \frac{\left(10adf(de-2cf)-b\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)x^2+c\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(-5adf-3bcf+4bde)}{3f}}{5f}$$

$$\Big\downarrow \text{406}$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\frac{\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+c\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(\ldots}{3\ldots}}{5f}$$

$$\Big\downarrow \text{320}$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\frac{\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}+x\sqrt{\ldots}}{5f}$$

$$\Big\downarrow \text{388}$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\frac{\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\ldots\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}\ldots}{5f}$$

$$\Big\downarrow \text{313}$$

$$\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}-\frac{\frac{\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)\text{EllipticF}\left(\ldots\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}\ldots}{5f}$$

3.36. $\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}dx$

input
```
Int[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

output
```
(b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*f) - (((4*b*d*e - 3*b*c*f - 5*a
*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + ((10*a*d*f*(d*e - 2*c*f)
- b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e +
 f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])) + (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[
f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f))/(5*f)
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.36.4 Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bdx^3\sqrt{dfx^4+cfx^2+dex^2+ce}}{5f}+\frac{\left(ad^2+2bcd-\frac{bd(4cf+4de)}{5f}\right)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{3df}+\frac{\left(c^2a-\frac{\left(ad^2+2bcd-\frac{bd(4cf+4\cdots}{5f}\cdots\right)}{3df}\cdots\right)}{\sqrt{\cdots}}\cdots\right)}{\cdots}$
risch	$\frac{x(3bdfx^2+5adf+6bcf-4bde)\sqrt{dx^2+c}\sqrt{fx^2+e}}{15f^2}+\frac{\left(\frac{15c^2af^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{6bc^2ef\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx\cdots}{e}}\cdots}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4\cdots}}\cdots\right)}{\cdots}$
default	$\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}\left(3\sqrt{-\frac{d}{c}}bd^2f^3x^7+5\sqrt{-\frac{d}{c}}ad^2f^3x^5+9\sqrt{-\frac{d}{c}}bcdf^3x^5-\sqrt{-\frac{d}{c}}bd^2ef^2x^5+5\sqrt{-\frac{d}{c}}acdf^3x^3+5\sqrt{-\frac{d}{c}}ad^2ef^2x^3+\cdots\right)}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*d/f*x^3
*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*d^2+2*b*c*d-1/5*b*d/f*(4*c*f+4
*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^2*a-1/3*(a*d^2+2*b*c*d
-1/5*b*d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1
+(c*f+d*e)/e/d)^(1/2))-(2*a*c*d+b*c^2-3/5*b*d/f*c*e-1/3*(a*d^2+2*b*c*d-1/5
*b*d/f*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*
(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e
/d)^(1/2))))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}\,dx =$$

$$-\frac{(8\,bd^2e^4-(13\,bcd+10\,ad^2)e^3f+(3\,bc^2+20\,acd)e^2f^2)\sqrt{df}\,x\sqrt{-\frac{e}{f}}E\left(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de}\right)-(8\,bd^2e^4+15}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b*d^2*e^4 - (13*b*c*d + 10*a*d^2)*e^3*f + (3*b*c^2 + 20*a*c*d)*e
^2*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e))
 - (8*b*d^2*e^4 + 15*a*c^2*f^4 - (13*b*c*d + 10*a*d^2)*e^3*f + (3*b*c^2 +
4*(5*a + b)*c*d)*e^2*f^2 - (6*b*c^2 + 5*a*c*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/
f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^2*e*f^3*x^4 + 8*b*
d^2*e^3*f - (13*b*c*d + 10*a*d^2)*e^2*f^2 + (3*b*c^2 + 20*a*c*d)*e*f^3 - (
4*b*d^2*e^2*f^2 - (6*b*c*d + 5*a*d^2)*e*f^3)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x
^2 + e))/(d*e*f^4*x)
```

3.36.6 Sympy [F]

$$\int \frac{(a+bx^2)\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)\left(c+dx^2\right)^{\frac{3}{2}}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

3.36.7 Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

3.36.8 Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2), x)
```

3.37 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

3.37.1 Optimal result

Integrand size = 30, antiderivative size = 282

$$\begin{aligned}\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = &-\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}\\ &+\frac{\sqrt{e}(2bde-bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &-\frac{\sqrt{e}(be-3af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-1/3*(-3*a*d*f-b*c*f+2*b*d*e)*x*(d*x^2+c)^(1/2)/d/f/(f*x^2+e)^(1/2)+1/3*(-
3*a*d*f-b*c*f+2*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1
/2)/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*f+
b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e
)^(1/2)/f
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{b\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)-ie(-2bde+bcf+3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+i(2be}{3\sqrt{\frac{d}{c}}f^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
(b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) - I*e*(-2*b*d*e + b*c*f + 3*a*d*f
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)] + I*(2*b*e - 3*a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/
c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.37.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(2bde-bcf-3adf)x^2+c(be-3af)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\int \frac{(2bde-bcf-3adf)x^2+c(be-3af)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{c(be-3af)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf-bcf+2bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}$$

$$\Big\downarrow \; 320$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{(-3adf-bcf+2bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3f}$$

$$\Big\downarrow \; 388$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{(-3adf-bcf+2bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3f}$$

$$\Big\downarrow \; 313$$

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf-bcf+2bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{3f}$$

input
```
Int[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
(b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) - ((2*b*d*e - b*c*f - 3*a*d*f)
*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(b*e - 3*a*f)*Sqrt
[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt
[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f)
```

3.37. $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.37.4 Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\,\Big(\frac{bx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3f}+\frac{(ac-\frac{ceb}{3f})\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\big(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\big)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(ad+bc-\frac{b(2cf+2de)}{3f})e\sqrt{1+\frac{d}{}}}{}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$
risch	$\frac{bx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3f}+\frac{\Big(\frac{3acf\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\big(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\big)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{bce\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\big(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\big)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(3adf+}{}}{3f\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,\Big(\sqrt{-\frac{d}{c}}\,bd\,f^2x^5+\sqrt{-\frac{d}{c}}\,bc\,f^2x^3+\sqrt{-\frac{d}{c}}\,bdef\,x^3+3\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\big)ac\,f^2-3\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2}{e}}}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*b/f*x*(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c-1/3*c*e/f*b)/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(a*d+b*c-1/3*b/f*(2*c*f+2*d*e))*e/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$
$$=\frac{(2\,bde^3-(bc+3\,ad)e^2f)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(2\,bde^3+bcef^2-3\,acf^3-(bc+3\,ad)e^2f)}{3\,def^3x}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*d*e^3 - (b*c + 3*a*d)*e^2*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(a
rcsin(sqrt(-e/f)/x), c*f/(d*e)) - (2*b*d*e^3 + b*c*e*f^2 - 3*a*c*f^3 - (b*
c + 3*a*d)*e^2*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x),
c*f/(d*e)) + (b*d*e*f^2*x^2 - 2*b*d*e^2*f + (b*c + 3*a*d)*e*f^2)*sqrt(d*x^
2 + c)*sqrt(f*x^2 + e))/(d*e*f^3*x)
```

3.37.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

3.37.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.37.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.38 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.38.1 Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
b*x*(d*x^2+c)^(1/2)/d/(f*x^2+e)^(1/2)-b*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(
1/2)*(d*x^2+c)^(1/2)/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(
1/2)+a*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/f^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(beE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(-be+af)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)\right)}{\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(b*e*EllipticE[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)] + (-(b*e) + a*f)*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)]))/(Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.38.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{406}$$

$$a\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx$$

$$\Big\downarrow \text{320}$$

$$b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{a\sqrt{e}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+ b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)$$

input
```
Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
b*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (a*Sqrt[e]*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.38.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.38.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be\right)\sqrt{\frac{f\,x^{2}+e}{e}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{f\sqrt{-\frac{d}{c}}\,(df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce)}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}-\frac{be\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\cdots}\right)\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}\,f}\right)}{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}$

input
```
int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*b*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*e)*((f*x^2+
e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(-d/c)^(
1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{df}be^2x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right) \mid \frac{cf}{de}) - \sqrt{dx^2+c}\sqrt{fx^2+e}bef - (be^2+af^2)\sqrt{df}x\sqrt{-\frac{e}{f}}F(\arcsin\left(\frac{\sqrt{-\cdots}}{x}\right)\cdots}{def^2x}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(d*f)*b*e^2*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e))
 - sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*b*e*f - (b*e^2 + a*f^2)*sqrt(d*f)*x*sqr
t(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)))/(d*e*f^2*x)
```

3.38.6 Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.38.7 Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.38.8 Giac [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.39 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

3.39.1 Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{\sqrt{e}(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+
d*e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-c*f+d*e)/c^(1/2)/d^(1/2)/(d
*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}(bc-ad)x(e+fx^2)+i(bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\right.\right.\right.}{d(-de+cf)\sqrt{}}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[d/c]*(Sqrt[d/c]*(b*c - a*d)*x*(e + f*x^2) + I*(b*c - a*d)*e*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)] - I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
F[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*(-(d*e) + c*f)*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

3.39.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{400}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(dx^2+c)^{3/2}}dx}{de-cf}$$

$$\Big\downarrow \text{313}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
-(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*
x^2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticF[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.39.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.39.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.60

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\, adf\, x^3+\sqrt{-\frac{d}{c}}\, bcf\, x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\, F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right) acf-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\, F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right) ade+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{}\right.}{\sqrt{-\frac{d}{c}}\, c(cf-de)(df\, x^4+cf\, x}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{\left(df\,x^2+de\right)x(ad-bc)}{dc(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\left(\frac{b}{d}+\frac{ad-bc}{dc}+\frac{e(ad-bc)}{c(cf-de)}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\, F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(ad-bc)}{}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

input
```
int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-d/c)^(1/2)*a*d*f*x^3+(-d/c)^(1/2)*b*c*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f-((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*
e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c*e-(-d/c)^(1/2)*a*d*e*x+(-d/c)^(1/2)*b*c*e*x)*(
f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/c/(c*f-d*e)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx =$$

$$-\frac{(bc^2d-acd^2)\sqrt{dx^2+c}\sqrt{fx^2+e}ex-((bcd^2-ad^3)ex^2+(bc^2d-acd^2)e)\sqrt{ce}\sqrt{-\frac{d}{c}}E(\arcsin\left(x\sqrt{-\frac{d}{c}}\right) \mid \frac{cf}{de}}{c^3d^2e^2-c^4def+(c}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-((b*c^2*d - a*c*d^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*e*x - ((b*c*d^2 - a*
d^3)*e*x^2 + (b*c^2*d - a*c*d^2)*e)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin
(x*sqrt(-d/c)), c*f/(d*e)) - (a*c^3*f + (a*c^2*d*f - (b*c^2*d + b*c*d^2 -
a*d^3)*e)*x^2 - (b*c^3 + b*c^2*d - a*c*d^2)*e)*sqrt(c*e)*sqrt(-d/c)*ellipt
ic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)))/(c^3*d^2*e^2 - c^4*d*e*f + (c^2*d^3
*e^2 - c^3*d^2*e*f)*x^2)
```

3.39.6 Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

3.39.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.39.8 Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.40 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

3.40.1 Optimal result

Integrand size = 30, antiderivative size = 284

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} \\ &+ \frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &- \frac{\sqrt{e}\sqrt{f}(2bce+ade-3acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-1/3*(-3*a*c*f+a*d*e+2*b*c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Elli
pticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/
2)*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2
+e)^(1/2)-1/3*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/
3*(2*a*d*(-2*c*f+d*e)+b*c*(c*f+d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/
2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2
+e)^(1/2)/c^(3/2)/(-c*f+d*e)^2/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(e+fx^2)\left(bc(2c^2f+d^2ex^2+cdfx^2)+ad(-5c^2f+2d^2ex^2+cd(3e-4\right.}{}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(2*c^2*f + d^2*e*x^2 + c*d*f*x^2) + a*d*(-5*
c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*e*(2*a*d*(d*e - 2*c*f) + b
*c*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(b*c*e + 2*a*d
*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
F[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*(d*e - c*f)^2*(c
+ d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

3.40.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \; 402$$

$$-\frac{\int -\frac{-((bc-ad)fx^2)+bce+2ade-3acf}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-((bc-ad)fx^2)+bce+2ade-3acf}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}$$

$$\Big\downarrow \text{400}$$

$$\frac{\frac{(2ad(de-2cf)+bc(cf+de))\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f(-3acf+ade+2bce)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{f(-3acf+ade+2bce)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
-1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(3/2)) + (
((2*a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*
c*e + a*d*e - 3*a*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2]))/(3*c*(d*e - c*f))
```

3.40. $\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx$

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.40.4 Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{x(ad-bc)\sqrt{df x^4+cf x^2+de x^2+ce}}{3d^2c(cf-de)\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(df x^2+de\right)x\left(4acdf-2ae d^2-c^2bf-bcde\right)}{3d c^2(cf-de)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{\left(-\frac{(ad-bc)f}{3dc(cf-de)}+\frac{4acdf-2ae d^2-c^2b}{3d(cf-de)c^2}\right.}{}\right.}{}$
default	Expression too large to display

3.40. $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

input
```
int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3/d^2/c/(c
*f-d*e)*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/3*(d
*f*x^2+d*e)/d/c^2/(c*f-d*e)^2*x*(4*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/((x^
2+c/d)*(d*f*x^2+d*e))^(1/2)+(-1/3*(a*d-b*c)/d*f/c/(c*f-d*e)+1/3/d/(c*f-d*e
)*(4*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/c^2+1/3*e/c^2/(c*f-d*e)^2*(4*a*c*d
*f-2*a*d^2*e-b*c^2*f-b*c*d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*
f+d*e)/e/d)^(1/2))-1/3*(4*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)
^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(272) = 544$.

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.15

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx =$$

$$-\frac{(((bcd^4+2\,ad^5)e^2+(bc^2d^3-4\,acd^4)ef)x^4+(bc^3d^2+2\,ac^2d^3)e^2+(bc^4d-4\,ac^3d^2)ef+2\,((bc^2d^3+2\,acd}{}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((((b*c*d^4 + 2*a*d^5)*e^2 + (b*c^2*d^3 - 4*a*c*d^4)*e*f)*x^4 + (b*c^
3*d^2 + 2*a*c^2*d^3)*e^2 + (b*c^4*d - 4*a*c^3*d^2)*e*f + 2*((b*c^2*d^3 + 2
*a*c*d^4)*e^2 + (b*c^3*d^2 - 4*a*c^2*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*e
lliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) + (3*a*c^5*f^2 + (3*a*c^3*d^2*f
^2 - (b*c*d^4 + 2*a*d^5)*e^2 - (2*b*c^3*d^2 + (a + b)*c^2*d^3 - 4*a*c*d^4)
*e*f)*x^4 - (b*c^3*d^2 + 2*a*c^2*d^3)*e^2 - (2*b*c^5 + (a + b)*c^4*d - 4*a
*c^3*d^2)*e*f + 2*(3*a*c^4*d*f^2 - (b*c^2*d^3 + 2*a*c*d^4)*e^2 - (2*b*c^4*
d + (a + b)*c^3*d^2 - 4*a*c^2*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic
_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - (((b*c^2*d^3 + 2*a*c*d^4)*e^2 + (b*c
^3*d^2 - 4*a*c^2*d^3)*e*f)*x^3 + (3*a*c^2*d^3*e^2 + (2*b*c^4*d - 5*a*c^3*d
^2)*e*f)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^5*d^3*e^3 - 2*c^6*d^2*e^2*
f + c^7*d*e*f^2 + (c^3*d^5*e^3 - 2*c^4*d^4*e^2*f + c^5*d^3*e*f^2)*x^4 + 2*
(c^4*d^4*e^3 - 2*c^5*d^3*e^2*f + c^6*d^2*e*f^2)*x^2)
```

3.40.6 Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

3.40.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.40.8 Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{b\,x^2+a}{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.41 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

3.41.1 Optimal result

Integrand size = 30, antiderivative size = 401

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)(c+dx^2)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2(c+dx^2)^{3/2}} + \frac{(bc(2d^2e^2-7cdef-3c^2f^2)+ad(8d^2e^2-23cdef+23c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{\sqrt{e}\sqrt{f}(bce(de-9cf)+a(4d^2e^2-11cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/15*(b*c*e*(-9*c*f+d*e)+a*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2))*(1/(1+f*x^2
/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2)
,(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-c*f+d*e)^3/(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2
)/c/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/15*(4*a*d*(-2*c*f+d*e)+b*c*(3*c*f+d*e))*x
*(f*x^2+e)^(1/2)/c^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*(b*c*(-3*c^2*f^2-7*
c*d*e*f+2*d^2*e^2)+a*d*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c
*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/(-c*f+d*e)^3/d^(1/2)/(d*x^2+c)^(1/2
)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \frac{-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2+c(-de+cf)(4ad(de-2cf)+bc\right.}{}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 + c*(-(d*e) +
c*f)*(4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2) + (a*d*(-8*d^2*
e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f + 3*c^2*f^2))
*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*(e*(a*d*(-8*d^2*e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*
e*f + 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e -
c*f)*(2*b*c*e*(d*e - 3*c*f) + a*(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Ell
ipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*c^3*Sqrt[d/c]*(d*e - c*f
)^3*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

3.41.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {402, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{402}$$

$$-\frac{\int -\frac{-3(bc-ad)fx^2+bce+4ade-5acf}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}(de-cf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3(bc-ad)fx^2+bce+4ade-5acf}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{5c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

$\big\downarrow$ 402

$$\frac{\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{3c(c+dx^2)^{3/2}(de-cf)}-\frac{\int -\frac{f(4ad(de-2cf)+bc(de+3cf))x^2+2bce(de-3cf)+a\left(8d^2e^2-19cdfe+15c^2f^2\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}}{5c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{f(4ad(de-2cf)+bc(de+3cf))x^2+2bce(de-3cf)+a\left(8d^2e^2-19cdfe+15c^2f^2\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{3c(c+dx^2)^{3/2}(de-cf)}}{5c(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

$\big\downarrow$ 400

$$\frac{\frac{\frac{\left(ad\left(23c^2f^2-23cdef+8d^2e^2\right)+bc\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)\int\frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}+x\sqrt{e+}}{5c(de-cf)}$$

$$\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

$\big\downarrow$ 313

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(23c^2f^2-23cdef+8d^2e^2\right)+bc\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{f\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+}}}{de-cf}}{3c(de-cf)}}{5c(de-cf)}$$

$$\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

$\big\downarrow$ 320

3.41. $\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

$$\frac{\frac{\frac{\sqrt{e+fx^2}\left(ad\left(23c^2f^2-23cdef+8d^2e^2\right)+bc\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)\text{Ellip}}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}}{5c(de-cf)}$$

$$\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\left(c+dx^2\right)^{5/2}(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
-1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(5/2)) + (
((4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2])/(3*c*(d*e -
c*f)*(c + d*x^2)^(3/2)) + (((b*c*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + a*d
*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c
 + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c*e
*(d*e - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]*
EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c*(d*e - c*f)))/(
5*c*(d*e - c*f))
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.41.4 Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{x(ad-bc)\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5d^3c(cf-de)\left(x^2+\frac{c}{d}\right)^3}-\frac{\left(8acdf-4ae\,d^2-3c^2bf-bcde\right)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{15c^2(cf-de)^2d^2\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(df\,x^2+de\right)x\left(23a\,c^2d\,f\right.}{15d}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5/d^3/c/(c
*f-d*e)*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3-1/15*(
8*a*c*d*f-4*a*d^2*e-3*b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)^2/d^2*x*(d*f*x^4+c*f*
x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/15*(d*f*x^2+d*e)/d/c^3/(c*f-d*e)^3*x*
(23*a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*e*f+2*b*c
*d^2*e^2)/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(-1/15*f*(8*a*c*d*f-4*a*d^2*e-3*
b*c^2*f-b*c*d*e)/d/c^2/(c*f-d*e)^2+1/15/d/(c*f-d*e)^2*(23*a*c^2*d*f^2-23*a
*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*e*f+2*b*c*d^2*e^2)/c^3+1/15*e
/c^3/(c*f-d*e)^3*(23*a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*
b*c^2*d*e*f+2*b*c*d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d
*e)/e/d)^(1/2))-1/15*(23*a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^
2-7*b*c^2*d*e*f+2*b*c*d^2*e^2)/c^3/(c*f-d*e)^3*e/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(
-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. 2(383) = 766.

Time = 0.14 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.13

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^3 - (7*b*c^2*d^5 + 23*a*c*d^6)*e^2*f - (3
*b*c^3*d^4 - 23*a*c^2*d^5)*e*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^3 -
 (7*b*c^3*d^4 + 23*a*c^2*d^5)*e^2*f - (3*b*c^4*d^3 - 23*a*c^3*d^4)*e*f^2)*
x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e^3 - (7*b*c^5*d^2 + 23*a*c^4*d^3)*e^2*f
 - (3*b*c^6*d - 23*a*c^5*d^2)*e*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^3 -
 (7*b*c^4*d^3 + 23*a*c^3*d^4)*e^2*f - (3*b*c^5*d^2 - 23*a*c^4*d^3)*e*f^2)*
x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - (1
5*a*c^7*f^3 + (15*a*c^4*d^3*f^3 + 2*(b*c*d^6 + 4*a*d^7)*e^3 + (b*c^3*d^4 +
 (4*a - 7*b)*c^2*d^5 - 23*a*c*d^6)*e^2*f - (9*b*c^4*d^3 + (11*a + 3*b)*c^3
*d^4 - 23*a*c^2*d^5)*e*f^2)*x^6 + 3*(15*a*c^5*d^2*f^3 + 2*(b*c^2*d^5 + 4*a
*c*d^6)*e^3 + (b*c^4*d^3 + (4*a - 7*b)*c^3*d^4 - 23*a*c^2*d^5)*e^2*f - (9*
b*c^5*d^2 + (11*a + 3*b)*c^4*d^3 - 23*a*c^3*d^4)*e*f^2)*x^4 + 2*(b*c^4*d^3
 + 4*a*c^3*d^4)*e^3 + (b*c^6*d + (4*a - 7*b)*c^5*d^2 - 23*a*c^4*d^3)*e^2*f
 - (9*b*c^7 + (11*a + 3*b)*c^6*d - 23*a*c^5*d^2)*e*f^2 + 3*(15*a*c^6*d*f^3
 + 2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^3 + (b*c^5*d^2 + (4*a - 7*b)*c^4*d^3 - 23
*a*c^3*d^4)*e^2*f - (9*b*c^6*d + (11*a + 3*b)*c^5*d^2 - 23*a*c^4*d^3)*e*f^
2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) -
 ((2*(b*c^2*d^5 + 4*a*c*d^6)*e^3 - (7*b*c^3*d^4 + 23*a*c^2*d^5)*e^2*f - (3
*b*c^4*d^3 - 23*a*c^3*d^4)*e*f^2)*x^5 + (5*(b*c^3*d^4 + 4*a*c^2*d^5)*e^3 -
 2*(6*b*c^4*d^3 + 29*a*c^3*d^4)*e^2*f - 9*(b*c^5*d^2 - 6*a*c^4*d^3)*e*f...
```

3.41.6 Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{7}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)/((c + d*x**2)**(7/2)*sqrt(e + f*x**2)), x)
```

3.41.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

3.41.8 Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

3.42 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

3.42.1 Optimal result

Integrand size = 30, antiderivative size = 501

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\,x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} + \frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(10adf(2de-3cf)-b(24d^2e^2-41cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-(-a*f+b*e)*x*(d*x^2+c)^(5/2)/e/f/(f*x^2+e)^(1/2)-1/15*(5*a*f*(3*c^2*f^2-1
3*c*d*e*f+8*d^2*e^2)-2*b*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*x*(d*x^2+c)
^(1/2)/e/f^3/(f*x^2+e)^(1/2)+1/15*(5*a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2)-
2*b*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e
)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(
d*x^2+c)^(1/2)/f^(7/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(
1/2)-1/15*(10*a*d*f*(-3*c*f+2*d*e)-b*(15*c^2*f^2-41*c*d*e*f+24*d^2*e^2))*(
1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*d*(-5*a*f+6*b*e)*x*(d*x^2+c)^(3/2)
*(f*x^2+e)^(1/2)/e/f^2-1/15*d*(b*e*(-23*c*f+24*d*e)-5*a*f*(-3*c*f+4*d*e))*
x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/f^3
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)\left(5af(-6cdef+3c^2f^2+d^2e(4e+fx^2))+be(-15c^2f^2+cdf\right.}{}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(5*a*f*(-6*c*d*e*f + 3*c^2*f^2 + d^2*e*(4*e + f
*x^2)) + b*e*(-15*c^2*f^2 + c*d*f*(41*e + 11*f*x^2) - 3*d^2*(8*e^2 + 2*e*f
*x^2 - f^2*x^4))) - I*d*e*(-5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + 2
*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) +
c*f)*(5*a*d*f*(-8*d*e + 9*c*f) + b*(48*d^2*e^2 - 64*c*d*e*f + 15*c^2*f^2))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)])/(15*Sqrt[d/c]*e*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.42.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(dx^2+c)^{3/2}(d(6be-5af)x^2+bce)}{\sqrt{fx^2+e}}dx}{ef}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx^2+c)^{3/2}(d(6be-5af)x^2+bce)}{\sqrt{fx^2+e}}dx}{ef}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int -\frac{\sqrt{dx^2+c}\left(d(be(24de-23cf)-5af(4de-3cf))x^2+ce(6bde-5bcf-5adf)\right)}{\sqrt{fx^2+e}}dx}{5f}+\frac{dx(c+dx^2)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}}{ef}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{dx(c+dx^2)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\int \frac{\sqrt{dx^2+c}\left(d(be(24de-23cf)-5af(4de-3cf))x^2+ce(6bde-5bcf-5adf)\right)}{\sqrt{fx^2+e}}dx}{5f}}{ef}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\frac{\int \frac{d\left(5af\left(8d^2e^2-13cdfe+3c^2f^2\right)-2be\left(24d^2e^2-44cdfe+19c^2f^2\right)\right)x^2+ce\left(10adf(2de-3cf)-b\left(24d^2e^2-41cdfe+15c^2f^2\right)\right.}{\sqrt{dx^2+c}\sqrt{fx^2+e}}}{3f}}{5f}}{ef}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\downarrow$ 406

$$\frac{\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\frac{ce\left(10adf(2de-3cf)-b\left(15c^2f^2-41cdef+24d^2e^2\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+d\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f\right.\right.}{3f}}{5f}}{ef}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\downarrow$ 320

$$\frac{\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\frac{d\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}\left(10adf(2de-3c\right.}{}}{3f}}{}}{ef}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\downarrow$ 388

$$\frac{\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\frac{d\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}\left(1\right.}{}}{3f}}{}}{ef}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\downarrow$ 313

3.42. $\displaystyle\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^{3/2}}\,dx$

$$\frac{\frac{dx(c+dx^2)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5f}-\frac{\frac{d\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{3f}}{}}{ef}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2])) + ((d*(6*b*e -
5*a*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*f) - ((d*(b*e*(24*d*e - 23*
c*f) - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (
d*(5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d
*e*f + 19*c^2*f^2))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sq
rt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3/2)
*(10*a*d*f*(2*d*e - 3*c*f) - b*(24*d^2*e^2 - 41*c*d*e*f + 15*c^2*f^2))*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f))/(5*f))
/(e*f)
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.42.4 Maple [A] (verified)

Time = 9.68 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.58

method	result
risch	$\frac{xd(3bdf\,x^2+5adf+11bcf-9bde)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{15f^3}+\frac{\left(-\frac{d\left(35acd\,f^2-25a\,d^2ef+23b\,c^2f^2-58bcdef+33b\,d^2e^2\right)e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\left(F\left(\ldots\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+c\ldots}}\ldots\right.}{\ldots}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)\left(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3\right)x}{e\,f^4\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{b\,d^2x^3\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5f^2}+\left(\frac{d^2(adf+3bc\ldots}{f^2}\ldots\right.\right.}{\ldots}$
default	Expression too large to display

input
```
int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*d*(3*b*d*f*x^2+5*a*d*f+11*b*c*f-9*b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/f^3+1/15/f^3*(-d*(35*a*c*d*f^2-25*a*d^2*e*f+23*b*c^2*f^2-58*b*c*d*e*
f+33*b*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d
)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))+(45*a*c^2*d*f
^3-50*a*c*d^2*e*f^2+15*a*d^3*e^2*f+15*b*c^3*f^3-56*b*c^2*d*e*f^2+54*b*c*d^
2*e^2*f-15*b*d^3*e^3)/f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))+(15*a*c^3*f^4-45*a*c^2*d*e*f^3+45*a*c*d^2*e^2*f^2-15*a*d^3*e^3
*f-15*b*c^3*e*f^3+45*b*c^2*d*e^2*f^2-45*b*c*d^2*e^3*f+15*b*d^3*e^4)/f*((d*
f*x^2+c*f)/e/(c*f-d*e)*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+(1/e-c*f/e/(c*f-d
*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+d/(c*f-
d*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f*x^2+e))^(1/2
)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.42. $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

3.42.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)\,(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{((48\,bd^3e^4f-15\,ac^2def^4-8\,(11\,bcd^2+5\,ad^3)e^3f^2+(38\,bc^2d+65\,acd^2)e^2f^3)x^3+(48\,bd^3e^5-15\,ac^2de^2f}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(((48*b*d^3*e^4*f - 15*a*c^2*d*e*f^4 - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*
f^2 + (38*b*c^2*d + 65*a*c*d^2)*e^2*f^3)*x^3 + (48*b*d^3*e^5 - 15*a*c^2*d*
e^2*f^3 - 8*(11*b*c*d^2 + 5*a*d^3)*e^4*f + (38*b*c^2*d + 65*a*c*d^2)*e^3*f
^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) -
((48*b*d^3*e^4*f - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*f^2 + (38*b*c^2*d + (65*a
+ 24*b)*c*d^2)*e^2*f^3 - ((15*a + 41*b)*c^2*d + 20*a*c*d^2)*e*f^4 + 15*(b*
c^3 + 2*a*c^2*d)*f^5)*x^3 + (48*b*d^3*e^5 - 8*(11*b*c*d^2 + 5*a*d^3)*e^4*f
 + (38*b*c^2*d + (65*a + 24*b)*c*d^2)*e^3*f^2 - ((15*a + 41*b)*c^2*d + 20*
a*c*d^2)*e^2*f^3 + 15*(b*c^3 + 2*a*c^2*d)*e*f^4)*x)*sqrt(d*f)*sqrt(-e/f)*e
lliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^3*e*f^4*x^6 + 48*b*d^3
*e^4*f - 15*a*c^2*d*e*f^4 - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*f^2 + (38*b*c^2*d
 + 65*a*c*d^2)*e^2*f^3 - (6*b*d^3*e^2*f^3 - (11*b*c*d^2 + 5*a*d^3)*e*f^4)*
x^4 + (24*b*d^3*e^3*f^2 - (47*b*c*d^2 + 20*a*d^3)*e^2*f^3 + (23*b*c^2*d +
35*a*c*d^2)*e*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^6*x^3 + d*
e^2*f^5*x)
```

3.42.6 Sympy [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2)**(3/2), x)
```

3.42.7 Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

3.42.8 Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2), x)
```

3.43 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

3.43.1 Optimal result

Integrand size = 30, antiderivative size = 358

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = &-\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}}\\ &-\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}}+\frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2}\\ &+\frac{(be(8de-7cf)-3af(2de-cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &-\frac{\sqrt{e}(4bde-3bcf-3adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-(-a*f+b*e)*x*(d*x^2+c)^(3/2)/e/f/(f*x^2+e)^(1/2)-1/3*(b*e*(-7*c*f+8*d*e)-
3*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^(1/2)+1/3*(b*e*(-7*c
*f+8*d*e)-3*a*f*(-c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Elli
pticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/f^(5/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3
*a*d*f-3*b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(
1/2)/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-3*a*f
+4*b*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/f^2
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)\left(3af(-de+cf)+be(4de-3cf+dfx^2)\right)-ide(-3af(-2de$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
(Sqrt[d/c]*f*x*(c + d*x^2)*(3*a*f*(-(d*e) + c*f) + b*e*(4*d*e - 3*c*f + d*
f*x^2)) - I*d*e*(-3*a*f*(-2*d*e + c*f) + b*e*(-8*d*e + 7*c*f))*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)
] - I*e*(-(d*e) + c*f)*(-8*b*d*e + 3*b*c*f + 6*a*d*f)*Sqrt[1 + (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqr
t[d/c]*e*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.43.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {401, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{\sqrt{dx^2+c}\left(d(4be-3af)x^2+bce\right)}{\sqrt{fx^2+e}}dx}{ef}-\frac{x(c+dx^2)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{dx^2+c}\left(d(4be-3af)x^2+bce\right)}{\sqrt{fx^2+e}}dx}{ef}-\frac{x(c+dx^2)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{d(be(8de-7cf)-3af(2de-cf))x^2+ce(4bde-3bcf-3adf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}}{ef}-\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}-\frac{\int \frac{d(be(8de-7cf)-3af(2de-cf))x^2+ce(4bde-3bcf-3adf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}}{ef}-\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}-\frac{ce(-3adf-3bcf+4bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+d(be(8de-7cf)-3af(2de-cf))\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}}{ef}-\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}-\frac{d(be(8de-7cf)-3af(2de-cf))\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(-3adf-3bcf+4bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\cdots}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}}{ef}\cdots$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}-\frac{d(be(8de-7cf)-3af(2de-cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(-3adf-3bcf+4bde)\,\text{EllipticF}\left(\text{arct}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}}{ef}\cdots$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 313$$

3.43. $\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{3/2}}\,dx$

$$\frac{\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3f}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}(-3adf-3bcf+4bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+d(be(8de-7cf)-3af(2de-cf))\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\cdots}{\cdots}\right.}{3f}}{ef}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*f*Sqrt[e + f*x^2])) + ((d*(4*b*e -
3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) - (d*(b*e*(8*d*e - 7*c*f)
- 3*a*f*(2*d*e - c*f))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]
*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3
/2)*(4*b*d*e - 3*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f
]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]))/(3*f))/(e*f)
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.43.4 Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.52

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(dfx^2+cf)(acf^2-adef-bcef+bde^2)x}{f^3e\sqrt{(x^2+\frac{e}{f})(dfx^2+cf)}}+\frac{bdx\sqrt{dfx^4+cfx^2+dex^2+ce}}{3f^2}+\frac{\left(\frac{2acdf^2-ad^2ef+bc^2f^2-2bcdef+bd^2e^2}{f^3}+(\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{bx\sqrt{dx^2+c}\sqrt{fx^2+e}\,d}{3f^2}+\frac{\left(-\frac{d(3adf+4bcf-5bde)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}\,f}+\frac{(6acdf^2\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}bd^2ef^2x^5+3\sqrt{-\frac{d}{c}}acdf^3x^3-3\sqrt{-\frac{d}{c}}ad^2ef^2x^3-2\sqrt{-\frac{d}{c}}bcdef^2x^3+4\sqrt{-\frac{d}{c}}bd^2e^2fx^3+6\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx\cdots}{}}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)
*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^3/e*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)
+1/3*b*d/f^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+((2*a*c*d*f^2-a*d^2*e*f
+b*c^2*f^2-2*b*c*d*e*f+b*d^2*e^2)/f^3+(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^
3*(c*f-d*e)/e-c/f^2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e-1/3*b*d/f^2*c*e)/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(d/f^2*(a*d*f
+2*b*c*f-b*d*e)-(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2*d/e-1/3*b*d/f^2*(2*c
*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/
2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \frac{((8\,bd^2e^3f+3\,acdef^3-(7\,bcd+6\,ad^2)e^2f^2)x^3+(8\,bd^2e^4+3\,acde^2f^2-(7\,bc\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((8*b*d^2*e^3*f + 3*a*c*d*e*f^3 - (7*b*c*d + 6*a*d^2)*e^2*f^2)*x^3 +
(8*b*d^2*e^4 + 3*a*c*d*e^2*f^2 - (7*b*c*d + 6*a*d^2)*e^3*f)*x)*sqrt(d*f)*s
qrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((8*b*d^2*e^3*f +
(3*a + 4*b)*c*d*e*f^3 - (7*b*c*d + 6*a*d^2)*e^2*f^2 - 3*(b*c^2 + a*c*d)*f^
4)*x^3 + (8*b*d^2*e^4 + (3*a + 4*b)*c*d*e^2*f^2 - (7*b*c*d + 6*a*d^2)*e^3*
f - 3*(b*c^2 + a*c*d)*e*f^3)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqr
t(-e/f)/x), c*f/(d*e)) + (b*d^2*e*f^3*x^4 - 8*b*d^2*e^3*f - 3*a*c*d*e*f^3
+ (7*b*c*d + 6*a*d^2)*e^2*f^2 - (4*b*d^2*e^2*f^2 - (4*b*c*d + 3*a*d^2)*e*f
^3)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^5*x^3 + d*e^2*f^4*x)
```

3.43.6 Sympy [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

3.43.7 Maxima [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

3.43.8 Giac [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

3.44 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.44.1 Optimal result

Integrand size = 30, antiderivative size = 258

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{(2be-af)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^(1/2)+(-a*f+2*b*e)*x*(d*x^2+c)
^(1/2)/e/f/(f*x^2+e)^(1/2)-(-a*f+2*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*
x^2+c)^(1/2)/f^(3/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/
2)+b*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}f(-be+af)x(c+dx^2)-ide(2be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{\vphantom{d}}}{\sqrt{\frac{d}{c}}ef^2\sqrt{c}}$$

input
```
Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
(Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(2*b*e - a*f)*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - I*e*(-2*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f^2*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])
```

3.44.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{d(2be-af)x^2+bce}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d(2be-af)x^2+bce}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{d(2be-af)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+bce\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\big\downarrow$ 320

$$\frac{d(2be-af)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\big\downarrow$ 388

$$\frac{d(2be-af)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

$\big\downarrow$ 313

$$\frac{d(2be-af)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{ef}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}$$

input
```
Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + (d*(2*b*e - a*f
)*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (b*e^(3/2)*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(e*f)
```

3.44. $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.44. $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.44.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(dfx^2+cf)(af-be)x}{ef^2\sqrt{\left(x^2+\frac{e}{f}\right)(dfx^2+cf)}}+\frac{\left(\frac{adf+bcf-bde}{f^2}+\frac{(cf-de)(af-be)}{f^2e}-\frac{c(af-be)}{fe}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+d}{ed}}\right.}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}\right.}{\sqrt{dx^2+c}\sqrt{fx^2+e}}$
default	$\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}adf^2x^3-\sqrt{-\frac{d}{c}}bdefx^3+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{c}{d}}\right.\right.}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)
*(a*f-b*e)/e/f^2*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+((a*d*f+b*c*f-b*d*e)/f^
2+(c*f-d*e)*(a*f-b*e)/f^2/e-c/f*(a*f-b*e)/e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2
)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(b*d/f-(a*f-b*e)/f*d/e)*e/(-d/c)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(E
llipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2)
,(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx =$$
$$-\frac{((2bde^2f-adef^2)x^3+(2bde^3-ade^2f)x)\sqrt{df}\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right) \mid \frac{cf}{de})-((2bde^2f-adef^2+bcf^3)x}{def}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
-(((2*b*d*e^2*f - a*d*e*f^2)*x^3 + (2*b*d*e^3 - a*d*e^2*f)*x)*sqrt(d*f)*sq
rt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((2*b*d*e^2*f - a*d
*e*f^2 + b*c*f^3)*x^3 + (2*b*d*e^3 - a*d*e^2*f + b*c*e*f^2)*x)*sqrt(d*f)*s
qrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (b*d*e*f^2*x^2 + 2
*b*d*e^2*f - a*d*e*f^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^4*x^3 + d*
e^2*f^3*x)
```

3.44.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

3.44.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.44.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.45 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx$

3.45.1 Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{(be-af)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(bc-ad)\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-c*f+d*e)/e^(1/
2)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1/(
1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e
)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)/f^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}f(-be+af)x(c+dx^2)-ide(be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{d}{c}}ef(-de+cf}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(b*e - a*f)*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f*(-(d*e) + c*f)*Sqrt[c
 + d*x^2]*Sqrt[e + f*x^2])
```

3.45.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 400}$$

$$\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{(bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[Arc
Tan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.45.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.45.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a d\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, b d e f\, x^{3}+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) b c e f-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) b d\, e^{2}-\sqrt{\frac{d\, x^{2}+c}{c}}}{\sqrt{-\frac{d}{c}}\, e f (c f-d e)(d f\, x^{4}-}$
elliptic	$\frac{\sqrt{(d\, x^{2}+c)(f\, x^{2}+e)}\, \left(\frac{\left(d f\, x^{2}+c f\right) x (a f-b e)}{f e (c f-d e) \sqrt{\left(x^{2}+\frac{e}{f}\right)\left(d f\, x^{2}+c f\right)}}+\frac{\left(\frac{b}{f}+\frac{a f-b e}{f e}-\frac{c (a f-b e)}{e (c f-d e)}\right) \sqrt{1+\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{f\, x^{2}}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{-1+\frac{c f+d e}{e d}}\right)}{\sqrt{-\frac{d}{c}}\, \sqrt{d f\, x^{4}+c f\, x^{2}+d e\, x^{2}+c e}}+\frac{d (a f-b e)}{}\right.}{\sqrt{d\, x^{2}+c}\, \sqrt{f\, x^{2}+e}}$

input
```
int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*d*f^2*x^3-(-d/c)^(1/2)*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*b*d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x-(-d/c)^(1/
2)*b*c*e*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/e/f/(c*f-d*e)/(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.23

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{(bcdef-acdf^2)\sqrt{dx^2+c}\sqrt{fx^2+e}x-(bd^2e^2-ad^2ef+(bd^2ef-ad^2f^2)x^2)}{}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
((b*c*d*e*f - a*c*d*f^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x - (b*d^2*e^2 -
a*d^2*e*f + (b*d^2*e*f - a*d^2*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(a
rcsin(x*sqrt(-d/c)), c*f/(d*e)) + (b*d^2*e^2 + (b*c^2 - a*c*d - a*d^2)*e*f
 + (b*d^2*e*f + (b*c^2 - a*c*d - a*d^2)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*ell
iptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)))/(c*d^2*e^3*f - c^2*d*e^2*f^2 + (
c*d^2*e^2*f^2 - c^2*d*e*f^3)*x^2)
```

3.45.6 Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

3.45.7 Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.45.8 Giac [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.46 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

3.46.1 Optimal result

Integrand size = 30, antiderivative size = 272

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx &= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}\\ &- \frac{\sqrt{f}(2bce-ade-acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &+ \frac{\sqrt{e}(bde+bcf-2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+(-2*a*d*f+b*c*f
+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2
)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e
)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*c*f-a*d*e+
2*b*c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/
2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*
e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}x(a(c^2f^2+cdf^2x^2+d^2e(e+fx^2))-bce(cf+d(e+2fx^2)))-i\right.}{}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[d/c]*(Sqrt[d/c]*x*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2)) - b
*c*e*(c*f + d*(e + 2*f*x^2))) - I*d*e*(2*b*c*e - a*(d*e + c*f))*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - I*(b*c - a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*e*(d*e - c*f)^2*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

3.46.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 402$$

$$-\frac{\int -\frac{c(be-af)-(bc-ad)fx^2}{\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{c(de-cf)}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c(be-af)-(bc-ad)fx^2}{\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{c(de-cf)}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \ 400$$

$$\frac{\frac{c(-2adf+bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{f(-acf-ade+2bce)\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{de-cf}}{c(de-cf)}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \ 313$$

$$\frac{\frac{c(-2adf+bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2bce)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{c(de-cf)}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \ 320$$

$$\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2bce)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{c(de-cf)}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
-(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) + (-((S
qrt[f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sq
rt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/
(c*(d*e - c*f))
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.46.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{2df\left(-\frac{(acf+ade-2bce)x^3}{2ce\left(c^2f^2-2cdef+d^2e^2\right)}-\frac{\left(c^2a\,f^2+a\,d^2e^2-b\,c^2ef-bcd\,e^2\right)x}{2ce\left(c^2f^2-2cdef+d^2e^2\right)df}\right)}{\sqrt{\left(x^4+\frac{(cf+de)x^2}{df}+\frac{ce}{df}\right)df}}+\frac{\left(\frac{a}{ce}-\frac{c^2a\,f^2+a\,d^2e^2-b\,c^2ef-bcd\,e^2}{ce\left(c^2f^2-2cdef+d^2e^2\right)}\right)\sqrt{1+\cdots}}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\cdots}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{\cdots}}$
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,acd\,f^2x^3+\sqrt{-\frac{d}{c}}\,a\,d^2ef\,x^3-2\sqrt{-\frac{d}{c}}\,bcdef\,x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdef+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-2*d*f*(-1/2*
(a*c*f+a*d*e-2*b*c*e)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2)*x^3-1/2*(a*c^2*f^2+a
*d^2*e^2-b*c^2*e*f-b*c*d*e^2)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2)/d/f*x)/((x^4
+(c*f+d*e)/d/f*x^2+c*e/d/f)*d*f)^(1/2)+(a/c/e-(a*c^2*f^2+a*d^2*e^2-b*c^2*e
*f-b*c*d*e^2)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+d*(a*c*f+a*d*e-2*b*c*e)/c/(c^2*f^2-2*c*
d*e*f+d^2*e^2)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/
2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(266) = 532$.

Time = 0.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.27

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{(ac^2d^2ef + (acd^3f^2 - (2\,bcd^3 - ad^4)ef)x^4 - (2\,bc^2d^2 - acd^3)e^2 + (ac^2d^2f^2 - (2\,bcd^3 - ad^4)e^2 - 2\,(bc^2d^2 -}{}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
-((a*c^2*d^2*e*f + (a*c*d^3*f^2 - (2*b*c*d^3 - a*d^4)*e*f)*x^4 - (2*b*c^2*
d^2 - a*c*d^3)*e^2 + (a*c^2*d^2*f^2 - (2*b*c*d^3 - a*d^4)*e^2 - 2*(b*c^2*d
^2 - a*c*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c
)), c*f/(d*e)) + (((b*c^2*d^2 + 2*b*c*d^3 - a*d^4)*e*f + (b*c^3*d - 2*a*c^
2*d^2 - a*c*d^3)*f^2)*x^4 + (b*c^3*d + 2*b*c^2*d^2 - a*c*d^3)*e^2 + (b*c^4
 - 2*a*c^3*d - a*c^2*d^2)*e*f + ((b*c^2*d^2 + 2*b*c*d^3 - a*d^4)*e^2 + 2*(
b*c^3*d - (a - b)*c^2*d^2 - a*c*d^3)*e*f + (b*c^4 - 2*a*c^3*d - a*c^2*d^2)
*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)
) - ((a*c^2*d^2*f^2 - (2*b*c^2*d^2 - a*c*d^3)*e*f)*x^3 - (b*c^3*d*e*f - a*
c^3*d*f^2 + (b*c^2*d^2 - a*c*d^3)*e^2)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))
/(c^3*d^3*e^4 - 2*c^4*d^2*e^3*f + c^5*d*e^2*f^2 + (c^2*d^4*e^3*f - 2*c^3*d
^3*e^2*f^2 + c^4*d^2*e*f^3)*x^4 + (c^2*d^4*e^4 - c^3*d^3*e^3*f - c^4*d^2*e
^2*f^2 + c^5*d*e*f^3)*x^2)
```

3.46.6 Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

3.46.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.46.8 Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.47 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}} dx$

3.47.1 Optimal result

Integrand size = 30, antiderivative size = 375

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}} dx = -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{f}(bce(de+7cf)+a(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}(ad(de-9cf)+bc(5de+3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/3*(2*a*d*
(-3*c*f+d*e)+b*c*(3*c*f+d*e))*x/c^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)
^(1/2)+1/3*(b*c*e*(7*c*f+d*e)+a*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^3/e^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*d*(-9*c*f+d*e)+b*c*(3
*c*f+5*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e
^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2
)/c^2/(-c*f+d*e)^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.39 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(-bce(3c^3f^2+d^3ex^2(e+fx^2)+cd^2fx^2(4e+7fx^2)+c^2df(5e+11f}{}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[d/c]*x*(-(b*c*e*(3*c^3*f^2 + d^3*e*x^2*(e + f*x^2) + c*d^2*f*x^2*(4*
e + 7*f*x^2) + c^2*d*f*(5*e + 11*f*x^2))) + a*(3*c^4*f^3 + 6*c^3*d*f^3*x^2
 - 2*d^4*e^2*x^2*(e + f*x^2) + c^2*d^2*f*(8*e^2 + 8*e*f*x^2 + 3*f^2*x^4) +
 c*d^3*e*(-3*e^2 + 4*e*f*x^2 + 7*f^2*x^4))) - I*d*e*(b*c*e*(d*e + 7*c*f) +
 a*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sq
rt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(
d*e) + c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)])/(3*c^2*Sqrt[d/c]*e*(-(d*e) + c*f)^3*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2
])
```

3.47.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {402, 25, 402, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 402$$

$$-\frac{\int -\frac{-3(bc-ad)fx^2+bce+2ade-3acf}{(dx^2+c)^{3/2}(fx^2+e)^{3/2}}dx}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3(bc-ad)fx^2+bce+2ade-3acf}{(dx^2+c)^{3/2}(fx^2+e)^{3/2}}dx}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{x(2ad(de-3cf)+bc(3cf+de))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{\int \frac{f\left(c(4bce-ade-3acf)-(2ad(de-3cf)+bc(de+3cf))x^2\right)}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x(2ad(de-3cf)+bc(3cf+de))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\int \frac{c(4bce-ade-3acf)-(2ad(de-3cf)+bc(de+3cf))x^2}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{\frac{x(2ad(de-3cf)+bc(3cf+de))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{c(ad(de-9cf)+bc(3cf+5de))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{de-cf}\right)}{c(de-cf)}}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\frac{x(2ad(de-3cf)+bc(3cf+de))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{c(ad(de-9cf)+bc(3cf+5de))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(\arctan\left(\cdots\right.\right.}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\cdots\right)}{c(de-cf)}}{3c(de-cf)}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

$$\Big\downarrow \text{ 320}$$

3.47. $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

$$\frac{\frac{x(2ad(de-3cf)+bc(3cf+de))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{\sqrt{e}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7c\cdots\right.}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\cdots}\right.}{c(de-cf)}}{3c(de-cf)}$$

$$\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
-1/3*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) + (
((2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(c*(d*e - c*f)*Sqrt[c + d*x^
2]*Sqrt[e + f*x^2]) - (f*(-(((b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c*d*e
*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(a*d*(d*e - 9*c*f) + b*c*(5*d*e + 3*
c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2])))/(c*(d*e - c*f)))/(3*c*(d*e - c*f))
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.47.4 Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.79

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{\left(df\,x^2+cf\right)fx(af-be)}{e(cf-de)^3\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{x(ad-bc)\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3dc(cf-de)^2\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df\,x^2+de\right)x\left(7acdf-2ae\,d^2-4c^2bf-bcde\right)}{3c^2(cf-de)^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)
*f/e/(c*f-d*e)^3*x*(a*f-b*e)/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+1/3/d/c/(c*f-
d*e)^2*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*
f*x^2+d*e)/c^2/(c*f-d*e)^3*x*(7*a*c*d*f-2*a*d^2*e-4*b*c^2*f-b*c*d*e)/((x^2
+c/d)*(d*f*x^2+d*e))^(1/2)+(1/(c*f-d*e)^2*f*(a*f-b*e)/e-c*f^2/e/(c*f-d*e)^
3*(a*f-b*e)+1/3*(a*d-b*c)*f/c/(c*f-d*e)^2-1/3/(c*f-d*e)^2*(7*a*c*d*f-2*a*d
^2*e-4*b*c^2*f-b*c*d*e)/c^2-1/3*d*e/c^2/(c*f-d*e)^3*(7*a*c*d*f-2*a*d^2*e-4
*b*c^2*f-b*c*d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^
(1/2))-(-d*f^2*(a*f-b*e)/e/(c*f-d*e)^3-1/3*d*f*(7*a*c*d*f-2*a*d^2*e-4*b*c^
2*f-b*c*d*e)/c^2/(c*f-d*e)^3)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)
^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1
+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))
)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1259 vs. $2(357) = 714$.

Time = 0.15 (sec) , antiderivative size = 1259, normalized size of antiderivative = 3.36

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((3*a*c^4*d^2*e*f^2 + (3*a*c^2*d^4*f^3 - (b*c*d^5 + 2*a*d^6)*e^2*f - 7
*(b*c^2*d^4 - a*c*d^5)*e*f^2)*x^6 + (6*a*c^3*d^3*f^3 - (b*c*d^5 + 2*a*d^6)
*e^3 - 3*(3*b*c^2*d^4 - a*c*d^5)*e^2*f - (14*b*c^3*d^3 - 17*a*c^2*d^4)*e*f
^2)*x^4 - (b*c^3*d^3 + 2*a*c^2*d^4)*e^3 - 7*(b*c^4*d^2 - a*c^3*d^3)*e^2*f
+ (3*a*c^4*d^2*f^3 - 2*(b*c^2*d^4 + 2*a*c*d^5)*e^3 - 3*(5*b*c^3*d^3 - 4*a*
c^2*d^4)*e^2*f - (7*b*c^4*d^2 - 13*a*c^3*d^3)*e*f^2)*x^2)*sqrt(c*e)*sqrt(-
d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) + (((b*c*d^5 + 2*a*d^6)*e
^2*f + (5*b*c^3*d^3 + (a + 7*b)*c^2*d^4 - 7*a*c*d^5)*e*f^2 + 3*(b*c^4*d^2
- 3*a*c^3*d^3 - a*c^2*d^4)*f^3)*x^6 + ((b*c*d^5 + 2*a*d^6)*e^3 + (5*b*c^3*
d^3 + (a + 9*b)*c^2*d^4 - 3*a*c*d^5)*e^2*f + (13*b*c^4*d^2 - 7*(a - 2*b)*c
^3*d^3 - 17*a*c^2*d^4)*e*f^2 + 6*(b*c^5*d - 3*a*c^4*d^2 - a*c^3*d^3)*f^3)*
x^4 + (b*c^3*d^3 + 2*a*c^2*d^4)*e^3 + (5*b*c^5*d + (a + 7*b)*c^4*d^2 - 7*a
*c^3*d^3)*e^2*f + 3*(b*c^6 - 3*a*c^5*d - a*c^4*d^2)*e*f^2 + (2*(b*c^2*d^4
+ 2*a*c*d^5)*e^3 + (10*b*c^4*d^2 + (2*a + 15*b)*c^3*d^3 - 12*a*c^2*d^4)*e^
2*f + (11*b*c^5*d - (17*a - 7*b)*c^4*d^2 - 13*a*c^3*d^3)*e*f^2 + 3*(b*c^6
- 3*a*c^5*d - a*c^4*d^2)*f^3)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(
x*sqrt(-d/c)), c*f/(d*e)) - ((3*a*c^3*d^3*f^3 - (b*c^2*d^4 + 2*a*c*d^5)*e^
2*f - 7*(b*c^3*d^3 - a*c^2*d^4)*e*f^2)*x^5 + (6*a*c^4*d^2*f^3 - (b*c^2*d^4
 + 2*a*c*d^5)*e^3 - 4*(b*c^3*d^3 - a*c^2*d^4)*e^2*f - (11*b*c^4*d^2 - 8*a*
c^3*d^3)*e*f^2)*x^3 - (3*a*c^2*d^4*e^3 + 3*b*c^5*d*e*f^2 - 3*a*c^5*d*f^...
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

3.47.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.47.8 Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.48 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

3.48.1 Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{(de-cf)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-(-c*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1
/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(-a*f+b*e)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1/2)/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)-ibc(-de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}{\sqrt{\frac{b}{a}}cd(-bc+ad)}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

3.48.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{400}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \text{313}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
-(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.48.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

3.48.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bcdf\, x^3+\sqrt{-\frac{b}{a}}\, b\, d^2 e\, x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) acdf-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) b\,c^2 f+\sqrt{\frac{b\,x^2+a}{a}}\right.}{\sqrt{-\frac{b}{a}}\, cd(ad-bc)(bd\,x^4}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{\left(bd\,x^2+ad\right)x(cf-de)}{dc(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(cf-de)}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

input
```
int((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*c*d*f*x^3+(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f-((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*b*c^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e-(-b/a)^(1/2)*a*c*d*f*x+(-b/a)^(1
/2)*a*d^2*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/c/d/(a*d-b*c)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.24

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(abd^2e-abcdf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)|}{ab^2c^3d-a^2bc^2d^2+}$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-((a*b*d^2*e - a*b*c*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (b^2*c*d*e -
 b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((a*b + b^2)*c*d*e + ((a*b + b^2)*d^2*e
 - (b^2*c*d + a^2*d^2)*f)*x^2 - (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(-b/a
)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*b^2*c^3*d - a^2*b*c^2*d^
2 + (a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^2)
```

3.48.6 Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.48.7 Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.48.8 Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.49 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c+dx^2)^{3/2}}\,dx$

3.49.1 Optimal result

Integrand size = 31, antiderivative size = 247

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\sqrt{a}\sqrt{b}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(-c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(a*d+b*c)/(d*x^2+c)^(1/2)+(-c*f+d*e)*Ellip
ticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)
*(d*x^2+c)^(1/2)/c/d/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+f*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de-cf)x(a-bx^2)+ibc(-de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(}{\sqrt{-\frac{b}{a}}cd(bc+}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c +
 a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {402, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}-\frac{\int -\frac{b(de-cf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b(de-cf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \; 399$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

3.49. $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c+dx^2)^{3/2}}\,dx$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + ((Sqrt[a
]*Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2
)/c]) + (Sqrt[a]*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a -
 b*x^2]*Sqrt[c + d*x^2]))/(c*(b*c + a*d))
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.49.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bcdf\, x^{3}-\sqrt{\frac{b}{a}}\, b\, d^{2} e\, x^{3}+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\, F\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) acdf+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\, F\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) b\,c^{2} f-\sqrt{\frac{-b\,x^{2}+a}{a}}}{\sqrt{\frac{b}{a}}\, cd(ad+bc)(-bd\,x^{4}+}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(d\,x^{2}+c)}\left(-\frac{\left(-bd\,x^{2}+ad\right)x(cf-de)}{dc(ad+bc)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(-bd\,x^{2}+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad+bc)}\right)\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, F\left(x\sqrt{\frac{b}{a}}, \sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}+\frac{b(cf-}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

input
```
int((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b*c*d*f*x^3-(b/a)^(1/2)*b*d^2*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*f+((-b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))
*b*c^2*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),
(-a*d/b/c)^(1/2))*b*c^2*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e-(b/a)^(1/2)*a*c*d*f*x+(b/a)^(1/
2)*a*d^2*e*x)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/c/d/(a*d+b*c)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.04

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(abd^2e-abcdf)\sqrt{-bx^2+a}\sqrt{dx^2+c}x+(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)}{}$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
((a*b*d^2*e - a*b*c*d*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x + (b^2*c*d*e -
 b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(b/a)*elliptic_e(a
rcsin(x*sqrt(b/a)), -a*d/(b*c)) + ((a*b - b^2)*c*d*e + ((a*b - b^2)*d^2*e
+ (b^2*c*d + a^2*d^2)*f)*x^2 + (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(b/a)*
elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a*b^2*c^3*d + a^2*b*c^2*d^2
+ (a*b^2*c^2*d^2 + a^2*b*c*d^3)*x^2)
```

3.49.6 Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.49.7 Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.49.8 Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.50 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}} \, dx$

3.50.1 Optimal result

Integrand size = 31, antiderivative size = 237

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx &= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} \\ &- \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} \\ &+ \frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
(c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(a*d+b*c)/(-d*x^2+c)^(1/2)-(c*f+d*e)*Ellipti
cE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)/(
a*d+b*c)/c^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*EllipticF(x*
d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c^(1
/2)/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.75 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.90

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de+cf)x(a+bx^2)-ibc(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{\frac{b}{a}}}{\sqrt{\frac{b}{a}}cd(bc+ad)}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]
 + I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a +
b*x^2]*Sqrt[c - d*x^2])
```

3.50.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {402, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{\int \frac{c(be-af)-b(de+cf)x^2}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \; 399$$

$$\frac{e(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \; 323$$

$$\frac{\frac{e\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

3.50. $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}}\,dx$

output
```
((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) + (-((Sqrt
[c]*(d*e + c*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*
x^2])) + (Sqrt[c]*(b*c + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*
x^2]*Sqrt[c - d*x^2]))/(c*(b*c + a*d))
```

3.50.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\, bcf\, x^3+\sqrt{\frac{d}{c}}\, bde\, x^3+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\, F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right) ade+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\, F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right) bce-\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{b\,x}\right.}{\sqrt{\frac{d}{c}}\, c(ad+bc)(-bd\,x^4-ad\,x^2}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(-d\,x^2+c)}\left(-\frac{\left(-bd\,x^2-ad\right)x(cf+de)}{dc(ad+bc)\sqrt{\left(x^2-\frac{c}{d}\right)\left(-bd\,x^2-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad+bc)}\right)\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\, F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+cb\,x^2+ac}}+\frac{(c}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}$

input
```
int((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d/c)^(1/2)*b*c*f*x^3+(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((b*x^2+
a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*d*e+((-d*x^2+c)/c)
^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*b*c*e
-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/a/
d)^(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)
^(1/2),(-b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(b
*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d+b*c)/(-b*d*x^4-a*d*x^2+b
*c*x^2+a*c)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{(acd^2e+ac^2df)\sqrt{bx^2+a}\sqrt{-dx^2+c}\,x-(acd^2e+ac^2df-(ad^3e+acd^2f)x^2)}{}$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
((a*c*d^2*e + a*c^2*d*f)*sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e +
 a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(arc
sin(x*sqrt(d/c)), -b*c/(a*d)) - (((b*c^2*d + a*d^3)*e - (a*c^2*d - a*c*d^2
)*f)*x^2 - (b*c^3 + a*c*d^2)*e + (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)*
elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)))/(a*b*c^4*d + a^2*c^3*d^2 - (a
*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

3.50.6 Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

3.50.7 Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.50.8 Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.51 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c-dx^2)^{3/2}}\,dx$

3.51.1 Optimal result

Integrand size = 32, antiderivative size = 242

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx &= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} \\ &+ \frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} \\ &+ \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-(c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-d*x^2+c)^(1/2)+(c*f+d*e)*Elli
pticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(-b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2
)/(-a*d+b*c)/c^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*Elliptic
F(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c
^(1/2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de+cf)x(a-bx^2)+ibc(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{}}{\sqrt{-\frac{b}{a}}cd(-bc+c}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*
c)] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*
d)*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

3.51.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {402, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 402$$

$$-\frac{\int -\frac{c(be+af)-b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c(be+af)-b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \; 399$$

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+e(bc-ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 323

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{e\sqrt{1-\frac{dx^2}{c}}(bc-ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 323

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 321

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 331

$$\frac{\frac{\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 330

$$\frac{\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

3.51. $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c-dx^2)^{3/2}}\,dx$

output
```
-(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((Sqr
t[c]*(d*e + c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^
2]) + (Sqrt[c]*(b*c - a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellip
ticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*S
qrt[c - d*x^2]))/(c*(b*c - a*d))
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.51.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(-\sqrt{\frac{d}{c}}\, bcf\, x^{3}-\sqrt{\frac{d}{c}}\, bde\, x^{3}+\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\, F\left(x\sqrt{\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right) ade-\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\, F\left(x\sqrt{\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right) bce-\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{-}\right.}{\sqrt{\frac{d}{c}}\, c(ad-bc)(bd\,x^{4}-ad\,x}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(-d\,x^{2}+c)}\left(-\frac{\left(bd\,x^{2}-ad\right)x(cf+de)}{dc(ad-bc)\sqrt{\left(x^{2}-\frac{c}{d}\right)\left(bd\,x^{2}-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad-bc)}\right)\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\, F\left(x\sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}+\frac{(c}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

input
```
int((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(d/c)^(1/2)*b*c*f*x^3-(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((-b*x^
2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*a*d*e-((-d*x^2+c)/c
)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*b*c*
e-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/a
/d)^(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/
c)^(1/2),(b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(
-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d-b*c)/(b*d*x^4-a*d*x^2-
b*c*x^2+a*c)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.06

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx =$$

$$-\frac{(acd^2e+ac^2df)\sqrt{-bx^2+a}\sqrt{-dx^2+c}x-(acd^2e+ac^2df-(ad^3e+acd^2f)x^2)\sqrt{ac}\sqrt{\frac{d}{c}}E(\arcsin\left(x\sqrt{\frac{d}{c}}\right)|}{abc^4d-a^2c^3d^2-}$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
-((a*c*d^2*e + a*c^2*d*f)*sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e
 + a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(a
rcsin(x*sqrt(d/c)), b*c/(a*d)) + (((b*c^2*d - a*d^3)*e + (a*c^2*d - a*c*d^
2)*f)*x^2 - (b*c^3 - a*c*d^2)*e - (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)
*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d)))/(a*b*c^4*d - a^2*c^3*d^2 - (a
*b*c^3*d^2 - a^2*c^2*d^3)*x^2)
```

3.51.6 Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

3.51.7 Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.51.8 Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,{(c-d\,x^2)}^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.52 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

3.52.1 Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} - \frac{\sqrt{2}b\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

output
```
b*x*(d*x^2+2)^(1/2)/d/(f*x^2+3)^(1/2)+1/2*a*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2
+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2
))*(d*x^2+2)^(1/2)*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(
1/2)-b*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)
/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(1/2)/
((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = -\frac{i\left(3bE\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-3b+af)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}\sqrt{d}f}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-3*b
+ a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))/(Sqrt[3]*S
qrt[d]*f)
```

3.52.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

$$\Big\downarrow \text{406}$$

$$a\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+b\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx$$

$$\Big\downarrow \text{320}$$

$$b\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

$$\Big\downarrow \text{388}$$

$$b\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+ b\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)$$

input
```
Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
b*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 +
d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTa
n[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2
)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

3.52.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.52.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

method	result
default	$\frac{\left(F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)ad-2F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)b+2E\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)b\right)\sqrt{2}}{2d\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(\frac{a\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{2\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}-\frac{b\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\left(F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)-E\left(\frac{x\sqrt{-3f}}{3}\right.\right.}{\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}\,d}\right.}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}$

input
```
int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a
*d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b
+2*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b)*
2^(1/2)/d/(-f)^(1/2)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx =$$
$$-\frac{9\sqrt{3}\sqrt{df}bx\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2f}{3d})-\sqrt{3}(af^2+9b)\sqrt{df}x\sqrt{-\frac{1}{f}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2f}{3d})-3\sqrt{}}{3\,df^2x}$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(9*sqrt(3)*sqrt(d*f)*b*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1
/f)/x), 2/3*f/d) - sqrt(3)*(a*f^2 + 9*b)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f
(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) - 3*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3
)*b*f)/(d*f^2*x)
```

3.52.6 Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

3.52.7 Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.52.8 Giac [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{b\,x^2+a}{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.53 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

3.53.1 Optimal result

Integrand size = 30, antiderivative size = 262

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}}+\frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f}+\frac{\sqrt{2}(6bd-2bf-3adf)\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}-\frac{\sqrt{2}(b-af)\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

output
```
-1/3*(-3*a*d*f+6*b*d-2*b*f)*x*(d*x^2+2)^(1/2)/d/f/(f*x^2+3)^(1/2)+1/3*(-3*
a*d*f+6*b*d-2*b*f)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(
1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2
)/d/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(-a*f+b)*(1/(3*f*x
^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/
2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/f^(3/2)/((d*x^2+2)/(f*x^2+
3))^(1/2)/(f*x^2+3)^(1/2)+1/3*b*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/f
```

3.53.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \frac{b\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}+i\sqrt{3}(6bd-2bf-3adf)E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}(3d-2f)(-2b+af)\operatorname{F}}{3\sqrt{d}f^2}$$

input
```
Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

output
```
(b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d - 2*b*f
- 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt
[3]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*
f)/(3*d)])/(3*Sqrt[d]*f^2)
```

3.53.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}(a+bx^2)}{\sqrt{fx^2+3}}\,dx$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\int -\frac{(6bd-3afd-2bf)x^2+6(b-af)}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}+\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{\int \frac{(6bd-3afd-2bf)x^2+6(b-af)}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{6(b-af)\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+(-3adf+6bd-2bf)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}$$

$$\Big\downarrow \text{320}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{(-3adf+6bd-2bf)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}}{3f}$$

$$\Big\downarrow \text{388}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{(-3adf+6bd-2bf)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}}{3f}$$

$$\Big\downarrow \text{313}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+(-3adf+6bd-2bf)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{3f}$$

input
```
Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

output
```
(b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) - ((6*b*d - 2*b*f - 3*a*d*f)*(
(x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*Ellipti
cE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (3*Sqrt[2]*(b - a*f)*Sqrt[2 + d*x^2]*
EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[f]*Sqrt[(2
+ d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]))/(3*f)
```

3.53. $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.53. $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

3.53.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(fx^2+3)(dx^2+2)}\left(\frac{bx\sqrt{dfx^4+3dx^2+2fx^2+6}}{3f}+\frac{\left(2a-\frac{2b}{f}\right)\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{2\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}-\frac{\left(ad+2b-\frac{b(6d+4f)}{3f}\right)\sqrt{3fx^2 \cdots}}{\cdots}\right.}{\sqrt{fx^2+3}\sqrt{dx^2+2}}$
risch	$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}+\frac{\left(-\frac{3b\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}+\frac{3af\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}-\cdots\right.}{3f\sqrt{f\cdots}}$
default	$\frac{\sqrt{dx^2+2}\sqrt{fx^2+3}\left(bd^2fx^5\sqrt{-f}+3\sqrt{2}\,E\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)adf\sqrt{dx^2+2}\sqrt{fx^2+3}+3bd^2x^3\sqrt{-f}+2bdfx^3\sqrt{-f}-6\sqrt{2}\,E\left(\frac{x\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/3*b/f*x*(d*
f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(2*a-2*b/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1
/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-
3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-(a*d+2*b-1/3*b/f*(6*d+4*f))/(-3*f
)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1
/2)/d*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-Elliptic
E(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$$
$$=\frac{3\sqrt{3}(6\,bd-(3\,ad+2\,b)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2\,f}{3\,d})+\sqrt{3}(2\,af^3-2\,bf^2-18\,bd+3\,(3\,ad+2\cdots}{3\,df^3x}$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(3)*(6*b*d - (3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_e
(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) + sqrt(3)*(2*a*f^3 - 2*b*f^2 - 18*
b*d + 3*(3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*
sqrt(-1/f)/x), 2/3*f/d) + (b*d*f^2*x^2 - 6*b*d*f + (3*a*d + 2*b)*f^2)*sqrt
(d*x^2 + 2)*sqrt(f*x^2 + 3))/(d*f^3*x)
```

3.53.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

3.53.7 Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

3.53.8 Giac [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+2}}{\sqrt{f\,x^2+3}}\,dx$$

input
```
int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2),x)
```

output
```
int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2), x)
```

3.54 $\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$

3.54.1 Optimal result

Integrand size = 30, antiderivative size = 356

$$\begin{aligned}\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx &= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} \\ &\quad+\frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df}+\frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} \\ &\quad-\frac{\sqrt{2}(5adf(3d+2f)-2b(9d^2-6df+4f^2))\sqrt{2+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\ &\quad-\frac{\sqrt{2}(3bd+2bf-10adf)\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\end{aligned}$$

output
```
1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*x*(d*x^2+2)^(1/2)/d^2/f/(
f*x^2+3)^(1/2)-1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*(1/(3*f*x^
2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2
),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d^2/f^(3/2)/((d*x^2+2)/(f*x
^2+3))^(1/2)/(f*x^2+3)^(1/2)-1/5*(-10*a*d*f+3*b*d+2*b*f)*(1/(3*f*x^2+9))^(
1/2)*(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(
4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1
/2)/(f*x^2+3)^(1/2)+1/5*b*x*(d*x^2+2)^(3/2)*(f*x^2+3)^(1/2)/d+1/15*(5*a*d*
f+3*b*d-4*b*f)*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/d/f
```

3.54.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$
$$=\frac{\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}(2bf+5adf+3bd(1+fx^2))+i\sqrt{3}(-5adf(3d+2f)+2b(9d^2-6df+4f^2))\,E}{15d^{3/2}f^2}$$

input
```
Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

output
```
(Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3*b*d*(1 +
 f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2 - 6*d*f + 4*f^2))*
EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3]*(3*d -
2*f)*(-6*b*d + 2*b*f + 5*a*d*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]],
(2*f)/(3*d)])/(15*d^(3/2)*f^2)
```

3.54.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx^2+2}\sqrt{fx^2+3}(a+bx^2)\ dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{\sqrt{dx^2+2}(3(2b-5ad)-(3bd+5afd-4bf)x^2)}{\sqrt{fx^2+3}}dx}{5d}+\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\int \frac{\sqrt{dx^2+2}(3(2b-5ad)-(3bd+5afd-4bf)x^2)}{\sqrt{fx^2+3}}dx}{5d}$$

$$\Big\downarrow \text{403}$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\int \frac{6(3bd-10afd+2bf)-\left(5adf(3d+2f)-2b\left(9d^2-6fd+4f^2\right)\right)x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{3f}}{5d}$$

$\downarrow$ 406

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{6(-10adf+3bd+2bf)\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx-\left(5adf(3d+2f)-2b(9d^2-6df+4f^2)\right)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{3f}}{5d}$$

$\downarrow$ 320

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b(9d^2-6df+4f^2)\right)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5a}{3f}}{5d}$$

$\downarrow$ 388

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b(9d^2-6df+4f^2)\right)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{\left(fx^2+3\right)^{3/2}}dx}{d}\right)}{3f}-\frac{x\sqrt{dx^2+2}}{}}{5d}$$

$\downarrow$ 313

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b(9d^2-6df+4f^2)\right)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right.}{3f}}{5d}$$

input
```
Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

3.54. $\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$

output
```
(b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (-1/3*((3*b*d - 4*b*f + 5*
a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/f + (-((5*a*d*f*(3*d + 2*f) - 2*
b*(9*d^2 - 6*d*f + 4*f^2))*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqr
t[2]*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(d*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]))) + (3*Sqrt[
2]*(3*b*d + 2*b*f - 10*a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)
/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3
 + f*x^2]))/(3*f))/(5*d)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.54.4 Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(f x^2+3)(d x^2+2)}\left(\frac{b x^3\sqrt{df x^4+3d x^2+2f x^2+6}}{5}+\frac{\left(adf+3bd+2bf-\frac{b(12d+8f)}{5}\right)x\sqrt{df x^4+3d x^2+2f x^2+6}}{3df}+\frac{\left(6a-\frac{2\left(adf+3bd+2bf-\frac{b(12d+\cdots}{5}\right.}{df}\right.\cdots}{2\sqrt{\cdots}}\right.\cdots}{\cdots}$
risch	$\frac{x(3bdf x^2+5adf+3bd+2bf)\sqrt{f x^2+3}\sqrt{d x^2+2}}{15df}+\frac{\left(-\frac{9bd\sqrt{3f x^2+9}\sqrt{2d x^2+4}F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\sqrt{df x^4+3d x^2+2f x^2+6}}-\frac{6bf\sqrt{3f x^2+9}\sqrt{2d x^2+4}F\left(\cdots\right.}{\sqrt{-3f}\sqrt{df x^4+3\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{f x^2+3}\sqrt{d x^2+2}\left(3b d^3f^2x^7\sqrt{-f}+5a d^3f^2x^5\sqrt{-f}+12b d^3f x^5\sqrt{-f}+8b d^2f^2x^5\sqrt{-f}+15a d^3f x^3\sqrt{-f}+10a d^2f^2x^3\sqrt{-f}+15\sqrt{2}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/5*b*x^3*(d*
f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/3*(a*d*f+3*b*d+2*b*f-1/5*b*(12*d+8*f))/d/
f*x*(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(6*a-2*(a*d*f+3*b*d+2*b*f-1/5*b*
(12*d+8*f))/d/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4
+3*d*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)
/f)^(1/2))-(3*a*d+2*a*f+12/5*b-1/3*(a*d*f+3*b*d+2*b*f-1/5*b*(12*d+8*f))/d/
f*(6*d+4*f))/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d
*x^2+2*f*x^2+6)^(1/2)/d*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/
f)^(1/2))-EllipticE(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right) \sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$

$$= \frac{3\sqrt{3}(18\,bd^2-2\,(5\,ad-4\,b)f^2-3\,(5\,ad^2+4\,bd)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2\,f}{3\,d}) + \sqrt{3}(4\,(5\,ad-b)}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*sqrt(3)*(18*b*d^2 - 2*(5*a*d - 4*b)*f^2 - 3*(5*a*d^2 + 4*b*d)*f)*s
qrt(d*f)*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) +
sqrt(3)*(4*(5*a*d - b)*f^3 - 54*b*d^2 + 6*((5*a - b)*d - 4*b)*f^2 + 9*(5*a
*d^2 + 4*b*d)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/
f)/x), 2/3*f/d) + (3*b*d^2*f^3*x^4 - 18*b*d^2*f + 2*(5*a*d - 4*b)*f^3 + 3*
(5*a*d^2 + 4*b*d)*f^2 + (3*b*d^2*f^2 + (5*a*d^2 + 2*b*d)*f^3)*x^2)*sqrt(d*
x^2 + 2)*sqrt(f*x^2 + 3))/(d^2*f^3*x)
```

3.54.6 Sympy [F]

$$\int \left(a+bx^2\right) \sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int \left(a+bx^2\right)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

3.54.7 Maxima [F]

$$\int \left(a+bx^2\right) \sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int \left(bx^2+a\right)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

3.54.8 Giac [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}\,dx$$

input
```
int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2),x)
```

output
```
int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2), x)
```

3.55 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$

3.55.1 Optimal result

Integrand size = 87, antiderivative size = 113

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

output
```
-1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+
b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))*(b-(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = -2i\sqrt{2}a\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt
[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt
[b^2 - 4*a*c])]
```

3.55.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {281, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b^2-4ac}-b+2cx^2}{\sqrt{\frac{2cx^2}{-\sqrt{b^2-4ac}-b}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}}dx$$

$$\Big\downarrow \; 281$$

$$-\left(\left(\sqrt{b^2-4ac}+b\right)\int \frac{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}dx\right)$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt[b^2
- 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b
 + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

3.55.3.1 Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.55.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2537 vs. $2(92) = 184$.

Time = 3.72 (sec) , antiderivative size = 2538, normalized size of antiderivative = 22.46

method	result	size
elliptic	Expression too large to display	2538

input
```
int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/
2)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*(-(2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2)
^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2-4*(-(2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c+(-(2*c*x^2+(-4
*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((-2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(
1/2)*b)*(1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16
*a^2*b*c^2-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*
a*c-b^2))^(1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c
^2-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)
*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4
*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2
)^(1/2)/(-4*a*c+b^2-8*c^2/(-b+(-4*a*c+b^2)^(1/2))*x^2*a+2*c/(-b+(-4*a*c+b^
2)^(1/2))*x^2*b^2-8*c^2*x^2/(-b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(-b-(-4*a*c+
b^2)^(1/2))*b^2-16*c^3*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))
*a+4*c^2*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*b^2)^(1/2)*El
lipticF(1/2*x*(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c...
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.27

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}+abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right)\mid-\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2+2\,ac}{2\,ac})+\sqrt{\frac{1}{2}}\left(\sqrt{b\cdots\right.}{\cdots}$$

input
```
integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) + a*b*x)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2
*a*c)/(a*c)) + sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b - b^2)*x - ((2*a
+ b)*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 +
 2*a*c)/(a*c)) + 2*a*c*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)*sqrt
(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c*x)
```

3.55.6 Sympy [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{-b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{-b-\sqrt{-4ac+b^2}}}\sqrt{\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-(-4*a*c+b**2)
**(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x**2 - sqrt
(-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b + 2*c*x**2 + sqrt(-4
*a*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

3.55.7 Maxima [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
 4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

3.55.8 Giac [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
 4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int -\frac{b-2\,c\,x^2+\sqrt{b^2-4\,a\,c}}{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

input
```
int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c
)^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)),x)
```

output
```
int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c
)^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)), x)
```

3.56 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

3.56.1 Optimal result

Integrand size = 81, antiderivative size = 526

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$+\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/
2)))^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/2*(1/(1+2*c*x^2/(b+(-
4*a*c+b^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(
b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x
^2/(b-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.19

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$
$$= -2i\sqrt{2}a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])]
```

3.56.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {281, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}dx$$

$$\Big\downarrow \text{281}$$

$$\left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx$$

$$\Big\downarrow \text{324}$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx+\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{320}$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}\cdots}}\right.$$

$$\Big\downarrow \text{388}$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{2c}\right)}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\cdots}}{\cdots}\right.$$

$$\downarrow 313$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\cdots\right.}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\right.}{b-\sqrt{b^2-4ac}}\right.$$

input
```
Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(b - Sqrt[b^2 - 4*a*c])*((2*c*(((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]) - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2
*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]))
)/(b - Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt
[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]))
```

3.56.3.1 Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.56.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. $2(499) = 998$.

Time = 1.37 (sec) , antiderivative size = 2477, normalized size of antiderivative = 4.71

method	result	size
elliptic	Expression too large to display	2477

input
```
int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2
)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2+4*(-(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c-(-(-2*c*x^2+(-
4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((2*c*x^
2+(-4*a*c+b^2)^(1/2)+b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(
1/2)*b)*(1/2*b/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+
(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3
/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*
b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4*a*c+b
^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+
b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4
*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+
(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2*((-4*a...
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.71

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}-abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left(\sqrt{b^2\cdots}\right.}{\cdots}$$

input
```
integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*x)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*
a*c)/(a*c)) - sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b + b^2)*x + ((2*a -
 b)*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^
2 - 4*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt
((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2
*a*c)/(a*c)) + 2*a*c*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b
*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c*x)
```

3.56.6 Sympy [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-4*a*c+b**2)**
(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2 - sqrt(-
4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*x**2 + sqrt(-4*a*c
 + b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

3.56.7 Maxima [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

3.56.8 Giac [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}\,\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

input
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/
2)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)),x)
```

output
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/
2)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)), x)
```

3.57 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$

3.57.1 Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{bx\sqrt{c+dx^2}}{2f} - \frac{(2bde-bcf-2adf)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}f^2} + \frac{(be-af)\sqrt{de-cf}\text{arctanh}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}$$

output
```
-1/2*(-2*a*d*f-b*c*f+2*b*d*e)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/f^2/d^(1/
2)+(-a*f+b*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2))*(-c*f+d*
e)^(1/2)/f^2/e^(1/2)+1/2*b*x*(d*x^2+c)^(1/2)/f
```

3.57.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{bfx\sqrt{c+dx^2} + \frac{2(be-af)\sqrt{-de+cf}\arctan\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}(e+fx^2)}{\sqrt{e}\sqrt{-de+cf}}\right)}{\sqrt{e}} + \frac{(2bde-bcf-2adf)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2f^2}$$

input
```
Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(b*f*x*Sqrt[c + d*x^2] + (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]*ArcTan[(-(f*x*S
qrt[c + d*x^2]) + Sqrt[d]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(d*e) + c*f])])/Sqrt
[e] + ((2*b*d*e - b*c*f - 2*a*d*f)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sq
rt[d])/(2*f^2)
```

3.57.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(2bde-bcf-2adf)x^2+c(be-2af)}{\sqrt{dx^2+c}(fx^2+e)}dx}{2f}+\frac{bx\sqrt{c+dx^2}}{2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\int \frac{(2bde-bcf-2adf)x^2+c(be-2af)}{\sqrt{dx^2+c}(fx^2+e)}dx}{2f}$$

$$\Big\downarrow \text{398}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\frac{(-2adf-bcf+2bde)\int \frac{1}{\sqrt{dx^2+c}}dx}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}}{2f}$$

$$\Big\downarrow \text{224}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\frac{(-2adf-bcf+2bde)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}}{2f}$$

$$\Big\downarrow \text{219}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{\sqrt{d}f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}}{2f}$$

$$\Big\downarrow \text{291}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{\sqrt{d}f}-\frac{2(be-af)(de-cf)\int\frac{1}{e-\frac{(de-cf)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{f}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{bx\sqrt{c+dx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{\sqrt{d}f}-\frac{2(be-af)\sqrt{de-cf}\text{arctanh}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f}}{2f}$$

input
```
Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(2*f) - (((2*b*d*e - b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(Sqrt[d]*f) - (2*(b*e - a*f)*Sqrt[d*e - c*f]*ArcTa
nh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f))/(2*f)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.57.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\sqrt{d x^2+c}\, b f x-\frac{(2adf+bcf-2bde)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)}{\sqrt{d}}+\frac{2(cf-de)(af-be)\arctan\left(\frac{e\sqrt{d x^2+c}}{x\sqrt{(cf-de)e}}\right)}{\sqrt{(cf-de)e}}}{2f^2}$
risch	$\frac{bx\sqrt{d x^2+c}}{2f}+\frac{\frac{(2adf+bcf-2bde)\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{f\sqrt{d}}-\frac{\left(ac f^2-adef-bcef+bd e^2\right)\ln\left(\frac{\frac{2cf-2de}{f}+\frac{2d\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{cf-de}{f}}\cdots}{x-\cdots}\right)}{\sqrt{-ef}\,f\sqrt{\frac{cf-de}{f}}\cdots}}{\cdots}$
default	$\frac{b\left(\frac{x\sqrt{d x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2\sqrt{d}}\right)}{f}+\frac{(af-be)\left(\sqrt{d\left(x-\frac{\sqrt{-ef}}{f}\right)^2+\frac{2d\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{cf-de}{f}}+\frac{\sqrt{d}\sqrt{-ef}\ln\left(\frac{\frac{d\sqrt{-ef}}{f}+d\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f^2*(-(d*x^2+c)^(1/2)*b*f*x-(2*a*d*f+b*c*f-2*b*d*e)/d^(1/2)*arctanh((
d*x^2+c)^(1/2)/x/d^(1/2))+2*(c*f-d*e)*(a*f-b*e)/((c*f-d*e)*e)^(1/2)*arctan
(e*(d*x^2+c)^(1/2)/x/((c*f-d*e)*e)^(1/2)))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 777, normalized size of antiderivative = 6.07

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$= \left[\frac{2\sqrt{dx^2+c}bdfx - (2bde-(bc+2ad)f)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right) - (bde-adf)\sqrt{\frac{de-cf}{e}}\,\mathrm{l}}{4df^2}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(
-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*d*e - a*d*f)*sqrt((d*e -
c*f)/e)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^
2 - 3*c^2*e*f)*x^2 - 4*(c*e^2*x + (2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*s
qrt((d*e - c*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x
^2 + c)*b*d*f*x + 2*(2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) - (b*d*e - a*d*f)*sqrt((d*e - c*f)/e)*log(((8*d^2*e^2 -
8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 - 3*c^2*e*f)*x^2 - 4*(c*
e^2*x + (2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*sqrt((d*e - c*f)/e))/(f^2*x
^4 + 2*e*f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - 2*(b*d*e
 - a*d*f)*sqrt(-(d*e - c*f)/e)*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d
*x^2 + c)*sqrt(-(d*e - c*f)/e)/((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x))
- (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c))/(d*f^2), 1/2*(sqrt(d*x^2 + c)*b*d*f*x + (2*b*d*e - (b*c + 2*a
*d)*f)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*e - a*d*f)*sqrt(
-(d*e - c*f)/e)*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)*sqrt(
-(d*e - c*f)/e)/((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x)))/(d*f^2)]
```

3.57.6 Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.58.1 Optimal result

Integrand size = 30, antiderivative size = 304

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = &-\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} \\ &+ \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}} \\ &+ \frac{b(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}} \\ &+ \frac{b(bc-ad)(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}} \\ &+ \frac{b(3b^2e^2-8abef+8a^2f^2)\,\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}\end{aligned}$$

output
```
1/2*b*(-a*d+b*c)*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(3/
2)+1/8*b*(8*a^2*f^2-8*a*b*e*f+3*b^2*e^2)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2)
)/d/f^(5/2)+b*(-a*d+b*c)^2*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^3/f^(1/2)-
(-a*d+b*c)^3*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^3/c^(1/2
)/(-c*f+d*e)^(1/2)-1/2*b^2*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/d^2/f-3/8*b^2*(-2*
a*f+b*e)*x*(f*x^2+e)^(1/2)/d/f^2+1/4*b^2*x*(b*x^2+a)*(f*x^2+e)^(1/2)/d/f
```

3.58.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \frac{\frac{b^2dx\sqrt{e+fx^2}(12adf+b(-3de-4cf+2dfx^2))}{f^2} + \frac{8(bc-ad)^3\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}} - \frac{b(24a^2d^2f^2-12abdf(de+2cf)+b^2(3d^2e^2+4}{f^{5/2}}}{8d^3}$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((b^2*d*x*Sqrt[e + f*x^2]*(12*a*d*f + b*(-3*d*e - 4*c*f + 2*d*f*x^2)))/f^2
 + (8*(b*c - a*d)^3*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))
/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*(24*a^2*d^2*f^
2 - 12*a*b*d*f*(d*e + 2*c*f) + b^2*(3*d^2*e^2 + 4*c*d*e*f + 8*c^2*f^2))*Lo
g[-(Sqrt[f]*x) + Sqrt[e + f*x^2]])/f^(5/2))/(8*d^3)
```

3.58.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {420, 318, 25, 299, 224, 219, 420, 299, 224, 219, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{420}$$
$$\frac{b\int \frac{(bx^2+a)^2}{\sqrt{fx^2+e}}dx}{d} - \frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$
$$\Big\downarrow \text{318}$$
$$\frac{b\left(\frac{\int -\frac{3b(be-2af)x^2+a(be-4af)}{\sqrt{fx^2+e}}dx}{4f} + \frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}\right)}{d} - \frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\downarrow 25$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\int \frac{3b(be-2af)x^2+a(be-4af)}{\sqrt{fx^2+e}}dx}{4f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\downarrow 299$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{\left(8a^2f^2-8abef+3b^2e^2\right)\int \frac{1}{\sqrt{fx^2+e}}dx}{2f}}{4f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\downarrow 224$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{\left(8a^2f^2-8abef+3b^2e^2\right)\int \frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{2f}}{4f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\downarrow 219$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{\left(8a^2f^2-8abef+3b^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\downarrow 420$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{\left(8a^2f^2-8abef+3b^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{bx^2+a}{\sqrt{fx^2+e}}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

3.58. $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

$$\Big\downarrow \; 299$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\int\frac{1}{\sqrt{fx^2+e}}dx}{2f}\right)}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\int\frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{2f}\right)}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 398$$

3.58. $\displaystyle\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{fx^2+e}}dx}{d}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}\right)}{d}$$

$\downarrow$ 224

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}\right)}{d}$$

$\downarrow$ 219

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}\right)}{d}$$

$\downarrow$ 291

3.58. $\int\frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}dx$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int\frac{1}{c-\frac{(cf-de)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}\right)}{d}\right)}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{b\left(\frac{bx(a+bx^2)\sqrt{e+fx^2}}{4f}-\frac{\frac{3bx\sqrt{e+fx^2}(be-2af)}{2f}-\frac{(8a^2f^2-8abef+3b^2e^2)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}}{4f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}\right)}{d}\right)}{d}$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
-(((b*c - a*d)*(-(((b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/
(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqr
t[f]*x)/Sqrt[e + f*x^2]])/(d*Sqrt[f])))/d) + (b*((b*x*Sqrt[e + f*x^2])/(2*
f) - ((b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*f^(3/2))))/d)
)/d) + (b*((b*x*(a + b*x^2)*Sqrt[e + f*x^2])/(4*f) - ((3*b*(b*e - 2*a*f)*x
*Sqrt[e + f*x^2])/(2*f) - ((3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcTanh[(Sq
rt[f]*x)/Sqrt[e + f*x^2]])/(2*f^(3/2)))/(4*f)))/d
```

3.58. $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.58.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\dfrac{f^{\frac{9}{2}}\left(ad-bc\right)^{3}\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^{2}+e}}{x\sqrt{\left(cf-de\right)c}}\right)+\frac{3\left(2\left(\frac{b^{2}c^{2}f^{2}}{3}-\left(af-\frac{be}{6}\right)bfdc+d^{2}\left(a^{2}f^{2}-\frac{1}{2}abfe+\frac{1}{8}b^{2}e^{2}\right)\right)f^{2}\operatorname{arctanh}\left(\frac{\sqrt{f\,x^{2}+e}}{x\sqrt{f}}\right)+b\sqrt{f\,x^{2}+\cdots}}{2}}{\sqrt{\left(cf-de\right)c}\,d^{3}f^{\frac{9}{2}}}$
risch	$\dfrac{b^{2}x\left(2bdf\,x^{2}+12adf-4bcf-3bde\right)\sqrt{f\,x^{2}+e}}{8f^{2}d^{2}}+\dfrac{\frac{b\left(24a^{2}d^{2}f^{2}-24abcd\,f^{2}-12ab\,d^{2}ef+8b^{2}c^{2}f^{2}+4b^{2}cdef+3b^{2}d^{2}e^{2}\right)\ln\left(\sqrt{f}\,x+\sqrt{f\,x^{2}\cdots}\right.}{d\sqrt{f}}}{\cdots}$
default	$\dfrac{b\left(\frac{b^{2}c^{2}\ln\left(\sqrt{f}\,x+\sqrt{f\,x^{2}+e}\right)}{\sqrt{f}}+b^{2}d^{2}\left(\frac{x^{3}\sqrt{f\,x^{2}+e}}{4f}-\frac{3e\left(\frac{x\sqrt{f\,x^{2}+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f\,x^{2}+e}\right)}{2f^{\frac{3}{2}}}\right)}{4f}\right)+\frac{3a^{2}d^{2}\ln\left(\sqrt{f}\,x+\sqrt{f\,x^{2}+e}\right)}{\sqrt{f}}-\frac{3abcd\cdots}{\cdots}\right.}{d^{3}}$

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((c*f-d*e)*c)^(1/2)*(f^(9/2)*(a*d-b*c)^3*arctanh(c*(f*x^2+e)^(1/2)/x/((c
*f-d*e)*c)^(1/2))+3/2*(2*(1/3*b^2*c^2*f^2-(a*f-1/6*b*e)*b*f*d*c+d^2*(a^2*f
^2-1/2*a*b*f*e+1/8*b^2*e^2))*f^2*arctanh((f*x^2+e)^(1/2)/x/f^(1/2))+b*(f*x
^2+e)^(1/2)*(-1/3*b*c*f+((1/6*b*x^2+a)*f-1/4*b*e)*d)*f^(5/2)*x*d)*b*((c*f-
d*e)*c)^(1/2))/d^3/f^(9/2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 1718, normalized size of antiderivative = 5.65

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e +
 c^2*f)*f^3*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*
d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f
)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (3*b^3*c*d^3*e^3 + (b^3*
c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b
*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(f)
*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) + 2*(2*(b^3*c*d^3*e*f^2 -
 b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d^2 - 12*a*b^2*c*d^3
)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f*x^2 + e))/(c*d^4*
e*f^3 - c^2*d^3*f^4), -1/16*(8*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*sqrt(c*d*e - c^2*f)*f^3*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*
c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*
e*f)*x)) - (3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^
3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(
f)*x - e) - 2*(2*(b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^
2*f + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^
2)*f^3)*x)*sqrt(f*x^2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4), 1/8*(2*(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e + c^2*f)*f^3*log(((d
^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*...
```

3.58.6 Sympy [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**3/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.58.7 Maxima [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^3}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.58.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^3}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.59 $\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.59.1 Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{2df}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}} - \frac{b(bc-ad)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}} - \frac{b(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}$$

output
```
-1/2*b*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(3/2)-b*(-a*d+b
*c)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(1/2)+(-a*d+b*c)^2*arctan(x*(
-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^2/c^(1/2)/(-c*f+d*e)^(1/2)+1/2*
b^2*x*(f*x^2+e)^(1/2)/d/f
```

3.59.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\frac{b^2dx\sqrt{e+fx^2}}{f}-\frac{2(bc-ad)^2\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{b(bde+2bcf-4adf)\log\left(-\sqrt{f}x+\sqrt{e+fx^2}\right)}{f^{3/2}}}{2d^2}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((b^2*d*x*Sqrt[e + f*x^2])/f - (2*(b*c - a*d)^2*ArcTan[(c*Sqrt[f] + d*x*(S
qrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e
 - c*f]) + (b*(b*d*e + 2*b*c*f - 4*a*d*f)*Log[-(Sqrt[f]*x) + Sqrt[e + f*x^
2]])/f^(3/2))/(2*d^2)
```

3.59.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {420, 299, 224, 219, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \frac{bx^2+a}{\sqrt{fx^2+e}}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\int \frac{1}{\sqrt{fx^2+e}}dx}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\int \frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{398}$$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{fx^2+e}}dx}{d}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$\downarrow$ 224

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$\downarrow$ 219

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int\frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}\right)}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int\frac{1}{c-\frac{(cf-de)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}\right)}{d}$$

$\downarrow$ 218

3.59. $\int\frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}dx$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}}{2f}-\frac{(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}\right)}{d}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
-(((b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e
+ f*x^2])])/(Sqrt[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e +
 f*x^2]])/(d*Sqrt[f])))/d) + (b*((b*x*Sqrt[e + f*x^2])/(2*f) - ((b*e - 2*a
*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*f^(3/2))))/d
```

3.59.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.59.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{f^{\frac{5}{2}}(ad-bc)^2\operatorname{arctanh}\left(\frac{c\sqrt{f x^2+e}}{x\sqrt{(cf-de)c}}\right)+\frac{\sqrt{(cf-de)c}\,b\left(\left((4ad-2bc)f^2-ebdf\right)\operatorname{arctanh}\left(\frac{\sqrt{f x^2+e}}{x\sqrt{f}}\right)+bd\sqrt{f x^2+e}\,x f^{\frac{3}{2}}\right)}{2}}{\sqrt{(cf-de)c}\,d^2 f^{\frac{5}{2}}}$
risch	$\frac{b^2x\sqrt{f x^2+e}}{2df}+\frac{\frac{b(4adf-2bcf-bde)\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{d\sqrt{f}}-\frac{f\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-}{d}}}{x-}\right.}{\sqrt{-cd}\,d\sqrt{-\frac{cf-de}{d}}}}{}$
default	$\frac{b\left(bd\left(\frac{x\sqrt{f x^2+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{2f^{\frac{3}{2}}}\right)+\frac{2ad\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}-\frac{bc\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}\right)}{d^2}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\right.}{}$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((c*f-d*e)*c)^(1/2)*(f^(5/2)*(a*d-b*c)^2*arctanh(c*(f*x^2+e)^(1/2)/x/((c
*f-d*e)*c)^(1/2))+1/2*((c*f-d*e)*c)^(1/2)*b*(((4*a*d-2*b*c)*f^2-e*b*d*f)*a
rctanh((f*x^2+e)^(1/2)/x/f^(1/2))+b*d*(f*x^2+e)^(1/2)*x*f^(3/2)))/d^2/f^(5
/2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 1111, normalized size of antiderivative = 6.69

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \left[-\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cde+c^2f}\,f^2\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\,\sqrt{-}}{d^2x^4+2\,cdx^2+c^2}\right.}{} \right.$$
$$-\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cde+c^2f}\,f^2\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\,\sqrt{-}}{d^2x^4+2\,cdx^2+c^2}\right.}{}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*
e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2
 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^
2*x^4 + 2*c*d*x^2 + c^2)) - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 +
 e)*x + (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*
b*c^2*d)*f^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*
d^3*e*f^2 - c^2*d^2*f^3), 1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*
e - c^2*f)*f^2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sq
rt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b^2*
c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - (b^2*c*d^2*e^2 + (b^2*c^2*d
 - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2)*sqrt(f)*log(-2*f*x^2
- 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^3*e*f^2 - c^2*d^2*f^3), -1/4*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*e^2 - 8*c*
d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e
 - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*
c*d*x^2 + c^2)) - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - 2*
(b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)
*f^2)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^3*e*f^2 - c^2*d^2*
f^3), 1/2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*e - c^2*f)*f^2*arctan(
1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d...
```

3.59.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**2/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.59.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.60 $\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.60.1 Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}+\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}$$

output
```
b*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(1/2)-(-a*d+b*c)*arctan(x*(-c*f+d
*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d/c^(1/2)/(-c*f+d*e)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\frac{(bc-ad)\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}-\frac{b\log\left(-\sqrt{f}x+\sqrt{e+fx^2}\right)}{\sqrt{f}}}{d}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
(((b*c - a*d)*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt
[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*Log[-(Sqrt[f]*x) + S
qrt[e + f*x^2]])/Sqrt[f])/d
```

3.60.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{398}$$

$$\frac{b\int \frac{1}{\sqrt{fx^2+e}}dx}{d}-\frac{(bc-ad)\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{fx^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}-\frac{(bc-ad)\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\int \frac{1}{c-\frac{(cf-de)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
-(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqr
t[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d*Sqr
t[f])
```

3.60.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.60.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\frac{(ad-bc)\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^2+e}}{x\sqrt{(cf-de)c}}\right)}{\sqrt{(cf-de)c}}+\frac{b\ \operatorname{arctanh}\left(\frac{\sqrt{f\,x^2+e}}{x\sqrt{f}}\right)}{\sqrt{f}}}{d}$
default	$\frac{b\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{d\sqrt{f}}-\frac{(-ad+bc)\ln\left(\frac{-\frac{2(cf-de)}{d}-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 f-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,d\sqrt{-\frac{cf-de}{d}}}$

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*((a*d-b*c)/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*
c)^(1/2))+b/f^(1/2)*arctanh((f*x^2+e)^(1/2)/x/f^(1/2)))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(75) = 150.

Time = 0.79 (sec) , antiderivative size = 737, normalized size of antiderivative = 8.10

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \left[\frac{\sqrt{-cde+c^2f}(bc-ad)f\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-cde+c^2f}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,(cd^2ef-c^2df^2)} \right.$$
$$-\frac{\sqrt{cde-c^2f}(bc-ad)f\arctan\left(\frac{\sqrt{cde-c^2f}((de-2\,cf)x^2-ce)\sqrt{fx^2+e}}{2\,((cdef-c^2f^2)x^3+(cde^2-c^2ef)x)}\right)-(bcde-bc^2f)\sqrt{f}\log\left(-2\,fx^2-2\,\sqrt{fx\right.}{2\,(cd^2ef-c^2df^2)}$$
$$-\frac{\sqrt{cde-c^2f}(bc-ad)f\arctan\left(\frac{\sqrt{cde-c^2f}((de-2\,cf)x^2-ce)\sqrt{fx^2+e}}{2\,((cdef-c^2f^2)x^3+(cde^2-c^2ef)x)}\right)+2\,(bcde-bc^2f)\sqrt{-f}\arctan\left(\frac{\sqrt{-f}x}{\sqrt{fx^2+e}}\right)}{2\,(cd^2ef-c^2df^2)}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2
*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3
 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2
)) + 2*(b*c*d*e - b*c^2*f)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f
)*x - e))/(c*d^2*e*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f
*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/
((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (b*c*d*e - b*c^2*f)*s
qrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^2*e*f - c^2*d
*f^2), 1/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f +
 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*
f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2
 + c^2)) - 4*(b*c*d*e - b*c^2*f)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e
)))/(c*d^2*e*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f*arcta
n(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*
e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b*c*d*e - b*c^2*f)*sqrt(
-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^2*e*f - c^2*d*f^2)]
```

3.60.6 Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.60.7 Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.60.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.61 $\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

output
```
arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/c^(1/2)/(-c*f+d*e)^(1/2
)
```

3.61.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

input
```
Integrate[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
-(ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e
 - c*f])]/(Sqrt[c]*Sqrt[d*e - c*f]))
```

3.61.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{c-\frac{x^2(cf-de)}{e+fx^2}}d\frac{x}{\sqrt{e+fx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

input
```
Int[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e -
c*f])
```

3.61.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.61.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^{2}+e}}{x\sqrt{(cf-de)c}}\right)}{\sqrt{(cf-de)c}}$
default	$\frac{\ln\left(\frac{-\frac{2(cf-de)}{d}-\frac{2f\sqrt{-cd}\,\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\,\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}f-\frac{2f\sqrt{-cd}\,\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}-\frac{cf-de}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,\sqrt{-\frac{cf-de}{d}}}-\frac{\ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-}}{}}{}\right)}{}$

input
```
int(1/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*c)^(1/2))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(39) = 78.

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$=\left[-\frac{\sqrt{-cde+c^2f}\,\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-cde+c^2f}\,\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,(cde-c^2f)},\ \frac{\arctan}{}\right.$$

input
```
integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^
2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt
(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))/(c*d*e - c^
2*f), 1/2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*
x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x))/sqrt(c*d*e - c
^2*f)]
```

3.61.6 Sympy [F]

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{f}\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d-de+2\,cf}{2\,\sqrt{cdef-c^2f^2}}\right)}{\sqrt{cdef-c^2f^2}}$$

input
```
integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(f)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/sqrt
(c*d*e*f - c^2*f^2))/sqrt(c*d*e*f - c^2*f^2)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{d\,e-c\,f}}{\sqrt{c}\,\sqrt{f\,x^2+e}}\right)}{\sqrt{-c\,(c\,f-d\,e)}} & \text{if } 0<d\,e-c\,f \\ \frac{\ln\left(\frac{\sqrt{c\,\left(f\,x^2+e\right)}+x\,\sqrt{c\,f-d\,e}}{\sqrt{c\,\left(f\,x^2+e\right)}-x\,\sqrt{c\,f-d\,e}}\right)}{2\,\sqrt{c\,(c\,f-d\,e)}} & \text{if } d\,e-c\,f<0 \\ \int \frac{1}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx & \text{if } d\,e-c\,f\notin\mathbb{R}\vee c\,f=d\,e \end{cases}$$

input
```
int(1/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
piecewise(0 < - c*f + d*e, atan((x*(- c*f + d*e)^(1/2))/(c^(1/2)*(e + f*x^
2)^(1/2)))/(-c*(c*f - d*e))^(1/2), - c*f + d*e < 0, log(((c*(e + f*x^2))^(
1/2) + x*(c*f - d*e)^(1/2))/((c*(e + f*x^2))^(1/2) - x*(c*f - d*e)^(1/2)))
/(2*(c*(c*f - d*e))^(1/2)), ~in(- c*f + d*e, 'real') | c*f == d*e, int(1/(
(c + d*x^2)*(e + f*x^2)^(1/2)), x))
```

3.62 $\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.62.1 Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\arctan\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}} - \frac{d\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

output
```
b*arctan(x*(-a*f+b*e)^(1/2)/a^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)/a^(1/2)/(-
a*f+b*e)^(1/2)-d*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+
b*c)/c^(1/2)/(-c*f+d*e)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{-\frac{b\arctan\left(\frac{a\sqrt{f}+bx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{a}\sqrt{be-af}}\right)}{\sqrt{a}\sqrt{be-af}} + \frac{d\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}}{bc-ad}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
(-((b*ArcTan[(a*Sqrt[f] + b*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[a]*Sqrt
[b*e - a*f])])/(Sqrt[a]*Sqrt[b*e - a*f])) + (d*ArcTan[(c*Sqrt[f] + d*x*(Sq
rt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e
- c*f]))/(b*c - a*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {407, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{407}$$

$$\frac{b\int \frac{1}{(bx^2+a)\sqrt{fx^2+e}}dx}{bc-ad}-\frac{d\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{bc-ad}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\int \frac{1}{a-\frac{(af-be)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{bc-ad}-\frac{d\int \frac{1}{c-\frac{(cf-de)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{bc-ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\arctan\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}}-\frac{d\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
(b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*(b*c -
a*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*
x^2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

3.62.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

3.62.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{d\,\operatorname{arctanh}\left(\frac{c\sqrt{f x^2+e}}{x\sqrt{(cf-de)c}}\right)\sqrt{(af-be)a}-b\,\operatorname{arctanh}\left(\frac{\sqrt{f x^2+e}\,a}{x\sqrt{(af-be)a}}\right)\sqrt{(cf-de)c}}{(ad-bc)\sqrt{(cf-de)c}\,\sqrt{(af-be)a}}$
default	$-\frac{b^2 d\ln\left(\frac{-\frac{2(af-be)}{b}+\frac{2f\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{af-be}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 f+\frac{2f\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{af-be}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(b\sqrt{-cd}+d\sqrt{-ab}\right)\left(d\sqrt{-ab}-b\sqrt{-cd}\right)\sqrt{-\frac{af-be}{b}}}+\frac{b^2 d\ln\left(-\frac{2(af-\cdots}{b}\cdots\right.}{\cdots}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*c)^(1/2))*((a*f-b*e)*a)^(1/2)-b*
arctanh((f*x^2+e)^(1/2)/x*a/((a*f-b*e)*a)^(1/2))*((c*f-d*e)*c)^(1/2))/(a*d
-b*c)/((c*f-d*e)*c)^(1/2)/((a*f-b*e)*a)^(1/2)
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(102) = 204$.

Time = 61.43 (sec) , antiderivative size = 1305, normalized size of antiderivative = 10.70

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*c*d*e - b*c^2*f)*sqrt(-a*b*e + a^2*f)*log(((b^2*e^2 - 8*a*b*e*f +
 8*a^2*f^2)*x^4 + a^2*e^2 - 2*(3*a*b*e^2 - 4*a^2*e*f)*x^2 + 4*((b*e - 2*a*
f)*x^3 - a*e*x)*sqrt(-a*b*e + a^2*f)*sqrt(f*x^2 + e))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + (a*b*d*e - a^2*d*f)*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*
e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e -
 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*
d*x^2 + c^2)))/((a*b^2*c^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*c*d^2)*
e*f + (a^2*b*c^3 - a^3*c^2*d)*f^2), 1/4*(2*(b*c*d*e - b*c^2*f)*sqrt(a*b*e
- a^2*f)*arctan(1/2*sqrt(a*b*e - a^2*f)*((b*e - 2*a*f)*x^2 - a*e)*sqrt(f*x
^2 + e)/((a*b*e*f - a^2*f^2)*x^3 + (a*b*e^2 - a^2*e*f)*x)) + (a*b*d*e - a^
2*d*f)*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c
^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqr
t(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a*b^2*c
^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*c*d^2)*e*f + (a^2*b*c^3 - a^3*c
^2*d)*f^2), -1/4*(2*(a*b*d*e - a^2*d*f)*sqrt(c*d*e - c^2*f)*arctan(1/2*sqr
t(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2
*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (b*c*d*e - b*c^2*f)*sqrt(-a*b*e + a^
2*f)*log(((b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*x^4 + a^2*e^2 - 2*(3*a*b*e^2 -
 4*a^2*e*f)*x^2 + 4*((b*e - 2*a*f)*x^3 - a*e*x)*sqrt(-a*b*e + a^2*f)*sqrt(
f*x^2 + e))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a*b^2*c^2*d - a^2*b*c*d^2)*...
```

3.62.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = -f^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2b-be+2\,af}{2\,\sqrt{abef-a^2f^2}}\right)}{\sqrt{abef-a^2f^2}(bcf-adf)} - \frac{d\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d-de+2\,cf}{2\,\sqrt{cdef-c^2f^2}}\right)}{\sqrt{cdef-c^2f^2}(bcf-adf)}\right)$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
-f^(3/2)*(b*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b - b*e + 2*a*f)/s
qrt(a*b*e*f - a^2*f^2))/(sqrt(a*b*e*f - a^2*f^2)*(b*c*f - a*d*f)) - d*arct
an(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/sqrt(c*d*e*f - c^
2*f^2))/(sqrt(c*d*e*f - c^2*f^2)*(b*c*f - a*d*f)))
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.63 $\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$

3.63.1 Optimal result

Integrand size = 30, antiderivative size = 203

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{b(b^2ce-3abde-2abcf+4a^2df)\arctan\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{d^2\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

output
```
1/2*b*(4*a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)*arctan(x*(-a*f+b*e)^(1/2)/a^
(1/2)/(f*x^2+e)^(1/2))/a^(3/2)/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)+d^2*arctan(x*
(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)^2/c^(1/2)/(-c*f+d*e)^
(1/2)+1/2*b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)
```

3.63.2 Mathematica [A] (verified)

Time = 15.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \frac{1}{2}\left(\frac{b^2x\sqrt{e+fx^2}}{a(-bc+ad)(-be+af)(a+bx^2)} + \frac{b(b^2ce+4a^2df-ab(3de+2cf))\arctan\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{2d^2\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}\right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((b^2*x*Sqrt[e + f*x^2])/(a*(-(b*c) + a*d)*(-(b*e) + a*f)*(a + b*x^2)) + (
b*(b^2*c*e + 4*a^2*d*f - a*b*(3*d*e + 2*c*f))*ArcTan[(Sqrt[b*e - a*f]*x)/(
Sqrt[a]*Sqrt[e + f*x^2])])/(a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (2*
d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c -
 a*d)^2*Sqrt[d*e - c*f]))/2
```

3.63.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {421, 25, 291, 218, 402, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{421}$$
$$\frac{d^2\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{(bc-ad)^2} - \frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^2\sqrt{fx^2+e}}dx}{(bc-ad)^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{d^2\int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}dx}{(bc-ad)^2} + \frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^2\sqrt{fx^2+e}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d^2\int \frac{1}{c-\frac{(cf-de)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^2\sqrt{fx^2+e}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^2\sqrt{fx^2+e}}dx}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx\sqrt{e+fx^2}(bc-ad)}{2a(a+bx^2)(be-af)}-\frac{\int -\frac{4dfa^2-b(3de+2cf)a+b^2ce}{(bx^2+a)\sqrt{fx^2+e}}dx}{2a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{4dfa^2-3bdea-2bcfa+b^2ce}{(bx^2+a)\sqrt{fx^2+e}}dx}{2a(be-af)}+\frac{bx\sqrt{e+fx^2}(bc-ad)}{2a(a+bx^2)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{(4a^2df-2abcf-3abde+b^2ce)\int \frac{1}{(bx^2+a)\sqrt{fx^2+e}}dx}{2a(be-af)}+\frac{bx\sqrt{e+fx^2}(bc-ad)}{2a(a+bx^2)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{(4a^2df-2abcf-3abde+b^2ce)\int \frac{1}{a-\frac{(af-be)x^2}{fx^2+e}}d\frac{x}{\sqrt{fx^2+e}}}{2a(be-af)}+\frac{bx\sqrt{e+fx^2}(bc-ad)}{2a(a+bx^2)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\arctan\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)(4a^2df-2abcf-3abde+b^2ce)}{2a^{3/2}(be-af)^{3/2}}+\frac{bx\sqrt{e+fx^2}(bc-ad)}{2a(a+bx^2)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

3.63. $\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$

output
```
(b*((b*(b*c - a*d)*x*Sqrt[e + f*x^2])/(2*a*(b*e - a*f)*(a + b*x^2)) + ((b^
2*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqr
t[a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*e - a*f)^(3/2))))/(b*c - a*d)^2 + (d
^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c -
a*d)^2*Sqrt[d*e - c*f])
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.63.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{\frac{d^{2}\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^{2}+e}}{x\sqrt{(cf-de)c}}\right)}{\sqrt{(cf-de)c}}+\frac{b\left(\frac{b(ad-bc)\sqrt{f\,x^{2}+e}\,x}{b\,x^{2}+a}-\frac{\left(4a^{2}df-2acfb-3abde+b^{2}ce\right)\operatorname{arctanh}\left(\frac{\sqrt{f\,x^{2}+e}\,a}{x\sqrt{(af-be)a}}\right)}{\sqrt{(af-be)a}}\right)}{2(af-be)a}}{(ad-bc)^{2}}$	169
default	Expression too large to display	1400

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*(d^2/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d
*e)*c)^(1/2))+1/2*b/(a*f-b*e)/a*(b*(a*d-b*c)*(f*x^2+e)^(1/2)*x/(b*x^2+a)-(
4*a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)/((a*f-b*e)*a)^(1/2)*arctanh((f*x^2+
e)^(1/2)/x*a/((a*f-b*e)*a)^(1/2))))
```

3.63.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.63.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2(dx^2+c)\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(177) = 354.

Time = 4.55 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.26

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{1}{2}\left(\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d-de+2\,cf}{2\,\sqrt{cdef-c^2f^2}}\right)}{(b^2c^2f^2-2\,abcdf^2+a^2d^2f^2)\sqrt{cdef-c^2f^2}} + \frac{(b^3ce-3\,ab^2de-2\,ab^2cf+4\,a^2bdf)\arctan\left(\right.}{(ab^3c^2ef^2-2\,a^2b^2cdef^2+a^3bd^2ef^2-a^2b^2c^2f^3+2\,a\right.}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*d^2*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/s
qrt(c*d*e*f - c^2*f^2))/((b^2*c^2*f^2 - 2*a*b*c*d*f^2 + a^2*d^2*f^2)*sqrt(
c*d*e*f - c^2*f^2)) + (b^3*c*e - 3*a*b^2*d*e - 2*a*b^2*c*f + 4*a^2*b*d*f)*
arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b - b*e + 2*a*f)/sqrt(a*b*e*f
 - a^2*f^2))/((a*b^3*c^2*e*f^2 - 2*a^2*b^2*c*d*e*f^2 + a^3*b*d^2*e*f^2 - a^
2*b^2*c^2*f^3 + 2*a^3*b*c*d*f^3 - a^4*d^2*f^3)*sqrt(a*b*e*f - a^2*f^2)) +
2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b^2*e - 2*(sqrt(f)*x - sqrt(f*x^2 + e))
^2*a*b*f - b^2*e^2)/((a*b^2*c*e*f^2 - a^2*b*d*e*f^2 - a^2*b*c*f^3 + a^3*d*
f^3)*((sqrt(f)*x - sqrt(f*x^2 + e))^4*b - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^
2*b*e + 4*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*f + b*e^2)))*f^(5/2)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.64 $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.64.1 Optimal result

Integrand size = 32, antiderivative size = 608

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\left(7ce-\frac{2de^2}{f}+\frac{3c^2f}{d}\right)x\sqrt{c+dx^2}}{15b\sqrt{e+fx^2}} + \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf} - \frac{\sqrt{e}(15a^2d^2f^2-5abdf(de+7cf)+b^2(-2d^2e^2+12cdef+23c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{de^{3/2}(-40abcdf+15a^2d^2f+b^2c(-de+34cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3cf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*d^2*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/b/f+1/15*d*(7*c*e-2*d*e^2/f+3*c^
2*f/d)*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*
f+b*d*e)*x*(d*x^2+c)^(1/2)/b^3/(f*x^2+e)^(1/2)+1/15*d*e^(3/2)*(-40*a*b*c*d
*f+15*a^2*d^2*f+b^2*c*(34*c*f-d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2
)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+
c)^(1/2)/b^3/c/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/1
5*(15*a^2*d^2*f^2-5*a*b*d*f*(7*c*f+d*e)+b^2*(23*c^2*f^2+12*c*d*e*f-2*d^2*e
^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^3/f^(3/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^3*e^(3/2)*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^
(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-a*d+b*c)*x*(d*x^2+c)^(1/2
)*(f*x^2+e)^(1/2)/b^2-2/15*d*(-3*c*f+d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2
)/b/f
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{-iabde(15a^2d^2f^2-5abdf(de+7cf)+b^2(-2d^2e^2+12cdef+23c^2f^2))\sqrt{1-\cdots}}{\cdots}$$

input
```
Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

output
```
((-I)*a*b*d*e*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + 7*c*f) + b^2*(-2*d^2*e^2
+ 12*c*d*e*f + 23*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(45*a^2*b*c*d^2*f^3 - 15*a^
3*d^3*f^3 + 5*a*b^2*d*f*(d^2*e^2 - c*d*e*f - 9*c^2*f^2) + b^3*(2*d^3*e^3 -
 13*c*d^2*e^2*f + 11*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1
 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*
Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^2)*(11*b*c*f - 5*a*d*f + b*d*(e + 3*f*x^2
)) - (15*I)*(b*c - a*d)^3*f*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a
*b^4*Sqrt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.64.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.531, Rules used = {420, 318, 25, 403, 27, 406, 320, 388, 313, 418, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{a+bx^2}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\int \left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}dx}{b}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{\int -\frac{\sqrt{fx^2+e}\left(2d(de-3cf)x^2+c(de-5cf)\right)}{\sqrt{dx^2+c}}dx}{5f}+\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\int \frac{\sqrt{fx^2+e}\left(2d(de-3cf)x^2+c(de-5cf)\right)}{\sqrt{dx^2+c}}dx}{5f}\right)}{b}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\frac{\int \frac{d\left(\left(2d^2e^2-7cdfe-3c^2f^2\right)x^2+ce(de-9cf)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{5f}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\int\frac{\left(2d^2e^2-7cdfe-3c^2f^2\right)x^2+ce(de-9cf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{5f}\right)}{b}$$

$\downarrow$ 406

$$\frac{(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+ce(de-9cf)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(de-}{5f}\right.}{b}$$

$\downarrow$ 320

$$\frac{(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c+}}{5f}\right.}{b}$$

$\downarrow$ 388

$$\frac{(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right.}{5f}\right.}{b}$$

$\downarrow$ 313

3.64. $\int\frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}dx$

$$\frac{(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left((-3c^2f^2-7cdef+2d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\text{arc}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+d\cdots}{c(e+f\cdots}}}\cdots\right.}{5f}\cdots\right.}{b}$$

$\downarrow$ 418

$$\frac{(bc-ad)\left(\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\int \frac{(bdx^2+2bc-ad)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}dx}{b^2}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left((-3c^2f^2-7cdef+2d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\text{arc}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+d\cdots}{c(e+f\cdots}}}\cdots\right.}{5f}\cdots\right.}{b}$$

$\downarrow$ 403

$$\frac{(bc-ad)\left(\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{\int \frac{d((bde+4bcf-3adf)x^2+(5bc-3ad)e)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left((-3c^2f^2-7cdef+2d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\text{arc}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+d\cdots}{c(e+f\cdots}}}\cdots\right.}{5f}\cdots\right.}{b}$$

$\downarrow$ 27

3.64. $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

$$\frac{(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\int\frac{(bde+4bcf-3adf)x^2+(5bc-3ad)e}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{b}+$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+d\right.}{c\left(e+f\right.}}}\right.}{5f}\right.}{b}$$

$\downarrow$ 406

$$\frac{(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\left(e(5bc-3ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf+4bcf+bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{1}{3}bx\sqrt{c+a}\right.}{b^2}\right.}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+d\right.}{c\left(e+f\right.}}}\right.}{5f}\right.}{b}$$

$\downarrow$ 320

3.64. $\int\frac{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\cdots\right.}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\text{EllipticF}\left(\text{arc}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+d\cdots\right.}{c\left(e+f\cdots\right.}}}\cdots\right.}{5f}\cdots\right.}{b}$$

$$\downarrow \; 388$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\cdots\right.}{b^2}\cdots\right.}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5f}-\frac{\frac{1}{3}\left(\left(-3c^2f^2-7cdef+2d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\text{EllipticF}\left(\text{arc}\cdots\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+d\cdots\right.}{c\left(e+f\cdots\right.}}}\cdots\right.}{5f}\cdots\right.}{b}$$

$$\downarrow \; 313$$

3.64. $\int \frac{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

$$\frac{(bc-ad)\left(\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\left(\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\right.\right.\right.}{b^2}\right)}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left((-3c^2f^2-7cdef+2d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\operatorname{EllipticF}\left(\operatorname{arc}\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+d}{c(e+f}}}\right.}{5f}\right.}{b}$$

$$\Big\downarrow \; 414$$

$$\frac{(bc-ad)\left(\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\operatorname{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\left(\frac{1}{3}\left(\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3\right.\right.}{}\right)}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5f}-\frac{\frac{1}{3}\left((-3c^2f^2-7cdef+2d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(de-9cf)\operatorname{EllipticF}\left(\operatorname{arc}\right.}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+d}{c(e+f}}}\right.}{5f}\right.}{b}$$

input
```
Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

3.64. $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

output
```
(d*((d*x*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*f) - ((2*(d*e - 3*c*f)*x*Sq
rt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2)*((
x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Elliptic
E[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3/2)*(d*e - 9*c*f)*Sqrt[c
+ d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/3)/(5*f)))/b + ((
b*c - a*d)*((d*((b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((b*d*e + 4*b*c*
f - 3*a*d*f)*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c +
d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + ((5*b*c - 3*a*d
)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
)/3))/b^2 + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a
*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*Sqrt[f]*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/b
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.64. $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.64. $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.64.4 Maple [A] (verified)

Time = 8.06 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{xd(-3bdf\,x^2+5adf-11bcf-bde)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{15f\,b^2}+\frac{\left(-\frac{d\left(15a^2d^2f^2-35abcd\,f^2-5ab\,d^2ef+23b^2c^2f^2+12b^2cdef-2b^2d^2e^2\right)e\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*d*(-3*b*d*f*x^2+5*a*d*f-11*b*c*f-b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/f/b^2+1/15/f/b^2*(-1/b*d*(15*a^2*d^2*f^2-35*a*b*c*d*f^2-5*a*b*d^2*e*
f+23*b^2*c^2*f^2+12*b^2*c*d*e*f-2*b^2*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x
*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+
d*e)/e/d)^(1/2)))-(15*a^3*d^3*f^2-45*a^2*b*c*d^2*f^2-15*a^2*b*d^3*e*f+45*a
*b^2*c^2*d*f^2+40*a*b^2*c*d^2*e*f-15*b^3*c^3*f^2-34*b^3*c^2*d*e*f+b^3*c*d^
2*e^2)/b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1
5*(a^4*d^3*f-3*a^3*b*c*d^2*f-a^3*b*d^3*e+3*a^2*b^2*c^2*d*f+3*a^2*b^2*c*d^2
*e-a*b^3*c^3*f-3*a*b^3*c^2*d*e+b^4*c^3*e)*f/b^2/a/(-d/c)^(1/2)*(1+d*x^2/c)
^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)))*((d*x^2+c)*(f*x^2+e))^(1/
2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.64.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.64.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(5/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

3.64.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.64.8 Giac [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.65 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.65.1 Optimal result

Integrand size = 32, antiderivative size = 400

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx &= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} \\ &+ \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} \\ &- \frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{d(5bc-3ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
1/3*(-3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)+1/3*d*(
-3*a*d+5*b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*
f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/b^2/c
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^2*e^(3
/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^2/c/f^(1
/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*d*f+4*b*c*f+
b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/
(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*x*(d*x^2+c)^(1/2)*(f
*x^2+e)^(1/2)/b
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{-iabde(bde+4bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\mid\frac{cf}{de}\right)-ia}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

output
```
((-I)*a*b*d*e*(b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-6*a*b*c*d*f
^2 + 3*a^2*d^2*f^2 + b^2*(-(d^2*e^2) + c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
+ f*(a*b^2*d*Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^2*(b*
e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

3.65.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {418, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{a+bx^2}dx$$

$$\Big\downarrow \ 418$$

$$\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\int \frac{(bdx^2+2bc-ad)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}dx}{b^2}$$

$$\Big\downarrow \ 403$$

$$\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{\int \frac{d\left((bde+4bcf-3adf)x^2+(5bc-3ad)e\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\int \frac{(bde+4bcf-3adf)x^2+(5bc-3ad)e}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 406$$

$$\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+$$
$$\frac{d\left(\frac{1}{3}\left(e(5bc-3ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf+4bcf+bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 320$$

$$\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}}{b^2}$$
$$\frac{(bc-ad)^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}$$

$$\Big\downarrow \ 388$$

$$\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}b}{b^2}$$

$$\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+$$

$$\frac{d\left(\frac{1}{3}\left(\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right.\right.\right.}{b^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+$$

$$\frac{d\left(\frac{1}{3}\left(\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right.\right.\right.}{b^2}$$

input
```
Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

output
```
(d*((b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((b*d*e + 4*b*c*f - 3*a*d*f)
*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + ((5*b*c - 3*a*d)*e^(3/2)*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/3))/b^2 +
((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.65. $\int\frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.65. $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.65.4 Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.11

method	result
risch	$\frac{dx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3b}-\frac{\left(\frac{\left(3a^3d^2f-6a^2bcdf-3a^2b\,d^2e+3a\,b^2c^2f+6a\,b^2cde-3b^3c^2e\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{-3a^2d^2 \cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/3/b*((3*a^3*d^2*f-6*a^2*b*c*d*
f-3*a^2*b*d^2*e+3*a*b^2*c^2*f+6*a*b^2*c*d*e-3*b^3*c^2*e)/b^2/a/(-d/c)^(1/2
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*
EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+1/b^2*(-3*a^2
*d^2*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-3*b^2
*c^2*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+3*a*b
*d^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-5*b^2
*d*c*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+6*a*b
*c*d*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*
b*d^2*f-4*b^2*c*d*f-b^2*d^2*e)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))
)))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.65.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.65.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

3.65.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.65.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.66 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.66.1 Optimal result

Integrand size = 32, antiderivative size = 321

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx &= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{de^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
f*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+d*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f
*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*(d*x^2+c)^(1/2)/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)
^(1/2)+(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Elliptic
Pi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2
+c)^(1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1
/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2
/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/b/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx =$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abdeE\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(bc-ad)\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d*e*EllipticE[I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.66.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {410, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 410}$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\int \frac{\sqrt{fx^2+e}}{\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\left(e\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+f\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{b}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\left(f\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\left(f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\left(\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\left(\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{b}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

output
```
(d*(f*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3/2)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/b + ((b*c - a*d)*e^(
3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

3.66. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.66. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

3.66.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.06

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2df+F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcf+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abde+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a^2df-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a\right.}{(df\,x^4+cf\,x^2+de\,x^2+ce)b^2\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)adf}{b^2\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)cf}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{de\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-}}\right.}{}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d*f+EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*c*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*
d*e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d*f-E
llipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*f-Ellipt
icPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*d*e+EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e)*(d*x^2+c)^(1/2)
*(f*x^2+e)^(1/2)*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)/b^2/(-d/c)^(1/2)/a
```

3.66.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.66.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

3.66.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.66.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.67 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.67.1 Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/
2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/f^(1
/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(be-af)\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\right.\right.}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*f*EllipticF[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.67.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 414}$$

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sq
rt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2])
```

3.67.3.1 Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.67.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)af+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)be\right)\sqrt{\frac{f\,x^2+e}{e}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{ba\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{f\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)f}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi}{a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*b*e)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d
*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.67.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.67.6 Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.67.7 Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.67.8 Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.68 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{3/2}}\,dx$

3.68.1 Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{3/2}}\,dx = -\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
b*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-
a*d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/2)/(d*x^2+c)
^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(ad\sqrt{\frac{d}{c}}ex+ad\sqrt{\frac{d}{c}}fx^3+iade\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right.\right.}{}$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[d/c]*(a*d*Sqrt[d/c]*e*x + a*d*Sqrt[d/c]*f*x^3 + I*a*d*e*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 + I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
I*a*c*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

3.68.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{416}$$

$$\frac{b\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\int \frac{\sqrt{fx^2+e}}{(dx^2+c)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \; 414$$

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f
)/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
+ d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.68.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

3.68.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.87

method	result
default	$\frac{\Big(\sqrt{-\frac{d}{c}}\, adf\, x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\Big)acf+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\Big)ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\Big(x\sqrt{-}}{ca\sqrt{-\frac{d}{c}}\,(ad-bc)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\,\Bigg(\frac{(df\,x^2+de)x}{c(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)(df\,x^2+de)}}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\Big)f}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,(ad-bc)}+\frac{de\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{}}{(ad-bc)c\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+}}}{}$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a*c*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*d
*e*x)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/a/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+
c*f*x^2+d*e*x^2+c*e)
```

3.68.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.68.6 Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.68.7 Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.68.8 Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.69 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.69.1 Optimal result

Integrand size = 32, antiderivative size = 401

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{\sqrt{d}(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
b^2*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e
^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/
(-a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d
*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/
2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+
b*c)/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*x*(f
*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-4*c*f+5*d*e)-a*d*(-c
*f+2*d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(3/2)/
(-a*d+b*c)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{ac\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\left(bc(6cde-5c^2f+5d^2ex^2-4cdfx^2)+ad(-3cde+2c^2f\right.}{}$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(a*c*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(6*c*d*e - 5*c^2*f + 5*d^2*e*x^2 - 4*c
*d*f*x^2) + a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2)) - I*a*d*e*
(a*d*(2*d*e - c*f) + b*c*(-5*d*e + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*
(-(d*e) + c*f)*(2*a*d^2*e + b*c*(-5*d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 - (3*I)*b*c^2*(b*e - a*f)*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(-(d*e) + c*f)*(c + d*x^2)^(3/2)*
Sqrt[e + f*x^2])
```

3.69.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {421, 401, 25, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\int \frac{(bdx^2+2bc-ad)\sqrt{fx^2+e}}{(dx^2+c)^{5/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}-\frac{\int-\frac{d\left((4bc-ad)fx^2+(5bc-2ad)e\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3cd}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int\frac{d\left((4bc-ad)fx^2+(5bc-2ad)e\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3cd}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int\frac{(4bc-ad)fx^2+(5bc-2ad)e}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 400$$

$$\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{(bc(5de-4cf)-ad(2de-cf))\int\frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{ef(bc-ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 313$$

$$\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{ef(bc-ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 320$$

3.69. $\displaystyle\int\frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

$\downarrow$ 414

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\left(\frac{\frac{\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}}\right)}{(bc-ad)^2}$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
-((d*(((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(c + d*x^2)^(3/2)) + (((b*c*(5*
d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x
^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - ((b*c - a*d)*e^(3/2)*Sqrt[f]*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c
)))/(b*c - a*d)^2) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*
f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt
[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.69. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.69. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. 2(466) = 932.

Time = 5.28 (sec) , antiderivative size = 1366, normalized size of antiderivative = 3.41

method	result	size
elliptic	Expression too large to display	1366
default	Expression too large to display	2067

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3/d/c*x/(a*
d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)/c
^2/(c*f-d*e)*x*(a*c*d*f-2*a*d^2*e-4*b*c^2*f+5*b*c*d*e)/(a*d-b*c)^2/((x^2+c
/d)*(d*f*x^2+d*e))^(1/2)-1/3*d^2/(c*f-d*e)/c/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*f+4/3*d/(c*f-d*e)/(a*d-b*
c)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*f-1
/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/c/(a*d-b*
c)^2*a*d*f+2/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))/c^2/(a*d-b*c)^2*a*e*d^2-5/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*
f+d*e)/e/d)^(1/2))/c/(a*d-b*c)^2*b*d*e+2/3*d^3/(c*f-d*e)/c^2/(a*d-b*c)^2*e
^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a-5/3*d^2
/(c*f-d*e)/c/(a*d-b*c)^2*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+
d*e)/e/d)^(1/2))*b+1/(a*d-b*c)^2/a*b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(...
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.69.7 Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.69.8 Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.70 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

3.70.1 Optimal result

Integrand size = 32, antiderivative size = 630

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}}$$
$$-\frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)(c+dx^2)^{3/2}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{\sqrt{d}(ad(8d^2e^2-13cdef+3c^2f^2)-2bc(9d^2e^2-14cdef+4c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{f}(bc(9de-11cf)-2ad(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
b^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e
^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/
(-a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*
d*e^(3/2)*(b*c*(-11*c*f+9*d*e)-2*a*d*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)
*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/
f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^2+
c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*d*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)
/(d*x^2+c)^(5/2)-1/15*d*(b*c*(-8*c*f+9*d*e)-a*d*(-3*c*f+4*d*e))*x*(f*x^2+e
)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*(a*d*(3*c^2*f^2-1
3*c*d*e*f+8*d^2*e^2)-2*b*c*(4*c^2*f^2-14*c*d*e*f+9*d^2*e^2))*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)^
2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b^2*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d
/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)/(c
*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.80 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{-ad\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(ad(4de\right.}{}$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

output
```
(-(a*d*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c
 - a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(-9*d*e + 8*c*f))*(c + d
*x^2) + (a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*
e^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*
f^2))*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*(a*d*e*(a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8
*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23
*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (d*e - c*f)*(-
(a*(2*a*b*c*d^2*e*(13*d*e - 14*c*f) + a^2*d^3*e*(-8*d*e + 9*c*f) + b^2*c^2
*(-33*d^2*e^2 + 49*c*d*e*f - 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)]) + 15*b^2*c^3*(b*e - a*f)*(-(d*e) + c*f)*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a
*d)^3*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

3.70.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {421, 401, 25, 27, 402, 25, 400, 313, 320, 416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\int \frac{(bdx^2+2bc-ad)\sqrt{fx^2+e}}{(dx^2+c)^{7/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}}-\frac{\int -\frac{d\left((8bc-3ad)fx^2+(9bc-4ad)e\right)}{\left(dx^2+c\right)^{5/2}\sqrt{fx^2+e}}dx}{5cd}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{d\left((8bc-3ad)fx^2+(9bc-4ad)e\right)}{\left(dx^2+c\right)^{5/2}\sqrt{fx^2+e}}dx}{5cd}+\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{(8bc-3ad)fx^2+(9bc-4ad)e}{\left(dx^2+c\right)^{5/2}\sqrt{fx^2+e}}dx}{5c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{x\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{3c\left(c+dx^2\right)^{3/2}(de-cf)}-\frac{\int -\frac{f(bc(9de-8cf)-ad(4de-3cf))x^2+e(bc(18de-19cf)-ad(8de-9cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}}{5c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\int \frac{f(bc(9de-8cf)-ad(4de-3cf))x^2+e(bc(18de-19cf)-ad(8de-9cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{3c\left(c+dx^2\right)^{3/2}(de-cf)}}{5c}+\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 400$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{ef(bc(9de-11cf)-2ad(2de-3cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(bc(9\cdots}{3c\left(c+\cdots\right.}}{5c}\cdots\right.}{(bc-ad)^2}$$

3.70. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

$$\Big\downarrow \text{313}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{ef(bc(9de-11cf)-2ad(2de-3cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}\cdots}{5c}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\,\text{EllipticF}\left(\cdots\right.}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\cdots}}{3c(de-cf)}\cdots}{5c}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \text{416}$$

$$\frac{b^2\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\,\text{EllipticF}\left(\cdots\right.}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\cdots}}{3c(de-cf)}\cdots}{5c}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \text{313}$$

3.70. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

$$\frac{b^2\left(\frac{b\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\text{EllipticF}(}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c(de-cf)}}{5c}\right.}{(bc-ad)^2}$$

$$\downarrow 414$$

$$\frac{b^2\left(\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\text{EllipticF}(}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c(de-cf)}}{5c}\right.}{(bc-ad)^2}$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

3.70. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

output
```
-((d*(((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*(c + d*x^2)^(5/2)) + (((b*c*(9*
d*e - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c
 + d*x^2)^(3/2)) + (-(((a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*c*(
9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c +
d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) - (e^(3/2)*Sqrt[f]*(b*c*(9*
d*e - 11*c*f) - 2*a*d*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c*(d*e - c*f)))/(5*c)))/(b*c - a*d)^
2) + (b^2*(-((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x
^2))/(e*(c + d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/
(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(b*c - a*d)^
2
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.70. $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3137 vs. $2(714) = 1428$.

Time = 6.96 (sec) , antiderivative size = 3138, normalized size of antiderivative = 4.98

method	result	size
elliptic	Expression too large to display	3138
default	Expression too large to display	6245

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-4/15/(-d/c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*f/(c*f-d*e)/c^2/(a*
d-b*c)^2*a*e*d^2+3/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d
)^(1/2))*f/(c*f-d*e)/c/(a*d-b*c)^2*b*d*e+13/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c^2/(a*d-b*c)^3*a^2*d^3*e*f+2
6/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*
e)/c^2/(a*d-b*c)^3*a*b*d^3*e^2-8/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))*f^2/(c*f-d*e)/(a*d-b*c)^2*b-b^3/(a*d-b*c)^3/a/(-d/
c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*e+1/15
*(d*f*x^2+d*e)/c^3/(c*f-d*e)^2*x*(3*a^2*c^2*d^2*f^2-13*a^2*c*d^3*e*f+8*a^2
*d^4*e^2-11*a*b*c^3*d*f^2+41*a*b*c^2*d^2*e*f-26*a*b*c*d^3*e^2+23*b^2*c^4*f
^2-58*b^2*c^3*d*e*f+33*b^2*c^2*d^2*e^2)/(a*d-b*c)^3/((x^2+c/d)*(d*f*x^2+d*
e))^(1/2)+1/5/d^2/c*x/(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c
/d)^3-23/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c...
```

3.70.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.70.6 Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

3.70.7 Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.70.8 Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{7/2}}\,dx=\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{7/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.71 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

3.71.1 Optimal result

Integrand size = 32, antiderivative size = 659

$$\begin{aligned}
\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx &= \frac{(bc-ad)^2 f^2 x\sqrt{c+dx^2}}{b^3 d\sqrt{e+fx^2}} \\
&+ \frac{2(bc-ad)f(2de-cf)x\sqrt{c+dx^2}}{3b^2 d\sqrt{e+fx^2}} \\
&+ \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} \\
&+ \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} \\
&- \frac{\sqrt{e}(15a^2d^2f^2-20abdf(de+cf)+3b^2(d^2e^2+9cdef+c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3 d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{e^{3/2}(15a^2d^2f+3b^2c(8de+3cf)-5abd(3de+5cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)^2 e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^3 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
(-a*d+b*c)^2*f^2*x*(d*x^2+c)^(1/2)/b^3/d/(f*x^2+e)^(1/2)+2/3*(-a*d+b*c)*f*
(-c*f+2*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)+1/15*(-2*c^2*f^2+7*c*
d*e*f+3*d^2*e^2)*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)+1/15*e^(3/2)*(15*a^
2*d^2*f+3*b^2*c*(3*c*f+8*d*e)-5*a*b*d*(5*c*f+3*d*e))*(1/(1+f*x^2/e))^(1/2)
*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/
f)^(1/2))*(d*x^2+c)^(1/2)/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)+(-a*d+b*c)^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x
^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*
e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1
/2)/(f*x^2+e)^(1/2)-1/15*(15*a^2*d^2*f^2-20*a*b*d*f*(c*f+d*e)+3*b^2*(c^2*f
^2+9*c*d*e*f+d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1
/2)/b^3/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*f*x*
(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/b+1/3*(-a*d+b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)/b^2+2/15*(-c*f+3*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{-iabe(15a^2d^2f^2-20abdf(de+cf)+3b^2(d^2e^2+9cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
((-I)*a*b*e*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 9*
c*d*e*f + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-15*a^3*d^2*f^3 + 15*a^2*b*d*f^2*(
d*e + 2*c*f) - 3*b^3*e*(d^2*e^2 + c*d*e*f - 7*c^2*f^2) + 5*a*b^2*f*(d^2*e^
2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*Sqrt[d/c]*x*(c + d*x^2
)*(e + f*x^2)*(-5*a*d*f + 3*b*(2*d*e + 2*c*f + d*f*x^2)) - (15*I)*(b*c - a
*d)^2*(b*e - a*f)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*
c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt[d/c]*f*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

3.71.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {420, 318, 403, 27, 406, 320, 388, 313, 418, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}{bx^2+a}dx}{b}+\frac{d\int \sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}dx}{b}$$

$$\Big\downarrow \text{318}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}{bx^2+a}dx}{b}+\frac{d\left(\frac{\int \frac{\sqrt{dx^2+c}\left(2f(3de-cf)x^2+e(5de-cf)\right)}{\sqrt{fx^2+e}}dx}{5d}+\frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{403}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}{bx^2+a}dx}{b}+\frac{d\left(\frac{\frac{\int \frac{f\left(\left(3d^2e^2+7cdfe-2c^2f^2\right)x^2+ce(9de-cf)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{5d}+\frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}{bx^2+a}dx}{b}+\frac{d\left(\frac{\frac{1}{3}\int \frac{\left(3d^2e^2+7cdfe-2c^2f^2\right)x^2+ce(9de-cf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{5d}+\frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}\right)}{b}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}(fx^2+e)^{3/2}}{bx^2+a}dx}{b} + d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+ce(9de-cf)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{5d}+\frac{fx(c+dx^2)^{3/2}\sqrt{e-}}{5d}\right.}{b}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}(fx^2+e)^{3/2}}{bx^2+a}dx}{b} + d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{5d}+\right.}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}(fx^2+e)^{3/2}}{bx^2+a}dx}{b} + d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c+dx^2}\sqrt{e+fx^2}(3}{5d}\right.}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}(fx^2+e)^{3/2}}{bx^2+a}dx}{b} + d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c}}{5d}\right.}{b}$$

3.71. $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

$$\downarrow 418$$

$$\frac{(bc-ad)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\int\frac{\sqrt{dx^2+c}(bfx^2+2be-af)}{\sqrt{fx^2+e}}dx}{b^2}\right)}{b}+$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c}}{5d}\right.}{b}$$

$$\downarrow 403$$

$$\frac{(bc-ad)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{\int\frac{f((4bde+bcf-3adf)x^2+c(5be-3af))}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{b}+$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c}}{5d}\right.}{b}$$

$$\downarrow 27$$

3.71. $\int\frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}dx$

$$\frac{(bc-ad)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{1}{3}\int\frac{(4bde+bcf-3adf)x^2+c(5be-3af)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{b}+$$

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2c^2f^2+7cdef+3d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c}}{5d}\right.}{b}$$

$$\Big\downarrow \; 406$$

$$\frac{(bc-ad)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{1}{3}\left(c(5be-3af)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf+bcf+4bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{1}{3}bx\sqrt{c+}\right.}{b^2}\right.}{b}$$

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2c^2f^2+7cdef+3d^2e^2\right)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c}}{5d}\right.}{b}$$

$$\Big\downarrow \; 320$$

3.71. $\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx$

$$\frac{(bc-ad)\left(\frac{f\left(\frac{1}{3}\left((-3adf+bcf+4bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}+\cdots\right.}{b}$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c}\cdots}{5d}\right.}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{(bc-ad)\left(\frac{f\left(\frac{1}{3}\left((-3adf+bcf+4bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\cdots\right)}{b^2}\right.}{b}$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c}\cdots}{5d}\right.}{b}$$

$$\Big\downarrow \text{313}$$

3.71. $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

$$\frac{(bc-ad)\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{1}{3}\left(\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+bcf+4bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{\cdots}}{\cdots}\right.\right.\right.}{b^2}\right)}{b}$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c\cdots}}{5d}\right.}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{(bc-ad)\left(\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\operatorname{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{f\left(\frac{1}{3}\left(\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3\cdots\right.\right.}{\cdots}\right)}{b}$$

$$\frac{d\left(\frac{\frac{1}{3}\left((-2c^2f^2+7cdef+3d^2e^2)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{2}{3}x\sqrt{c\cdots}}{5d}\right.}{b}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

3.71. $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

output
```
(d*((f*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*d) + ((2*(3*d*e - c*f)*x*Sq
rt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*((
x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Elliptic
E[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3/2)*(9*d*e - c*f)*Sqrt[c
+ d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/3)/(5*d)))/b + ((
b*c - a*d)*((f*((b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((4*b*d*e + b*c*
f - 3*a*d*f)*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c +
d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(5*b*e
 - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/
3))/b^2 + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d
), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])))/b
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.71. $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.71.4 Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x\left(-3bdf\,x^2+5adf-6bcf-6bde\right)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{15b^2}+\frac{\left(-\frac{\left(15a^2d^2f^2-20abcd\,f^2-20ab\,d^2ef+3b^2c^2f^2+27b^2cdef+3b^2d^2e^2\right)e\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(-3*b*d*f*x^2+5*a*d*f-6*b*c*f-6*b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(
1/2)/b^2+1/15/b^2*(-1/b*(15*a^2*d^2*f^2-20*a*b*c*d*f^2-20*a*b*d^2*e*f+3*b^
2*c^2*f^2+27*b^2*c*d*e*f+3*b^2*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(
1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)
^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/
d)^(1/2)))-(15*a^3*d^2*f^2-30*a^2*b*c*d*f^2-30*a^2*b*d^2*e*f+15*a*b^2*c^2*
f^2+55*a*b^2*c*d*e*f+15*a*b^2*d^2*e^2-24*b^3*c^2*e*f-24*b^3*c*d*e^2)/b^2/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+(15*a^4*d^2*f
^2-30*a^3*b*c*d*f^2-30*a^3*b*d^2*e*f+15*a^2*b^2*c^2*f^2+60*a^2*b^2*c*d*e*f
+15*a^2*b^2*d^2*e^2-30*a*b^3*c^2*e*f-30*a*b^3*c*d*e^2+15*b^4*c^2*e^2)/b^2/
a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)))
*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.71.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.71.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

3.71.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.71.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.72 $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx$

3.72.1 Optimal result

Integrand size = 32, antiderivative size = 403

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx &= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} \\
&+ \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} \\
&- \frac{\sqrt{e}\sqrt{f}(4bde+bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
\end{aligned}$$

output
```
1/3*f*(-3*a*d*f+b*c*f+4*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)-1/3
*(-3*a*d*f+b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Elliptic
E(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(
d*x^2+c)^(1/2)/b^2/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(
-3*a*f+5*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/
e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/
2)/b^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*f*x*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/b+c^(3/2)*(-a*f+b*e)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/
d/e)^(1/2))*(f*x^2+e)^(1/2)/a/b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e
/(d*x^2+c))^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.64 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{ab^2c\sqrt{\frac{d}{c}}efx + ab^2d\sqrt{\frac{d}{c}}efx^3 + ab^2c\sqrt{\frac{d}{c}}f^2x^3 + ab^2d\sqrt{\frac{d}{c}}f^2x^5 - iabe(4bde + bc}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(a*b^2*c*Sqrt[d/c]*e*f*x + a*b^2*d*Sqrt[d/c]*e*f*x^3 + a*b^2*c*Sqrt[d/c]*f
^2*x^3 + a*b^2*d*Sqrt[d/c]*f^2*x^5 - I*a*b*e*(4*b*d*e + b*c*f - 3*a*d*f)*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)] - I*a*(3*a^2*d*f^2 - 3*a*b*f*(d*e + c*f) + b^2*e*(-(d*e) + 4*c
*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] - (3*I)*b^3*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a*b
^2*d*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b^2*c*e*f*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - (6*I)*a^2*b*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*b*c*f^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a^3*d*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])
/(3*a*b^3*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.72.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {418, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \ 418$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\int \frac{\sqrt{dx^2+c}\left(bfx^2+2be-af\right)}{\sqrt{fx^2+e}}dx}{b^2}$$

$$\Big\downarrow \ 403$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{\int \frac{f\left((4bde+bcf-3adf)x^2+c(5be-3af)\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{1}{3}\int \frac{(4bde+bcf-3adf)x^2+c(5be-3af)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 406$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+\frac{f\left(\frac{1}{3}\left(c(5be-3af)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf+bcf+4bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}$$

$$\Big\downarrow \ 320$$

$$\frac{f\left(\frac{1}{3}\left((-3adf+bcf+4bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{\vphantom{x}}\right.}{b^2}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}$$

$$\Big\downarrow \ 388$$

$$\frac{f\left(\frac{1}{3}\left((-3adf+bcf+4bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int\frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}b\right.}{b^2}$$

$$\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}$$

$\Big\downarrow$ 313

$$\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b^2}+$$

$$\frac{f\left(\frac{1}{3}\left(\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+bcf+4bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right.\right.\right.}{b^2}$$

$\Big\downarrow$ 414

$$\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+$$

$$\frac{f\left(\frac{1}{3}\left(\frac{\sqrt{e}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+bcf+4bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right.\right.\right.}{b^2}$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(f*((b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/3 + ((4*b*d*e + b*c*f - 3*a*d*f)
*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(5*b*e - 3*a*f)*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sq
rt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/3))/b^2 + (c
^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqr
t[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

3.72. $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx$

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.72. $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx$

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.72.4 Maple [A] (verified)

Time = 8.26 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.10

method	result
risch	$\frac{fx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3b} - \frac{\left(\frac{\left(3a^3d\,f^2-3a^2bc\,f^2-6a^2bdef+6a\,b^2cef+3a\,b^2d\,e^2-3c\,e^2b^3\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+ -\frac{3a^2d \ldots}{\ldots}\right.}{\ldots}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/3/b*((3*a^3*d*f^2-3*a^2*b*c*f^
2-6*a^2*b*d*e*f+6*a*b^2*c*e*f+3*a*b^2*d*e^2-3*b^3*c*e^2)/b^2/a/(-d/c)^(1/2
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*
EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+1/b^2*(-3*a^2
*d*f^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-3*b^2
*d*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+3*a*b
*c*f^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-5*b^2
*c*e*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+6*a*b
*d*e*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*
b*d*f^2-b^2*c*f^2-4*b^2*d*e*f)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))
)))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.72.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.72.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

3.72.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.72.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.73 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.73.1 Optimal result

Integrand size = 32, antiderivative size = 328

$$\begin{aligned}\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} \\ &- \frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
f^2*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1
+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^
(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+
e)^(1/2)+e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipt
icPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x
^2+c)^(1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+
e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2
)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/b/c/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.56

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abefE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(be-af)\left(af\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(b\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*e*f*EllipticE[I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.73.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{f\int \frac{\sqrt{fx^2+e}}{\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{f\left(e\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+f\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{b}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{f\left(f\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{f\left(f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{f\left(\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f\left(\frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+f\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{b}$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(f*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (e^(3/2)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/b + (e^(3/2)*(b*e -
a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

3.73. $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.73.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.73.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^{2}f^{2}+F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a^{2}f^{2}-2\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\right.}{a\sqrt{-\frac{d}{c}}\,b^{2}(df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce)}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\left(-\frac{f^{2}\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^{2}\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}+\frac{f\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)e}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}+\frac{fe\sqrt{1+\frac{d\,x^{2}}{c}}}{b\sqrt{-}}\right.}{}$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*f^2+EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*e*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*
e*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*f^2-2
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*e*f+Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*e^2)*((f*x^2+
e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(
1/2)/b^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.73.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(e+fx^2\right)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.73.6 Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.73.7 Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.73.8 Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.74 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.74.1 Optimal result

Integrand size = 32, antiderivative size = 224

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{(de-cf)\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^
(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/a/c/(-a*d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-
(-c*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/
2)/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.20

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(abd\sqrt{\frac{d}{c}}e^2x - abc\sqrt{\frac{d}{c}}efx + abd\sqrt{\frac{d}{c}}efx^3 - abc\sqrt{\frac{d}{c}}f^2x^3 - iabe(-de+cf\right.}{}$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[d/c]*(a*b*d*Sqrt[d/c]*e^2*x - a*b*c*Sqrt[d/c]*e*f*x + a*b*d*Sqrt[d/c
]*e*f*x^3 - a*b*c*Sqrt[d/c]*f^2*x^3 - I*a*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
+ I*a*(-(a*c*f^2) + b*e*(-(d*e) + 2*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*e^2*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)] - (2*I)*a*b*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a^2
*c*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*
Sqrt[e + f*x^2])
```

3.74.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {417, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{417}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{fx^2+e}}{(dx^2+c)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \text{313}$$

$$\frac{(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \text{414}$$

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*
x^2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi
[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c
 - a*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.74.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(274) = 548$.

Time = 4.00 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.81

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\,abc\,f^2x^3+\sqrt{-\frac{d}{c}}\,abdef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2c\,f^2-2\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcef+\sqrt{\right.}{}$
elliptic	Expression too large to display

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-d/c)^(1/2)*a*b*c*f^2*x^3+(-d/c)^(1/2)*a*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*f^2-
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*e*f-((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2-
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*e*
f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/
d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e^2-(-d/c)^(1/2)*a*b*c*e*f*x+(-d/c)^(1/
2)*a*b*d*e^2*x)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/a/(-d/c)^(1/2)/c/(a*d-b*
c)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.74.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.74.6 Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.74.7 Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.74.8 Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.75 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.75.1 Optimal result

Integrand size = 32, antiderivative size = 391

$$\begin{aligned}\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} \\ &- \frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &+ \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
b*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*
f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(
1/2)/a/c/(-a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1
/2)+1/3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2
)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2
/(-a*d+b*c)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-c*f+d*e)
*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-c*f+5*d*e)-2*a*
d*(c*f+d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+
b*c)^2/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.03 (sec) , antiderivative size = 999, normalized size of antiderivative = 2.55

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{3a^2cd^2\sqrt{\frac{d}{c}}e^2x - 6abc^3\left(\frac{d}{c}\right)^{3/2}e^2x + 2abc^3\sqrt{\frac{d}{c}}efx + a^2c^3\left(\frac{d}{c}\right)^{3/2}efx - 5abcd^2\sqrt{}}{}$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(3*a^2*c*d^2*Sqrt[d/c]*e^2*x - 6*a*b*c^3*(d/c)^(3/2)*e^2*x + 2*a*b*c^3*Sqr
t[d/c]*e*f*x + a^2*c^3*(d/c)^(3/2)*e*f*x - 5*a*b*c*d^2*Sqrt[d/c]*e^2*x^3 +
 2*a^2*d^3*Sqrt[d/c]*e^2*x^3 + 5*a^2*c*d^2*Sqrt[d/c]*e*f*x^3 - 5*a*b*c^3*(
d/c)^(3/2)*e*f*x^3 + 2*a*b*c^3*Sqrt[d/c]*f^2*x^3 + a^2*c^3*(d/c)^(3/2)*f^2
*x^3 - 5*a*b*c*d^2*Sqrt[d/c]*e*f*x^5 + 2*a^2*d^3*Sqrt[d/c]*e*f*x^5 + 2*a^2
*c*d^2*Sqrt[d/c]*f^2*x^5 + a*b*c^3*(d/c)^(3/2)*f^2*x^5 + I*a*e*(b*c*(-5*d*
e + c*f) + 2*a*d*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f
)*(5*b*c*e - 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^3*e
^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b*c^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - (3*I)*a^2*c^3*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[
(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*d*e^2*x^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b*c^2*d*e*f*x^2*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)] - (3*I)*a^2*c^2*d*f^2*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*c^2*...
```

3.75.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {419, 25, 401, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 419}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}-\frac{\int -\frac{\sqrt{fx^2+e}\left(-\left((be-af)x^2d^2\right)+aed^2-bc(2de-cf)\right)}{(dx^2+c)^{5/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{fx^2+e}\left(-\left((be-af)x^2d^2\right)+aed^2-bc(2de-cf)\right)}{(dx^2+c)^{5/2}}dx}{(bc-ad)^2}+\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}+\frac{-\frac{\int \frac{d\left(f(bc(4de-cf)-ad(de+2cf))x^2+e(bc(5de-2cf)-ad(2de+cf))\right)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3cd}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}+\frac{-\frac{\int \frac{f(bc(4de-cf)-ad(de+2cf))x^2+e(bc(5de-2cf)-ad(2de+cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{-\frac{(bc(5de-cf)-2ad(cf+de))\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx-ef(bc-ad)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}+\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 313}$$

3.75. $\int \frac{\left(e+fx^2\right)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}dx$

$$-\frac{\frac{\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-ef(bc-ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}+\frac{b(be-af)\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{b(be-af)\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(bc-ad)^2}+\frac{-\frac{\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \; 414$$

$$\frac{be^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{-\frac{\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{3c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(bc-ad)^2}$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-1/3*((b*c - a*d)*(d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(c + d*x^2)^(3/2)) -
(((b*c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*Sqrt[c + d*x^2]*
Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt
[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c))/(b*c - a*d)^2
 + (b*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.75. $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.75. $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(456) = 912$.

Time = 5.19 (sec) , antiderivative size = 1645, normalized size of antiderivative = 4.21

method	result	size
elliptic	Expression too large to display	1645
default	Expression too large to display	1879

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3*(c*f-d*e
)/d^2/(a*d-b*c)/c*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d
*f*x^2+d*e)*(2*a*c*d*f+2*a*d^2*e+b*c^2*f-5*b*c*d*e)/d/(a*d-b*c)^2/c^2*x/((
x^2+c/d)*(d*f*x^2+d*e))^(1/2)-1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+
(c*f+d*e)/e/d)^(1/2))/d/(a*d-b*c)^2*c*b*f^2+2/3/(a*d-b*c)^2/c^2*e^2/(-d/c)
^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*d^2-5/3/(a*d-b*c
)^2/c*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*
d-2/3/(a*d-b*c)^2/c^2*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)
/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e
)/e/d)^(1/2))*a*d^2-1/3/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*
x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))*b*f+5/3/(a*d-b*c)^2/c*e^2/(-d/c)^(1/2)*(1+d*x^2/
c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*d-2/3/(a*d-b*c)^2/c*e/(-d/c)^(1/
2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*d*f-1/3/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2...
```

3.75.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.75.8 Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.76 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

3.76.1 Optimal result

Integrand size = 32, antiderivative size = 639

$$\begin{aligned}
\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx &= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} \\
&-\frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}} \\
&-\frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&+\frac{(ad(8d^2e^2-3cdef-2c^2f^2)-3bc(6d^2e^2-6cdef+c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&+\frac{e^{3/2}\sqrt{f}(3bc(3de-2cf)-ad(4de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+\frac{b^2e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
b^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)
^(1/2)/a/c/(-a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^
(1/2)+1/15*e^(3/2)*(3*b*c*(-2*c*f+3*d*e)-a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))
^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-
d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-c*f+d*e)*x*(f*x^2+e)^(1/2)
/c/(-a*d+b*c)/(d*x^2+c)^(5/2)-1/15*(3*b*c*(-c*f+3*d*e)-2*a*d*(c*f+2*d*e))*
x*(f*x^2+e)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/15*(a*d*(-2*c^2*f^2-3
*c*d*e*f+8*d^2*e^2)-3*b*c*(c^2*f^2-6*c*d*e*f+6*d^2*e^2))*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*
f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)/d^(1/2)/(d*x
^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b*(-a*f+b*e)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*
f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)
/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(3bc(-3c\right.}{}$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

output
```
(-(a*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c -
 a*d)*(-(d*e) + c*f)*(3*b*c*(-3*d*e + c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x
^2) + (a^2*d^2*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + 3*b^2*c^2*(11*d^2*e^2
 - 11*c*d*e*f + c^2*f^2) + 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2)
)*(c + d*x^2)^2)) + I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*(a*e*(-3*b^2*c^2*(11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + a^2*d^2*(-8*d^2*e
^2 + 3*c*d*e*f + 2*c^2*f^2) - 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f
^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a*(3*b^
2*c^2*e*(11*d*e - 8*c*f) + a^2*d^2*e*(8*d*e + c*f) + a*b*c*(-26*d^2*e^2 -
7*c*d*e*f + 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
15*b*c^3*(b*e - a*f)^2*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*
f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e - c*f)*(c + d*x^2)^(5/
2)*Sqrt[e + f*x^2])
```

3.76.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {419, 25, 401, 27, 402, 25, 400, 313, 320, 416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{419}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}-\frac{\int -\frac{\sqrt{fx^2+e}(-((be-af)x^2d^2)+aed^2-bc(2de-cf))}{(dx^2+c)^{7/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{fx^2+e}(-((be-af)x^2d^2)+aed^2-bc(2de-cf))}{(dx^2+c)^{7/2}}dx}{(bc-ad)^2}+\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}+\frac{-\frac{\int \frac{d\left(f(bc(8de-3cf)-ad(3de+2cf))x^2+e(bc(9de-4cf)-ad(4de+cf))\right)}{\left(dx^2+c\right)^{5/2}\sqrt{fx^2+e}}dx}{5cd}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}+\frac{-\frac{\int \frac{f(bc(8de-3cf)-ad(3de+2cf))x^2+e(bc(9de-4cf)-ad(4de+cf))}{\left(dx^2+c\right)^{5/2}\sqrt{fx^2+e}}dx}{5c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}+\frac{-\frac{\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{f(de-cf)(3bc(3de-cf)-2ad(2de+cf))x^2+e(de-cf)(9bc(2de-cf)-ad(8de+cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}}{5c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-}{5c(c+dx^2)^{5/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}+\frac{-\frac{\frac{\int \frac{f(de-cf)(3bc(3de-cf)-2ad(2de+cf))x^2+e(de-cf)(9bc(2de-cf)-ad(8de+cf))}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{3c\left(c+dx^2\right)^{3/2}}}{5c}-\frac{x\sqrt{e+fx^2}(bc-ad)(de-c}{5c(c+dx^2)^{5/2}}}{(bc-ad)^2}$$

$$\Big\downarrow \; 400$$

$$\frac{-\frac{\frac{-\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx-ef(3bc(3de-2cf)-ad(4de-cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{x\sqrt{e+fx^2}(3bc(3de-cf}{3c\left(c+dx^2\right.}}{5c}}{(bc-ad)^2}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \; 313$$

3.76. $\int \frac{\left(e+fx^2\right)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

$$-\frac{\frac{\frac{-ef(3bc(3de-2cf)-ad(4de-cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{x\sqrt{e\cdots}}{\cdots}}{5c}}{(bc-ad)^2}$$

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}$$

$\big\downarrow$ 320

$$\frac{b(be-af)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{3/2}}dx}{(bc-ad)^2}+$$
$$-\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\,\text{EllipticF}\left(\text{arcta}\cdots\right.}{c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}\cdots}{5c}}{(bc-ad)^2}$$

$\big\downarrow$ 416

$$\frac{b(be-af)\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(bc-ad)^2}+$$
$$-\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\,\text{EllipticF}\left(\text{arcta}\cdots\right.}{c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}\cdots}{5c}}{(bc-ad)^2}$$

$\big\downarrow$ 313

$$\frac{b(be-af)\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}\right)}{(bc-ad)^2}+$$
$$-\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\,\text{EllipticF}\left(\text{arcta}\cdots\right.}{c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}\cdots}{5c}}{(bc-ad)^2}$$

3.76. $\displaystyle\int \frac{\left(e+fx^2\right)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

$$\Big\downarrow \; 414$$

$$-\frac{\frac{-\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\,\text{EllipticF}\left(\text{arcta}\right.}{c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3c(de-cf)}}{5c}}{(bc-ad)^2}$$

$$\frac{b(be-af)\left(\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}\right)}{(bc-ad)^2}$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

output
```
(-1/5*((b*c - a*d)*(d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(c + d*x^2)^(5/2)) -
(((3*b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(3*c*(c +
 d*x^2)^(3/2)) + (-(((a*d*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - 3*b*c*(6*d
^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))])) - (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*c*f) - a
*d*(4*d*e - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(c*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
)/(3*c*(d*e - c*f)))/(5*c))/(b*c - a*d)^2 + (b*(b*e - a*f)*(-((Sqrt[d]*Sqr
t[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqr
t[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) +
 (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)
/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(b*c - a*d)^2
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.76. $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3111 vs. $2(723) = 1446$.

Time = 6.88 (sec) , antiderivative size = 3112, normalized size of antiderivative = 4.87

method	result	size
elliptic	Expression too large to display	3112
default	Expression too large to display	6211

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5*(c*f-d*e
)/d^3/(a*d-b*c)/c*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3-2/15/(
c*f-d*e)/c/(a*d-b*c)^3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)
/e/d)^(1/2))*a^2*d^2*f^2-1/5/(c*f-d*e)/c^2/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a^2*d^3*f-26/15/(c*f-d*e)/c
^2/(a*d-b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^
(1/2))*a*b*d^3-11/5/(c*f-d*e)/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/
2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b^2*d*f-1/(a*d-b*c)^3*a*b/(-d/c)^(1/2)*(
1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ell
ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f^2+2/(a*d-b*c)^
3*b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*
e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/
2))*f*e-1/(a*d-b*c)^3/a*b^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*e^2+1/15*(2*a*c*d*f+4*a*d^2*e+3*b*c^2*f-9*b*c*d*
e)/(a*d-b*c)^2/d^2/c^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^...
```

3.76.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.76.8 Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}\,dx=\int \frac{\left(f\,x^2+e\right)^{3/2}}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.77 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

3.77.1 Optimal result

Integrand size = 32, antiderivative size = 621

$$\begin{aligned}\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx &= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} \\ &+ \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} - \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{2d\sqrt{e}(de-2cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &- \frac{d\sqrt{e}(de-3cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

output
```
d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)-2/3*d*(-2*c*f+d*e)*x*(d
*x^2+c)^(1/2)/b/f/(f*x^2+e)^(1/2)+2/3*d*(-2*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)
*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/
f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)/(f*x^2+e)^(1/2)-1/3*d*(-3*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/
2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2
)*(d*x^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2
)-d*(-a*d+b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/
f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+d*(-a*d+b*c)*(1/(1
+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)
^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)/b/f+c^(3/2)*(-a*d+b*c)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*
x^2+e)^(1/2)/a/b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/
2)
```

3.77.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{-iabd^2e(-2bde+7bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-i}{}$$

input
```
Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((-I)*a*b*d^2*e*(-2*b*d*e + 7*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*d*(3*a^2
*d^2*f^2 + 3*a*b*d*f*(d*e - 3*c*f) + b^2*(2*d^2*e^2 - 8*c*d*e*f + 9*c^2*f^
2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] + f*(a*b^2*c*d*(d/c)^(3/2)*x*(c + d*x^2)*(e + f*x^2) - (3
*I)*(b*c - a*d)^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*
c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f^2*Sq
rt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.77.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 599, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {420, 318, 25, 406, 320, 388, 313, 420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{fx^2+e}}dx}{b}$$

$$\Big\downarrow \text{318}$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{\int -\frac{2d(de-2cf)x^2+c(de-3cf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\int \frac{2d(de-2cf)x^2+c(de-3cf)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}\right)}{b}$$

$$\Big\downarrow \text{406}$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{c(de-3cf)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+2d(de-2cf)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}\right)}{b}$$

$$\Big\downarrow \text{320}$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{2d(de-2cf)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}\right)}{b}$$

$$\Big\downarrow \; 388$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{3f}\right)}{b}$$

$$\Big\downarrow \; 313$$

$$\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

$$\Big\downarrow \; 420$$

3.77. $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx$

$$\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{fx^2+e}}dx}{b}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

$\downarrow$ 324

$$\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(c\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{b}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

$\downarrow$ 320

$$\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{b}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

3.77. $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx$

$\downarrow$ 388

$$\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{b}\right)}{b}+\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

$\downarrow$ 313

$$\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{b}\right)}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{3f}\right)}{b}$$

$\downarrow$ 414

3.77. $\int \frac{\left(c+dx^2\right)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

$$\frac{(bc-ad)\left(\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\operatorname{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{\ldots}}{\ldots}\right.\right.}{b}\right.}{b}$$

$$\frac{d\left(\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\frac{\sqrt{e}\sqrt{c+dx^2}(de-3cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+2d(de-2cf)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{3f}\right)}{b}$$

input
```
Int[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
(d*((d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) - (2*d*(d*e - 2*c*f)*((x*S
qrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f)))/b + ((b*c -
a*d)*((d*(d*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*Sqrt[c
+ d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/b + (c^(3/2)*(b*
c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (c*f)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])))/b
```

3.77. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.77. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.77.4 Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.19

method	result
risch	$\frac{d^2 x\sqrt{d x^2+c}\,\sqrt{f x^2+e}}{3bf} - \frac{\left(\frac{3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)f\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{d\left(-\frac{3a^2d^2f\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f}{}}}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4}}\right.}{}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,a\,b^2d^3f^2x^5+\sqrt{-\frac{d}{c}}\,a\,b^2c\,d^2f^2x^3+\sqrt{-\frac{d}{c}}\,a\,b^2d^3ef\,x^3+3\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^3d^3f^2-9\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f-1/3/f/b*(3*(a^3*d^3-3*a^2*b*
c*d^2+3*a*b^2*c^2*d-b^3*c^3)*f/b^2/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),
b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+d/b^2*(-3*a^2*d^2*f/(-d/c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-9*b^2*c^2*f/(-d/c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+b^2*d*c*e/(-d/c)^(1/2)*(1+d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+9*a*b*c*d*f/(-d/c)^(1/2)*(1+d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*b*d^2*f-7*b^2*c*d*f+2*b^2*d^
2*e)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-El
lipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f*x^2+e))^
(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.77.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.77.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(5/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

3.77.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.77.8 Giac [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.78 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

3.78.1 Optimal result

Integrand size = 32, antiderivative size = 319

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
d*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)-d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(
1/2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(
1/2)+d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b*c)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/b/e/d^(1/2)/(d*x^2+c)^(
1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abd^2eE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\mid\frac{cf}{de}\right)-ad(bde-2bcf+adf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\right.\right.}{ab^2\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d^2*e*EllipticE[I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - a*d*(b*d*e - 2*b*c*f + a*d*f)*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)^2*f*EllipticPi[(b*c)/(a*d
), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*f*Sqrt[c + d*x^
2]*Sqrt[e + f*x^2])
```

3.78.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}dx$$
$$\Big\downarrow \text{ 420}$$
$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b}+\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{fx^2+e}}dx}{b}$$
$$\Big\downarrow \text{ 324}$$
$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(c\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{b}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{b}$$

$$\Big\downarrow \textbf{388}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{b}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{b}+\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{b}$$

$$\Big\downarrow \textbf{414}$$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+d\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{b}$$

input
```
Int[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

3.78. $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

output
```
(d*(d*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/b + (c^(3/2)*(b*c - a*
d)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (c*f)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e
*(c + d*x^2))])
```

3.78.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.78. $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

3.78.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2f+2F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcdf-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\right.}{a\,b^2f\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{d^2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^2\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{2d\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)c}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{d^2e\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-}}\right.}{}$

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*f+2*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*f-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*b*d^2*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*f-2*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*f+EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*f)*((f*x^2+e)/e)^(1
/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/b^2/f/(-d/c)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.78.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.78.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

3.78.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.78.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.79 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

3.79.1 Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx=\frac{c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/d^(1
/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx=$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(bc-ad)\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\ldots\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*EllipticF[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.79.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{414}$$

$$\frac{c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

output
```
(c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2)
)/(e*(c + d*x^2))])
```

3.79.3.1 Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.79.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ad-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)ad+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)bc\right)\sqrt{\frac{f\,x^2+e}{e}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}}{ba\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{d\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)d}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi}{a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4-}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d-EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*b*c)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f
*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.79.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.79.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

3.79.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.79.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.80 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.80.1 Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

output
```
EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

3.80.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d),
ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

3.80.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

3.80.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\Pi\left(x\sqrt{-\frac{d}{c}}, \frac{bc}{ad}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{f\,x^{2}+e}{e}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{f\,x^{2}+e}\,\sqrt{d\,x^{2}+c}}{a\sqrt{-\frac{d}{c}}\,(df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce)}$	118
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}}, \frac{bc}{ad}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}\,a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}$	133

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((f*x^2+e)/e)
^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.80.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.80.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.80.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.80.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.81 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

3.81.1 Optimal result

Integrand size = 32, antiderivative size = 344

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx &= -\frac{d^{3/2}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\\ &\quad-\frac{d\sqrt{e}(bde-2bcf+adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &\quad+\frac{b^2c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

output
```
-d*(a*d*f-2*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF
(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^
(1/2)/c/(-a*d+b*c)^2/(-c*f+d*e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)-d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d
+b*c)/(-c*f+d*e)/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b
^2*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/(-a
*d+b*c)^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(acd\left(\frac{d}{c}\right)^{3/2}ex + acd\left(\frac{d}{c}\right)^{3/2}fx^3 + iad^2e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\right.}{}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[d/c]*(a*c*d*(d/c)^(3/2)*e*x + a*c*d*(d/c)^(3/2)*f*x^3 + I*a*d^2*e*Sq
rt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)] + I*a*d*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*d*e*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)] - I*b*c^2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi
[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*(
d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.81.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {421, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 421$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{(bc-ad)^2} - \frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \ 400$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{d(bc-ad)\int \frac{\sqrt{fx^2+e}}{(dx^2+c)^{3/2}}dx}{de-cf}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{b^2c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\left(\frac{\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\right)}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
-((d*((Sqrt[d]*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))]) + (Sqrt[e]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[c +
d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]
*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(b*c
 - a*d)^2) + (b^2*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt
[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

3.81. $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

3.81.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.81.4 Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\, a\, d^{2} f\, x^{3}+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a c d f-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a\, d^{2} e+\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, E\left(\right.\right.}{c a (a d-b c) \sqrt{-}}$
elliptic	$\frac{\sqrt{(d\, x^{2}+c)(f\, x^{2}+e)}\, \left(-\frac{\left(d f\, x^{2}+d e\right) d x}{c (c f-d e)(a d-b c) \sqrt{\left(x^{2}+\frac{c}{d}\right)\left(d f\, x^{2}+d e\right)}}+\frac{\sqrt{1+\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{f\, x^{2}}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{-1+\frac{c f+d e}{e d}}\right) d}{\sqrt{-\frac{d}{c}}\, \sqrt{d f\, x^{4}+c f\, x^{2}+d e\, x^{2}+c e}\, c (a d-b c)}+\frac{d^{2} e \sqrt{1+\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{f\, x^{2}}{e}}\, E}{c (c f-d e)(a d-b c) \sqrt{-\frac{d}{c}}}\right.}{\sqrt{d\, x^{2}+c}\, \sqrt{f\, x^{2}+e}}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-d/c)^(1/2)*a*d^2*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e+((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e
-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*d*e-(-d
/c)^(1/2)*a*d^2*e*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/a/(a*d-b*c)/(-d/c)^
(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.81.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.81.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

3.81.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.81.8 Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.82 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

3.82.1 Optimal result

Integrand size = 32, antiderivative size = 435

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} - \frac{d^{3/2}(bc(5de-7cf)-2ad(de-2cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d\sqrt{e}\sqrt{f}(ad(de-3cf)-2bc(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^2\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

output
```
-1/3*d*(a*d*(-3*c*f+d*e)-2*b*c*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*
x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/
2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^
2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d^2*x*(f*x^2+e)^(1/2)/c/(-a*d+
b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*d^(3/2)*(b*c*(-7*c*f+5*d*e)-2*a*d*(-2*
c*f+d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c
)^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b^2*Ellip
ticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)
^(1/2)*(1+f*x^2/e)^(1/2)/a/(-a*d+b*c)^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^
(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{acd\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\left(bc(-6cde+8c^2f-5d^2ex^2+7cdfx^2)+ad(-\right.}{}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
(a*c*d*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(-6*c*d*e + 8*c^2*f - 5*d^2*e*x^2 +
7*c*d*f*x^2) + a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*a*d
^2*e*(2*a*d*(d*e - 2*c*f) + b*c*(-5*d*e + 7*c*f))*(c + d*x^2)*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 + I*a*d*(-(d*e) + c*f)*(a*d*(2*d*e - 3*c*f) + b*c*(-5*d*e + 6*c*f))*(c +
d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*(d*e - c*f)^2*(c + d*x^2)*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(d*e - c*f)^2*(c + d*x^2)
^(3/2)*Sqrt[e + f*x^2])
```

3.82.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {421, 402, 25, 400, 313, 320, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 421$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^{5/2}\sqrt{fx^2+e}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{dx\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}-\frac{\int -\frac{d(bc-ad)fx^2+bc(5de-6cf)-ad(2de-3cf)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{d(bc-ad)fx^2+bc(5de-6cf)-ad(2de-3cf)}{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}dx}{3c(de-cf)}+\frac{dx\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 400$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{f\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{d(bc(5de-7cf)-2ad(de-2cf))\int \frac{\sqrt{fx^2+e}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}}{3c(de-cf)}+\frac{dx\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 313$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{f\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{dx\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{dx\sqrt{e+fx^2}(bc-ad)}{3c(c+dx^2)^{3/2}(de-cf)}\right)}{(bc-ad)^2}$$

3.82. $\displaystyle\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

$\Big\downarrow$ 413

$$\frac{b^2\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{\sqrt{c+dx^2}(bc-ad)^2} - \frac{d\left(\frac{\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{dx\cdots}{3c(\cdots}\right.}{(bc-ad)^2}$$

$\Big\downarrow$ 413

$$\frac{b^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2} - \frac{d\left(\frac{\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{dx\cdots}{3c(\cdots}\right.}{(bc-ad)^2}$$

$\Big\downarrow$ 412

$$\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2} - \frac{d\left(\frac{\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(ad(de-3cf)-b\left(4cde-6c^2f\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}}{3c(de-cf)}+\frac{dx\cdots}{3c(\cdots}\right.}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

3.82. $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

output
```
-((d*((d*(b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)
) + ((Sqrt[d]*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sqrt[e + f*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(d*e - c
*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (Sqrt[e]*Sqrt
[f]*(a*d*(d*e - 3*c*f) - b*(4*c*d*e - 6*c^2*f))*Sqrt[c + d*x^2]*EllipticF[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(d*e - c*f)*Sqrt[(e*(c +
 d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*c*(d*e - c*f))))/(b*c - a*d
)^2) + (b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b
*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a
*d)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(460) = 920$.

Time = 6.55 (sec) , antiderivative size = 1325, normalized size of antiderivative = 3.05

method	result	size
elliptic	Expression too large to display	1325
default	Expression too large to display	2062

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3/c/(c*f-d
*e)*x/(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/3*(d*f*x
^2+d*e)*d/c^2/(c*f-d*e)^2*x*(4*a*c*d*f-2*a*d^2*e-7*b*c^2*f+5*b*c*d*e)/(a*d
-b*c)^2/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)-1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)
*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d*f/c/(c*f-d*e)/(a*d-b*c)+4/3/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*d^2/c/(a*d-b*c
)^2*a*f-2/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*
x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/
(c*f-d*e)*d^3/c^2/(a*d-b*c)^2*a*e-7/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*
x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*d/(a*d-b*c)^2*b*f+5/3/(-d/c)^(1/2)*(1+
d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*d^2/c/(a*d-b*c)^2*
b*e+4/3*d^3/(c*f-d*e)^2/c/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+
f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2
),(-1+(c*f+d*e)/e/d)^(1/2))*a*f-2/3*d^4/(c*f-d*e)^2/c^2/(a*d-b*c)^2*e^2/(-
d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*
e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a-7/3*d^2/(...
```

3.82.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.82.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

3.82.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.82.8 Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.83 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

3.83.1 Optimal result

Integrand size = 32, antiderivative size = 980

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}}$$
$$+\frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{3ef(be-af)^2\sqrt{e+fx^2}}$$
$$+\frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}}+\frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2}$$
$$+\frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2}$$
$$-\frac{(bc-ad)\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
(-c*f+d*e)*x*(d*x^2+c)^(3/2)/e/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(
-3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/(f*x^2+e)^(1/2)+1
/3*(b*e*(-c^2*f^2-7*c*d*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2
))*x*(d*x^2+c)^(1/2)/e/f/(-a*f+b*e)^2/(f*x^2+e)^(1/2)-1/3*(b*e*(-c^2*f^2-7
*c*d*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^
(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d
*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/f^(3/2)/(-a*f+b*e)^2/e^(1/2)/(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(2*a*d*f*(-3*c*f+2*d*e)-b*(-3*c^2*f^
2-2*c*d*e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(
1/2)/f^(3/2)/(-a*f+b*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+
1/3*d*(-3*a*d+5*b*c)*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*
x^2+c)^(1/2)/b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x
^2+e)^(1/2)+(-a*d+b*c)^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*E
llipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))
*(d*x^2+c)^(1/2)/a/b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)/(f*x^2+e)^(1/2)-1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*f+b*d*e)*(1/(1+f*x^2/e))^
(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d
*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2...
```

3.83.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.83 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.36

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{-iabde(-ad^2ef+b(2d^2e^2-2cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\mathrm{arcsinh}\Big(}{}$$

input
```
Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

output
```
((-I)*a*b*d*e*(-(a*d^2*e*f) + b*(2*d^2*e^2 - 2*c*d*e*f + c^2*f^2))*Sqrt[1
+ (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(
d*e)] - I*a*d^2*e*(b*e - a*f)*(-2*b*d*e + 3*b*c*f - a*d*f)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
f*(a*b^2*Sqrt[d/c]*(d*e - c*f)^2*x*(c + d*x^2) + I*(b*c - a*d)^3*e*f*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*e*f^2*(b*e - a*f)*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

3.83.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 828, normalized size of antiderivative = 0.84, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {419, 25, 401, 27, 403, 406, 320, 388, 313, 418, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{419}$$

$$\frac{b(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}-\frac{\int -\frac{\left(dx^2+c\right)^{3/2}\left(-\left((bc-ad)x^2f^2\right)+acf^2+be(de-2cf)\right)}{\left(fx^2+e\right)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(dx^2+c\right)^{3/2}\left(-\left((bc-ad)x^2f^2\right)+acf^2+be(de-2cf)\right)}{\left(fx^2+e\right)^{3/2}}dx}{(be-af)^2}+\frac{b(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}+\frac{\frac{x\left(c+dx^2\right)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\int \frac{f\sqrt{dx^2+c}\left(c(bc-ad)ef-d(af(4de-3cf)-be(3de-2cf))x^2\right)}{\sqrt{fx^2+e}}dx}{ef}}{(be-af)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}+\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\int \frac{\sqrt{dx^2+c}\left(c(bc-ad)ef-d(af(4de-3cf)-be(3de-2cf))x^2\right)}{\sqrt{fx^2+e}}dx}{e}}{(be-af)^2}$$

$\downarrow$ 403

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{\int \frac{ce\left(2adf(2de-3cf)-b\left(3d^2e^2-2cdfe-3c^2f^2\right)\right)-d\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-af\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3f}-\frac{dx\sqrt{c+d}}{}}{e}}{(be-af)^2}$$

$$\frac{b(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}$$

$\downarrow$ 406

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{ce\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)-af\left(3c^2f^2-13cdef+\right.\right.}{3f}}{e}}{(be-af)^2}$$

$$\frac{b(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}$$

$\downarrow$ 320

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{e}}{(be-af)^2}$$

$$\frac{b(bc-ad)\int \frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}$$

$\downarrow$ 388

3.83. $\displaystyle\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{e}}{(be-af)^2}$$

$$\frac{b(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}$$

$\downarrow$ 313

$$\frac{b(bc-ad)\int\frac{(dx^2+c)^{3/2}\sqrt{fx^2+e}}{bx^2+a}dx}{(be-af)^2}+$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 418

$$\frac{b(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\int\frac{(bdx^2+2bc-ad)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}dx}{b^2}\right)}{(be-af)^2}+$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 403

3.83. $\int\frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}dx$

$$\frac{b(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{\int\frac{d\left((bde+4bcf-3adf)x^2+(5bc-3ad)e\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{3d}+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{(be-af)^2}+$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 27

$$\frac{b(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\int\frac{(bde+4bcf-3adf)x^2+(5bc-3ad)e}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right)}{(be-af)^2}+$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 406

$$\frac{b(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\left(e(5bc-3ad)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+(-3adf+4bcf+bde)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)+\frac{1}{3}bx\sqrt{c+\right.}{b^2}\right.}{(be-af)^2}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 320

3.83. $\displaystyle\int\frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}dx$

$$\frac{b(bc-ad)\left(\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}bx\sqrt{c+dx^2}\sqrt{e+fx^2}\right)}{b^2}\right.}{(be-af)^2}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$$\Big\downarrow \; 388$$

$$\frac{b(bc-ad)\left(\frac{d\left(\frac{1}{3}\left((-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)+\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)+\frac{1}{3}bx\sqrt{c+dx}\right.}{b^2}\right.}{(be-af)^2}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$$\Big\downarrow \; 313$$

3.83. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

$$\frac{b(bc-ad)\left(\frac{(bc-ad)^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{b^2}+\frac{d\left(\frac{1}{3}\left(\frac{e^{3/2}\sqrt{c+dx^2}(5bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+(-3adf+4bcf+bde)\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\right.\right.\right.}{b^2}\right.}{(be-af)^2}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(be-af)(de-cf)}{e\sqrt{e+fx^2}}-\frac{\frac{e^{3/2}\sqrt{c+dx^2}\left(2adf(2de-3cf)-b\left(-3c^2f^2-2cdef+3d^2e^2\right)\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-d\left(be\left(-c^2f^2-7cdef+6d^2e^2\right)\right.}{3f}}{(be-af)^2}$$

$\downarrow$ 414

$$\frac{\frac{(be-af)(de-cf)x(dx^2+c)^{3/2}}{e\sqrt{fx^2+e}}-\frac{\frac{e^{3/2}\left(2adf(2de-3cf)-b\left(3d^2e^2-2cdfe-3c^2f^2\right)\right)\sqrt{dx^2+c}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-d\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-a\right.}{3f}}{(be-af)^2}$$

$$\frac{b(bc-ad)\left(\frac{e^{3/2}\sqrt{dx^2+c}\operatorname{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)(bc-ad)^2}{ab^2c\sqrt{f}\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{d\left(\frac{1}{3}b\sqrt{dx^2+c}\sqrt{fx^2+e}x+\frac{1}{3}\left(\frac{(5bc-3ad)\sqrt{dx^2+c}\operatorname{EllipticF}\left(\arctan\right.\right.}{c\sqrt{f}\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{f}}\right.\right.}{}\right.}{(be-af)^2}$$

input
```
Int[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

3.83. $\int\frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

output
```
(((b*e - a*f)*(d*e - c*f)*x*(c + d*x^2)^(3/2))/(e*Sqrt[e + f*x^2]) - (-1/3
*(d*(a*f*(4*d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])/f + (-(d*(b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2*e^2
- 13*c*d*e*f + 3*c^2*f^2))*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqr
t[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))) +
(e^(3/2)*(2*a*d*f*(2*d*e - 3*c*f) - b*(3*d^2*e^2 - 2*c*d*e*f - 3*c^2*f^2))
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))/(3*f))/e)
/(b*e - a*f)^2 + (b*(b*c - a*d)*((d*((b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
/3 + ((b*d*e + 4*b*c*f - 3*a*d*f)*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2])
 - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
])) + ((5*b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]))/3))/b^2 + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*El
lipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
a*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(
b*e - a*f)^2
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.83. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.83. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

3.83.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.08

method	result	size
default	Expression too large to display	1063
elliptic	Expression too large to display	1255

input
```
int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*b^2*c^2*d*f^3*x^3-2*(-d/c)^(1/2)*a*b^2*c*d^2*e*f^2*x^3+(-d
/c)^(1/2)*a*b^2*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*e*f^2+3*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*
e*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*b*d^3*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f+2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*
d^3*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a^2*b*d^3*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*e*f^2+2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*
c*d^2*e^2*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*e*f
^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*d^2*e*f^2+3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1
/2))*a*b^2*c^2*d*e*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*e*f^2+(-d/c)^...
```

3.83.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.83.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(5/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

3.83.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.83.8 Giac [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.84 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

3.84.1 Optimal result

Integrand size = 32, antiderivative size = 223

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{(de-cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\operatorname{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
(-c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/
2)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b
*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*
e)/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{ab\sqrt{\frac{d}{c}}f(de-cf)x(c+dx^2) - iabde(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\text{arcsin}}{}$$

input
```
Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

output
```
(a*b*Sqrt[d/c]*f*(d*e - c*f)*x*(c + d*x^2) - I*a*b*d*e*(-(d*e) + c*f)*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - I*a*d^2*e*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)^2*e*f*Sqrt[1
 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)])/(a*b*Sqrt[d/c]*e*f*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])
```

3.84.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {417, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{417}$$

$$\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{be-af} + \frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{be-af}$$

$$\Big\downarrow \text{313}$$

$$\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{be-af} + \frac{\sqrt{c+dx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$\Big\downarrow \text{414}$$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{c+dx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1
- (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e
*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

3.84.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(273) = 546$.

Time = 4.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.66

method	result
default	$\dfrac{\Bigg(\sqrt{-\frac{d}{c}}\,abcd\,f^{2}x^{3}-\sqrt{-\frac{d}{c}}\,ab\,d^{2}ef\,x^{3}+\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{f\,x^{2}+e}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\Big)a^{2}d^{2}ef-\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{f\,x^{2}+e}{e}}\,F\Big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\Big)ab\,d^{2}e^{2}-}{}$
elliptic	$\dfrac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\,\Bigg(\frac{\left(df\,x^{2}+cf\right)(cf-de)x}{f(af-be)e\sqrt{\left(x^{2}+\frac{e}{f}\right)\left(df\,x^{2}+cf\right)}}+\frac{\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)d^{2}}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}\,bf}-\frac{d\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,E\Big(x\sqrt{-\frac{d}{c}}}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x}}}{}$

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*b*c*d*f^2*x^3-(-d/c)^(1/2)*a*b*d^2*e*f*x^3+((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2
*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*b*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e^2-((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*a^2*d^2*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*e*f-((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*e*f+(-d/c)^(1/2)*a*b*c^2*f^2*x-(-d/c)^(1/2
)*a*b*c*d*e*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/a/(-d/c)^(1/2)/e/f/(a*f
-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.84.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

3.84.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.84.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.85 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

3.85.1 Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{bc^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
-(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*
x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/2)
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b*c^(3/2)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-
b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c
)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

3.85.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}fx(c+dx^2) - iade\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right) - i(bc}{a\sqrt{\frac{d}{c}}e(be-af)\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

output
```
(-(a*Sqrt[d/c]*f*x*(c + d*x^2)) - I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)*e*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt
[e + f*x^2])
```

3.85.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 416$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{be-af} - \frac{f\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{be-af}$$

$$\Big\downarrow \; 313$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{be-af} - \frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$\Big\downarrow \; 414$$

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

output
```
-((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c
 + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

3.85.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

3.85.4 Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.36

method	result
default	$\frac{\Big(\sqrt{-\frac{d}{c}}\, adf\, x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\Big(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\Big) ade+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\Big(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\Big) ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\Big(x}{ea\sqrt{-\frac{d}{c}}\,(af-be)(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\,\Bigg(\frac{\left(df\,x^2+cf\right)x}{e(af-be)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}-\frac{d\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\Big(x\sqrt{-\frac{d}{c}},\frac{bc}{ad}}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de}}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d*e-((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*c*f*x)*(f*x^2+e)^(1/2)*(d*x^2
+c)^(1/2)/e/a/(-d/c)^(1/2)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.85.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\left(e+fx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.85.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

3.85.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.85.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.86 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

3.86.1 Optimal result

Integrand size = 32, antiderivative size = 344

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx = \frac{f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2
)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*
e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b^2*e^(3/2)*(1/
(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2
/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*f+b*e)^2/f^
(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d*f-b*c*f+2*b*d*
e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+
f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*f+
b*e)^2/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.86.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}f^2x(c+dx^2) - iadef\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right.\right.}{a\sqrt{\frac{d}{c}}e(-be+af}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(-(a*Sqrt[d/c]*f^2*x*(c + d*x^2)) - I*a*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e)
 + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(-(b*e) + a*f)*(d*e - c
*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.86.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {421, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2} - \frac{f\int \frac{bfx^2+2be-af}{\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2} - \frac{f\left(\frac{(-adf-bcf+2bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf} - \frac{f(be-af)\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{de-cf}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\left(\frac{(-adf-bcf+2bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{b^2\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\left(\frac{\sqrt{e}\sqrt{c+dx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{ 414}$$

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{f\left(\frac{\sqrt{e}\sqrt{c+dx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
-((f*(-((Sqrt[f]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt
[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(
b*e - a*f)^2) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*S
qrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

3.86. $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

3.86.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.86. $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

3.86.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a d f^2 x^3-\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\, E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right) a d e f-\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) bcef+\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{f x^2+e}{e}}\,\Pi}{ae(af-be)\sqrt{-\frac{d}{c}}\,(cf-de)(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{\left(df\,x^2+cf\right) f x}{e(cf-de)(af-be)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right) f}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,e(af-be)}-\frac{\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,F\left(x\sqrt{}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x}}\right.}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a*d*f^2*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e
*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x)*(f*x^2+e)^(1
/2)*(d*x^2+c)^(1/2)/a/e/(a*f-b*e)/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)
```

3.86.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.86.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

3.86.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.86.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

3.87.1 Optimal result

Integrand size = 32, antiderivative size = 539

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{f}(2bc^2f-ad(de+cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d^2\sqrt{e}(bde-3bcf+2adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^3c^{3/2}\sqrt{e+fx^2}\,\mathrm{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
-d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-d^2*(2*a*d*
f-3*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/
2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/
(-a*d+b*c)^2/(-c*f+d*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e
)^(1/2)-b^2*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b
*c)^2/(-a*f+b*e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-d
*(2*b*c^2*f-a*d*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c
)^(1/2)/c/(-a*d+b*c)^2/(-c*f+d*e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)/(f*x^2+e)^(1/2)+b^3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elli
pticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f
*x^2+e)^(1/2)/a/(-a*d+b*c)^2/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^
2+e)/e/(d*x^2+c))^(1/2)
```

3.87.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.13 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{\frac{adx\left(-adf\left(c^2f^2+cdf^2x^2+d^2e(e+fx^2)\right)+b\left(c^3f^3+c^2df^3x^2+d^3e^2(e+fx^2)\right)\right)}{c}+iad\sqrt{}}{}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
((a*d*x*(-(a*d*f*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2))) + b*(c^3*f^3
 + c^2*d*f^3*x^2 + d^3*e^2*(e + f*x^2))))/c + I*a*d*Sqrt[d/c]*e*(-(a*d*f*(
d*e + c*f)) + b*(d^2*e^2 + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*c*d*(d/c)^(3/2)*e*
(b*e - a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipti
cF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*Sqrt[d/c]*e*(d*e - c*f)^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*d*(-(b*c) + a*d)*e*(b*e - a*f)*(d*e - c*f
)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.87.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {421, 402, 25, 400, 313, 320, 416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^{3/2}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{\int -\frac{d(bc-ad)fx^2+c(bde-2bcf+adf)}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{d(bc-ad)fx^2+c(bde-2bcf+adf)}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{f\left(2bc^2f-ad(cf+de)\right)\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{de-cf}+\frac{cd(2adf-3bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{cd(2adf-3bcf+bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}+\frac{\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(cf+de)\right)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$\big\downarrow$ 320

$$\frac{b^2\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(cf+de)\right)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$\big\downarrow$ 416

$$\frac{b^2\left(\frac{b\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\sqrt{fx^2+e}}dx}{be-af}-\frac{f\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{be-af}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(cf+de)\right)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$\big\downarrow$ 313

3.87. $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

$$\frac{b^2\left(\frac{b\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}dx}{be-af}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{f}\sqrt{c+dx^2}(2bc^2f-ad(cf+de))E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-\ldots}\right)}{(bc-ad)^2}$$

$\downarrow$ 414

$$\frac{b^2\left(\frac{bc^{3/2}\sqrt{e+fx^2}\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\sqrt{f}\sqrt{c+dx^2}(2bc^2f-ad(cf+de))E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}}{c(de-cf)}+\frac{dx(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-\ldots}\right)}{(bc-ad)^2}$$

input

```
Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output

```
-((d*((d*(b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) +
((Sqrt[f]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*(d*e - c*f)*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*(b*d*e - 3*b*c*f + 2
*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]))/(c*(d*e - c*f))))/(b*c - a*d)^2) + (b^2*(-((Sqrt[f]*Sqrt[c + d*x
^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*(b*e
 - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (b*c^(3/
2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e +
f*x^2))/(e*(c + d*x^2))])))/(b*c - a*d)^2
```

3.87. $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.87.4 Maple [A] (verified)

Time = 6.60 (sec) , antiderivative size = 956, normalized size of antiderivative = 1.77

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a^{2} c\, d^{2} f^{3} x^{3}+\sqrt{-\frac{d}{c}}\, a^{2} d^{3} e\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, c^{2} d\, f^{3} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, d^{3} e^{2} f\, x^{3}-\sqrt{\frac{d\,x^{2}+c}{c}}\, \sqrt{\frac{f\,x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a^{2} c\, d^{2} e\, f^{2}+\sqrt{\right.}{}}{}$
elliptic	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d/c)^(1/2)*a^2*c*d^2*f^3*x^3+(-d/c)^(1/2)*a^2*d^3*e*f^2*x^3-(-d/c)^(1/2
)*a*b*c^2*d*f^3*x^3-(-d/c)^(1/2)*a*b*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2
+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^3*e^3-((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^2*c*d^2*e*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e*f^2+((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b
*d^3*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2)
,b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e*f^2-2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1
/2))*b^2*c^2*d*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*d^2*e^3+(-d/c)^(1/2)
*a^2*c^2*d*f^3*x+(-d/c)^(1/2)*a^2*d^3*e^2*f*x-(-d/c)^(1/2)*a*b*c^3*f^3*x-(
-d/c)^(1/2)*a*b*d^3*e^3*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/e/(c*f-d*e)^2
/a/(-d/c)^(1/2)/(a*d-b*c)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

3.87.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.87.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

3.87.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.87.8 Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.88 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

3.88.1 Optimal result

Integrand size = 32, antiderivative size = 814

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^2f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{f}(bc(5d^2e^2-7cdef-6c^2f^2)-ad(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{b^2\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^4e^{3/2}\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)-1/3*d^2
*(b*c*(-9*c*f+5*d*e)-2*a*d*(-3*c*f+d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(
d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+b^2*f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/
e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*
(d*x^2+c)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/e^(1/2)/(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b^4*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^
2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e
/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*d+b*c)^2/(-a*f+b*e)^2/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*(b*c*(-6*c^2*f^2-7*c*d*e*f+
5*d^2*e^2)-a*d*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(
1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/e^(1/2)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-b^2*(-a*d*f-b*c*f+2*b*d*e)*(1/(1
+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)
^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)^2/(
-a*f+b*e)^2/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3
*d^2*(b*c*(-15*c*f+7*d*e)-a*d*(-9*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2
/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))
*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

3.88.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.59 (sec) , antiderivative size = 1645, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
((-I)*a*d*e*(2*a*b*d*(d*e - 3*c*f)*(d*e + c*f)^2 + a^2*d^2*f*(-2*d^2*e^2 +
 7*c*d*e*f + 3*c^2*f^2) + b^2*c*(-5*d^3*e^3 + 10*c*d^2*e^2*f + 3*c^3*f^3))
*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)] + (Sqrt[d/c]*(6*a*b^2*c^2*d^5*e^4*x - 3*a^2*b*c*
d^6*e^4*x - 11*a*b^2*c^3*d^4*e^3*f*x + 2*a^2*b*c^2*d^5*e^3*f*x + 3*a^3*c*d
^6*e^3*f*x + 11*a^2*b*c^3*d^4*e^2*f^2*x - 8*a^3*c^2*d^5*e^2*f^2*x - 3*a*b^
2*c^6*d*f^4*x + 6*a^2*b*c^5*d^2*f^4*x - 3*a^3*c^4*d^3*f^4*x + 5*a*b^2*c*d^
6*e^4*x^3 - 2*a^2*b*d^7*e^4*x^3 - 4*a*b^2*c^2*d^5*e^3*f*x^3 - a^2*b*c*d^6*
e^3*f*x^3 + 2*a^3*d^7*e^3*f*x^3 - 11*a*b^2*c^3*d^4*e^2*f^2*x^3 + 12*a^2*b*
c^2*d^5*e^2*f^2*x^3 - 4*a^3*c*d^6*e^2*f^2*x^3 + 11*a^2*b*c^3*d^4*e*f^3*x^3
 - 8*a^3*c^2*d^5*e*f^3*x^3 - 6*a*b^2*c^5*d^2*f^4*x^3 + 12*a^2*b*c^4*d^3*f^
4*x^3 - 6*a^3*c^3*d^4*f^4*x^3 + 5*a*b^2*c*d^6*e^3*f*x^5 - 2*a^2*b*d^7*e^3*
f*x^5 - 10*a*b^2*c^2*d^5*e^2*f^2*x^5 + 2*a^2*b*c*d^6*e^2*f^2*x^5 + 2*a^3*d
^7*e^2*f^2*x^5 + 10*a^2*b*c^2*d^5*e*f^3*x^5 - 7*a^3*c*d^6*e*f^3*x^5 - 3*a*
b^2*c^4*d^3*f^4*x^5 + 6*a^2*b*c^3*d^4*f^4*x^5 - 3*a^3*c^2*d^5*f^4*x^5 - I*
a*c*d^2*Sqrt[d/c]*e*(b*e - a*f)*(-(d*e) + c*f)*(2*a*d*(d*e - 3*c*f) + b*c*
(-5*d*e + 9*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^4*d^3*Sqrt[d/c]*e
^4*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - (9*I)*b^3*c^7*(d/c)^(5/2)*e^3*f*Sqrt[1 ...
```

3.88.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 791, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {421, 402, 25, 402, 27, 400, 313, 320, 421, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^{5/2}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{3c(c+dx^2)^{3/2}\sqrt{e+fx^2}(de-cf)}-\frac{\int-\frac{3d(bc-ad)fx^2+bc(5de-6cf)-ad(2de-3cf)}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^{3/2}}dx}{3c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int\frac{3d(bc-ad)fx^2+bc(5de-6cf)-ad(2de-3cf)}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^{3/2}}dx}{3c(de-cf)}+\frac{dx(bc-ad)}{3c(c+dx^2)^{3/2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{\int\frac{f\left(c(2bc(de-3cf)+ad(de+3cf))-d(bc(5de-9cf)-2ad(de-3cf))x^2\right)}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}}{3c(de-cf)}+\frac{dx(bc-ad)}{3c(c+dx^2)^{3/2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\int\frac{c(2bc(de-3cf)+ad(de+3cf))-d(bc(5de-9cf)-2ad(de-3cf))x^2}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{c(de-cf)}}{3c(de-cf)}+\frac{dx(bc-ad)}{3c(c+dx^2)^{3/2}\sqrt{e+fx^2}(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{400}$$

3.88. $\int\frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{cd(bc(7de-15cf)-ad(de-9cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)\int}{de-cf}\right.}{c(de-cf)}}{3c(de-cf)}\right.}{(bc-ad)^2}$$

$\downarrow$ 313

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{cd(bc(7de-15cf)-ad(de-9cf))\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\left(-3c^2f^2-7cdef+2d^2\right.\right.}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx\right.}{c\left(e+fx\right.}}}\right.}{c(de-cf)}}{3c(de-cf)}\right.}{(bc-ad)^2}$$

$\downarrow$ 320

$$\frac{b^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{3/2}}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f}}\right.}{c(de-cf)}}{3c(de-cf)}\right.}{(bc-ad)^2}$$

$\downarrow$ 421

3.88. $\displaystyle\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

$$\frac{b^2\left(\frac{b^2\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\int \frac{bfx^2+2be-af}{\sqrt{dx^2+c}\left(fx^2+e\right)^{3/2}}dx}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\cdots\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f\cdots}}\cdots\right.}{c(de-cf)}\cdots}{3c(de-cf)}\cdots\right.}{(bc-ad)^2}\cdots$$

$\big\downarrow$ 400

$$\frac{b^2\left(\frac{b^2\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\left(\frac{(-adf-bcf+2bde)\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{f(be-af)\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{de-cf}\right)}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\cdots\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f\cdots}}\cdots\right.}{c(de-cf)}\cdots}{3c(de-cf)}\cdots\right.}{(bc-ad)^2}\cdots$$

$\big\downarrow$ 313

3.88. $\displaystyle\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

$$\frac{b^2\left(\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\left(\frac{(-adf-bcf+2bde)\int\frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{de-cf}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f}}\right)}{c(de-cf)}}{3c(de-cf)}\right)}{(bc-ad)^2}$$

$\downarrow$ 320

$$\frac{b^2\left(\frac{b^2\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}dx}{(be-af)^2}-\frac{f\left(\frac{\sqrt{e}\sqrt{c+dx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f}}\right)}{c(de-cf)}}{3c(de-cf)}\right)}{(bc-ad)^2}$$

$\downarrow$ 414

3.88. $\displaystyle\int\frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

$$b^2\left(\frac{\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{f\left(\frac{\sqrt{e}\sqrt{c+dx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(be-af)}{\sqrt{e}\sqrt{e+fx^2}(\ldots}\ldots\right)}{(be-af)^2}\ldots}{(bc-ad)^2}\right.$$

$$d\left(\frac{\frac{dx(bc(5de-9cf)-2ad(de-3cf))}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{f\left(\frac{d\sqrt{e}\sqrt{c+dx^2}(bc(7de-15cf)-ad(de-9cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{c+dx^2}\left(bc\left(-6c^2f^2-7cdef+5d^2e^2\right)-ad\ldots\right.}{\sqrt{e}\sqrt{f}\sqrt{e+f\ldots}}\right)}{c(de-cf)}}{3c(de-cf)}\ldots}{(bc-ad)^2}\right.$$

input

```
Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output

```
-((d*((d*(b*c - a*d)*x)/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]
) + ((d*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(c*(d*e - c*f)*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2]) - (f*(-(((b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*
f^2) - a*d*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f
)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (d*Sqrt[e]*(b*
c*(7*d*e - 15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(c*(d*e - c*f)))/(3*c*(d*e - c*
f))))/(b*c - a*d)^2) + (b^2*(-((f*(-((Sqrt[f]*(b*e - a*f)*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*(d*e - c
*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (Sqrt[e]*(2*
b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])))/(b*e - a*f)^2) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*
EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])
/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2])))/(b*c - a*d)^2
```

3.88. $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. 2(923) = 1846.

Time = 7.73 (sec) , antiderivative size = 2127, normalized size of antiderivative = 2.61

method	result	size
elliptic	Expression too large to display	2127
default	Expression too large to display	4115

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*d/c/(c*f-
d*e)*x/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/
2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)*d^2/c^2/(c*f-d*e)^2*x*(7*a*c*d*f-2*a*d^2*
e-10*b*c^2*f+5*b*c*d*e)/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/((x^2+
c/d)*(d*f*x^2+d*e))^(1/2)+(d*f*x^2+c*f)*f^3/e/(c*f-d*e)^3*x/(a*f-b*e)/((x^
2+e/f)*(d*f*x^2+c*f))^(1/2)-7/3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*f*d^4/(c*f-d*e)^2/c/(a*d-b*c)
/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*a*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e
)/e/d)^(1/2))+2/3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*d^5/(c*f-d*e)^2/c^2/(a*d-b*c)/(a*c*d*f-a*
d^2*e-b*c^2*f+b*c*d*e)*a*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)
)-7/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*
e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^3/(c
*f-d*e)/c/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*a*f+2/3/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^4/(c*f-d*e)/c^2/(a*d-b
*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*a*e-5/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/
2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^3/(c*f-d*e)/c/(a*d-b*c)/(a*c*d*f-a*d^2
*e-b*c^2*f+b*c*d*e)*b*e+10/3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/...
```

3.88.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.88.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

3.88.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.88.8 Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.89 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

3.89.1 Optimal result

Integrand size = 28, antiderivative size = 242

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}} + \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(3a-7b)\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{(a-2b)(a-b)\sqrt{2+x^2}\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

output
```
-(a-2*b)*x*(x^2+2)^(1/2)/b^2/(x^2+1)^(1/2)+1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/
2)/b-1/6*(3*a-7*b)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*(x^2+2)^(1/2)/b^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+1/2*(a-2*b)*(a-b)*(1/(
x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+2)^(1/2)/
a/b^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+(a-2*b)*(1/(x^2+1))^(1/2)*EllipticE(
x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b^2/((x^2+2)/(x^2+1))^(
1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{ab^2x\sqrt{1+x^2}\sqrt{2+x^2}+3ia(a-2b)bE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-ia(3a^2-9ab+7b}{}$$

input
```
Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
(a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*EllipticE[I*Arc
Sinh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*EllipticF[I*ArcSinh[x/Sq
rt[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (12*
I)*a^2*b*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (15*I)*a*b^2*Ellip
ticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*
ArcSinh[x/Sqrt[2]], 2])/(3*a*b^3)
```

3.89.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {418, 25, 403, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^{3/2}\sqrt{x^2+2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{418}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}+\frac{\int -\frac{\sqrt{x^2+2}(-bx^2+a-2b)}{\sqrt{x^2+1}}dx}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\int \frac{\sqrt{x^2+2}(-bx^2+a-2b)}{\sqrt{x^2+1}}dx}{b^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\int \frac{3(a-2b)x^2+2(3a-5b)}{\sqrt{x^2+1}\sqrt{x^2+2}}dx-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \text{406}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(2(3a-5b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+3(a-2b)\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(3(a-2b)\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \text{388}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx\right)+\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{2\sqrt{x^2+1}(a-b)^2\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{ab^2\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}-\frac{\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

3.89. $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

input
```
Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
-((-1/3*(b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]) + (3*(a - 2*b)*((x*Sqrt[2 + x^2]
)/Sqrt[1 + x^2] - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/(Sqrt[
1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])) + (Sqrt[2]*(3*a - 5*b)*Sqrt[2 + x^2]*
EllipticF[ArcTan[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/3)/b
^2) + (2*(a - b)^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]]
, -1])/(a*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

3.89.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\sqrt{x^2+1}\,\sqrt{x^2+2}}{3b}-\frac{\left(\frac{3i(a-2b)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2b\sqrt{x^4+3x^2+2}}+\frac{i\left(3a^2-12ab+13b^2\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b^2\sqrt{x^4+3x^2+2}} \cdots\right.}{3b\sqrt{x^2+1}\,\sqrt{x^2+2}}$
default	$-\frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(-a\,b^2x^5+3i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2b+7i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2} \cdots\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\,\left(\frac{x\sqrt{x^4+3x^2+2}}{3b}-\frac{7i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{6b\sqrt{x^4+3x^2+2}}+\frac{ia^2\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^3\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi \cdots}{a\sqrt{x^4+3x^2 \cdots}} \cdots\right.}{\cdots}$

input
```
int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/2)/b-1/3/b*(3/2*I*(a-2*b)/b*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1
/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))+1/2*I*(3*a^2-12*a*b+13*b^2)/b^2*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x
*2^(1/2),2^(1/2))-I*(3*a^3-12*a^2*b+15*a*b^2-6*b^3)/b^2/a*2^(1/2)*(1+1/2*x
^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b
/a,2^(1/2)))*((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)
```

3.89.5 Fricas [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

3.89.6 Sympy [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{(x^2+1)^{\frac{3}{2}}\sqrt{x^2+2}}{a+bx^2}\,dx$$

input
```
integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((x**2 + 1)**(3/2)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

3.89.7 Maxima [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

3.89.8 Giac [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{(x^2+1)^{3/2}\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

input
```
int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.90 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

3.90.1 Optimal result

Integrand size = 28, antiderivative size = 192

$$\begin{aligned}\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx &= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} \\ &\quad + \frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} \\ &\quad - \frac{(a-2b)\sqrt{2+x^2}\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}\end{aligned}$$

output
```
x*(x^2+2)^(1/2)/b/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*(x^2+2)^(1/2)/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-1/2*(a-
2*b)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+
2)^(1/2)/a/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-(1/(x^2+1))^(1/2)*EllipticE(x
/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b/((x^2+2)/(x^2+1))^(1/2
)
```

3.90.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{i\left(-abE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(a-2b)\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+(-a+b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\right.\right.\right.}{ab^2}$$

input
```
Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
(I*(-(a*b*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]) + (a - 2*b)*(a*EllipticF[I*A
rcSinh[x/Sqrt[2]], 2] + (-a + b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]],
 2])))/(a*b^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {409, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{409}$$

$$\frac{\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}dx}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{324}$$

$$\frac{\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{320}$$

$$\frac{\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{-\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx+\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{\sqrt{x^2+2}(a-2b)\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

input
```
Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
((x*Sqrt[2 + x^2])/Sqrt[1 + x^2] - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan
[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*Elli
pticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/
b - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

3.90.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 409
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f/e, 0] &&  !Sim
plerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.90.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

method	result
default	$\frac{i\left(F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2-2F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba-a^2\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+3\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ba-2\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b^2-E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba\right)}{a\,b^2}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{b\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a}{2\sqrt{x^4+3x^2+2}\,b^2}-\frac{ia\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^2\sqrt{x^4+3x^2+2}}-\frac{2i \cdots}{\cdots}\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

input
```
int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
I*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2-2*EllipticF(1/2*I*x*2^(1/2),2^(1
/2))*b*a-a^2*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))+3*EllipticPi(1/2*I*
x*2^(1/2),2*b/a,2^(1/2))*b*a-2*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b
^2-EllipticE(1/2*I*x*2^(1/2),2^(1/2))*b*a)/a/b^2
```

3.90.5 Fricas [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

3.90.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

input
```
integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

3.90.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

3.90.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

input
```
int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.91 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx$

3.91.1 Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\frac{2\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

output
```
2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/2)
,1-2*b/a,I)*(x^2+1)^(1/2)/a/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx =-\frac{i\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)-(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{ab}$$

input
```
Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

output
```
((-I)*(a*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (a - 2*b)*EllipticPi[(2*b)/a
, I*ArcSinh[x/Sqrt[2]], 2]))/(a*b)
```

3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 414$$

$$\frac{2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

input
```
Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

output
```
(2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(
1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

3.91.3.1 Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.91.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result
default	$-\frac{i\left(aF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-a\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+2b\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)\right)}{ab}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{a\sqrt{x^4+3x^2+2}}\right)}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-I*(a*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-a*EllipticPi(1/2*I*x*2^(1/2),2*b/
a,2^(1/2))+2*b*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))/a/b
```

3.91.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^4 + (a + b)*x^2 + a), x)
```

3.91.6 Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(a+bx^2)\,\sqrt{x^2+1}}\,dx$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

3.91.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

3.91.8 Giac [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,(b\,x^2+a)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)), x)
```

3.92 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx$

3.92.1 Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \frac{\sqrt{2}\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

output
```
-2*b*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1
/2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)
+(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^
(1/2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \frac{\frac{x}{\sqrt{\frac{1+x^2}{2+x^2}}} + iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right) - \frac{i(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{a}}{a-b}$$

input
```
Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

output
```
(x/Sqrt[(1 + x^2)/(2 + x^2)] + I*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (I*(
a - 2*b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/a)/(a - b)
```

3.92.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}(a+bx^2)}dx$$

$$\Big\downarrow \ 416$$

$$\frac{\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx}{a-b}-\frac{b\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{a-b}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{b\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{a-b}$$

$$\Big\downarrow \ 414$$

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

input
```
Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

output
```
(Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*S
qrt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, Arc
Tan[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

3.92.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

3.92.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

method	result
default	$\frac{\left(iE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a\sqrt{x^2+2}\,\sqrt{x^2+1}-i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)a\sqrt{x^2+2}\,\sqrt{x^2+1}+2i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b\sqrt{x^2+2}\,\sqrt{x^2+1}+a\,x^3+2ax\right)\sqrt{x^2+1}\,\sqrt{x^2+}}{a(x^4+3x^2+2)(a-b)}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{(x^2+2)x}{(a-b)\sqrt{(x^2+1)(x^2+2)}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{(a-b)\sqrt{x^4+3x^2+2}}+\frac{2ib\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x}}{(a-b)a\sqrt{}}\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+2)^(1/2)*(x^2+1)^(1/2)-I*Elli
pticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+2)^(1/2)*(x^2+1)^(1/2)+2*I*El
lipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)+a*x^
3+2*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/a/(x^4+3*x^2+2)/(a-b)
```

3.92.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^
2 + a), x)
```

3.92.6 Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(a+bx^2)\,(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 2)/((a + b*x**2)*(x**2 + 1)**(3/2)), x)
```

3.92.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

3.92.8 Giac [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}\,(b\,x^2+a)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)), x)
```

3.93 $\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)}\,dx$

3.93.1 Optimal result

Integrand size = 28, antiderivative size = 215

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)}\,dx = \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\sqrt{2}\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{2b^2\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

output
```
2*b^2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(
1/2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)^2/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1
/2)+1/3*x*(x^2+2)^(1/2)/(a-b)/(x^2+1)^(3/2)+(a-2*b)*(1/(x^2+1))^(1/2)*Elli
pticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/(a-b)^2/((x^2+2)/
(x^2+1))^(1/2)-1/3*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*(x^2+2)^(1/2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \frac{4a^2x\sqrt{1+x^2}\sqrt{2+x^2} - 7abx\sqrt{1+x^2}\sqrt{2+x^2} + 3a^2x^3\sqrt{1+x^2}\sqrt{2+x^2} - 6ab}{}$$

input

```
Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

output

```
(4*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 7*a*b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]
 + 3*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 6*a*b*x^3*Sqrt[1 + x^2]*Sqrt[2
+ x^2] + (3*I)*a*(a - 2*b)*(1 + x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]
- I*a*(a - b)*(1 + x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3*I)*a*b*E
llipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2*EllipticPi[(2*b)/a
, I*ArcSinh[x/Sqrt[2]], 2] + (6*I)*a*b*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x
/Sqrt[2]], 2] - (12*I)*b^2*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2
] + (3*I)*a*b*x^4*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2
*x^4*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/(3*a*(a - b)^2*(1 + x^2
)^2)
```

3.93.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {421, 25, 401, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{(x^2+1)^{5/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 421$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2} - \frac{\int -\frac{\sqrt{x^2+2}(-bx^2+a-2b)}{(x^2+1)^{5/2}}dx}{(a-b)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\int \frac{\sqrt{x^2+2}(-bx^2+a-2b)}{(x^2+1)^{5/2}}dx}{(a-b)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{x\sqrt{x^2+2}(a-b)}{3(x^2+1)^{3/2}}-\frac{1}{3}\int -\frac{(a-4b)x^2+2(2a-5b)}{(x^2+1)^{3/2}\sqrt{x^2+2}}dx}{(a-b)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\int \frac{(a-4b)x^2+2(2a-5b)}{(x^2+1)^{3/2}\sqrt{x^2+2}}dx+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\left(3(a-2b)\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx-2(a-b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-2(a-b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}+\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}(a-b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{2b^2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}(a-b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

input
```
Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

3.93. $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx$

output
```
(((a - b)*x*Sqrt[2 + x^2])/(3*(1 + x^2)^(3/2)) + ((3*Sqrt[2]*(a - 2*b)*Sqr
t[2 + x^2]*EllipticE[ArcTan[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x
^2)]) - (Sqrt[2]*(a - b)*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[1
+ x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/3)/(a - b)^2 + (2*b^2*Sqrt[1 + x^2]*Ell
ipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(
2 + x^2)]*Sqrt[2 + x^2])
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

3.93.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{x\sqrt{x^4+3x^2+2}}{3(a-b)\left(x^2+1\right)^2}+\frac{\left(x^2+2\right)x(a-2b)}{(a-b)^2\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}\,(a-b)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,aE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)^2\sqrt{x^4+3x^2+2}}-\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$
default	$-\frac{iF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2\sqrt{x^2+2}\,\sqrt{x^2+1}-3iE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2\sqrt{x^2+2}\,\sqrt{x^2+1}+6i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b^2\sqrt{x^2+2}\,\sqrt{x^2+1}-3i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ab}{}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)*(1/3*x/(a-b)*(x^4+3*x^
2+2)^(1/2)/(x^2+1)^2+(x^2+2)*x*(a-2*b)/(a-b)^2/((x^2+1)*(x^2+2))^(1/2)-1/6
*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2
*I*x*2^(1/2),2^(1/2))/(a-b)+1/2*I/(a-b)^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*a*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-I/(a-b)^2*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*b*EllipticE(1/2*I
*x*2^(1/2),2^(1/2))+I/(a-b)^2*b*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x
^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))-2*I/(a-b)^2/a*
b^2*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))
```

3.93.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^8 + (a + 3*b)*x^6 + 3*(a + b)*x^
4 + (3*a + b)*x^2 + a), x)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

output
```
Timed out
```

3.93.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

3.93.8 Giac [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(x^2+1)^{5/2}\,(b\,x^2+a)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)), x)
```

3.94 $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$

3.94.1 Optimal result

Integrand size = 32, antiderivative size = 298

$$\begin{aligned}\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx &= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}}-\frac{\sqrt{2}\sqrt{f}\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\\ &+\frac{3d\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\\ &+\frac{3(2b-ad)\sqrt{2+dx^2}\,\text{EllipticPi}\left(1-\frac{3b}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\end{aligned}$$

output
```
f*x*(d*x^2+2)^(1/2)/b/(f*x^2+3)^(1/2)+3/2*d*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2
+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2
))*(d*x^2+2)^(1/2)/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)
^(1/2)+3/2*(-a*d+2*b)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticPi(x
*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1-3*b/a/f,1/2*(4-6*d/f)^(1/2))*(d*x^2+2
)^(1/2)/a/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(1
/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2
+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*f^(1/2)*(d*x^2+2)^(1/2)/b/((d*x^2+2
)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$$
$$= \frac{i\left(-3abdE\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-2b+ad)\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(3b-af)\,\text{EllipticPi}\right.\right.}{\sqrt{3}ab^2\sqrt{d}}$$

input
```
Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

output
```
(I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-2*
b + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (3*
b - a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*
d)])))/(Sqrt[3]*a*b^2*Sqrt[d])
```

3.94.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {410, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}\,dx$$
$$\Big\downarrow \text{ 410}$$
$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\int \frac{\sqrt{fx^2+3}}{\sqrt{dx^2+2}}dx}{b}$$
$$\Big\downarrow \text{ 324}$$
$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(3\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+f\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx\right)}{b}$$
$$\Big\downarrow \text{ 320}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(f\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)\right)}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{3\sqrt{dx^2+2}(2b-ad)\,\text{EllipticPi}\left(1-\frac{3b}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{d\left(\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)\right)}{b}$$

input
```
Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

output
```
(d*(f*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(
2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (3*Sqrt[2 + d*x^2]*EllipticF[A
rcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])))/b + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*E
llipticPi[1 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/
(Sqrt[2]*a*b*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

3.94. $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$

3.94.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.94.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\left(F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)a^{2}df-a^{2}\Pi\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)df-3F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)dba-2fE\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right.}{2a\,b^{2}\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^{2}+3)(d\,x^{2}+2)}\left(-\frac{\sqrt{3f\,x^{2}+9}\,\sqrt{2d\,x^{2}+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)a\,d\,f}{2b^{2}\sqrt{-3f}\,\sqrt{d\,f\,x^{4}+3d\,x^{2}+2f\,x^{2}+6}}+\frac{3\sqrt{3f\,x^{2}+9}\,\sqrt{2d\,x^{2}+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\,\sqrt{d\,f\,x^{4}+3d\,x^{2}+2f\,x^{2}+6}}+\frac{f\sqrt{3f\,x}}{}\right.}{}$

input
```
int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*
a^2*d*f-a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(
1/2)*3^(1/2)/(-f)^(1/2))*d*f-3*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1
/2)*3^(1/2)*(d/f)^(1/2))*d*b*a-2*f*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*
2^(1/2)*3^(1/2)*(d/f)^(1/2))*b*a+3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b
/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a+2*EllipticPi(1/3*x*3
^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*f*b*a
-6*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1
/2)/(-f)^(1/2))*b^2)*2^(1/2)/a/b^2/(-f)^(1/2)
```

3.94.5 Fricas [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

3.94.6 Sympy [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

3.94.7 Maxima [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

3.94.8 Giac [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}{b\,x^2+a}\,dx$$

input
```
int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2),x)
```

output
```
int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2), x)
```

3.95 $\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx$

3.95.1 Optimal result

Integrand size = 32, antiderivative size = 93

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \frac{2\sqrt{3+fx^2}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output
```
2/3*(1/(2*d*x^2+4))^(1/2)*(2*d*x^2+4)^(1/2)*EllipticPi(x*d^(1/2)*2^(1/2)/(
2*d*x^2+4)^(1/2),1-2*b/a/d,1/3*(9-6*f/d)^(1/2))*(f*x^2+3)^(1/2)/a*3^(1/2)/
d^(1/2)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = -\frac{i\left(ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(2b-ad)\,\text{EllipticPi}\left(\frac{2b}{ad},i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}ab\sqrt{d}}$$

input
```
Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

output
```
((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (2*b -
 a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
))/(Sqrt[3]*a*b*Sqrt[d])
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}}{\sqrt{fx^2+3}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{414}$$

$$\frac{2\sqrt{fx^2+3}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

input
```
Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

output
```
(2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]]
, 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2
 + d*x^2)])
```

3.95.3.1 Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

3.95.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)ad-\Pi\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)ad+2\Pi\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)b\right)\sqrt{2}}{2ab\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(\frac{\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}-\frac{\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{d\,x^2}{2}}\,\Pi\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{af},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)d}{b\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}+\frac{2\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\cdots}}{a\sqrt{-\frac{f}{3}}\,\sqrt{\cdots}}\right.}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}$

input
```
int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a
*d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1
/2)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1
/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*b)*2^(1/2)/a/b/(-f)^(1/2)
```

3.95.5 Fricas [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*f*x^4 + (a*f + 3*b)*x^2 + 3*a)
, x)
```

3.95.6 Sympy [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(a+bx^2)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

3.95.7 Maxima [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

3.95.8 Giac [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+2}}{(b\,x^2+a)\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)), x)
```

3.96 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

3.96.1 Optimal result

Integrand size = 32, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

output
```
1/3*EllipticPi(1/2*x*(-d)^(1/2)*2^(1/2),2*b/a/d,1/3*6^(1/2)*(f/d)^(1/2))/a
*3^(1/2)/(-d)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3
]*a*Sqrt[-d])
```

3.96.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \ 412$$

$$\frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3
]*a*Sqrt[-d])
```

3.96.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.96.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\Pi\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)\sqrt{2}}{2a\sqrt{-f}}$	53
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\,\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{d\,x^2}{2}}\,\Pi\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{af},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,a\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}$	115

input
```
int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(
1/2)/(-f)^(1/2))*2^(1/2)/a/(-f)^(1/2)
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

3.96.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.96.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.97 $\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx$

3.97.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx=-\frac{\sqrt{1+\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

output
```
-EllipticF(x,(-b/a)^(1/2))*(1+b*x^2/a)^(1/2)/(b*x^2+a)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx=-\frac{\sqrt{\frac{a+bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

3.97.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {281, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x^2-1)\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 281$$

$$-\int \frac{1}{\sqrt{1-x^2}\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \; 323$$

$$-\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{1-x^2}\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 321$$

$$-\frac{\sqrt{\frac{bx^2}{a}+1}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

3.97.3.1 Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.97.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{b\,x^{2}+a}{a}}\, F\left(x, \sqrt{-\frac{b}{a}}\right)}{\sqrt{b\,x^{2}+a}}$	35
elliptic	$-\frac{\sqrt{-\left(x^{2}-1\right)\left(b\,x^{2}+a\right)}\,\sqrt{1+\frac{b\,x^{2}}{a}}\, F\left(x, \sqrt{-1-\frac{-a+b}{a}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{4}-a\,x^{2}+b\,x^{2}+a}}$	77

input
```
int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\,\sqrt{a+bx^2}}\,dx = -\frac{F(\arcsin{(x)}\mid -\frac{b}{a})}{\sqrt{a}}$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-elliptic_f(arcsin(x), -b/a)/sqrt(a)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{F\left(\operatorname{asin}(x)\middle|-\frac{b}{a}\right)}{\sqrt{a}} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-elliptic_f(asin(x), -b/a)/sqrt(a), (x > -1) & (x < 1)))
```

3.97.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

3.97.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(-1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)),x)
```

output
```
-int(1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.98 $\int \frac{a+bx^2}{\sqrt{c+dx^2}(e+fx^2)^2} dx$

3.98.1 Optimal result

Integrand size = 28, antiderivative size = 113

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)} - \frac{(bce-2ade+acf)\text{arctanh}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

output
```
-1/2*(a*c*f-2*a*d*e+b*c*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1
/2))/e^(3/2)/(-c*f+d*e)^(3/2)+1/2*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/(-c*f+d*e
)/(f*x^2+e)
```

3.98.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\frac{\sqrt{e}(be-af)x\sqrt{c+dx^2}}{(de-cf)(e+fx^2)} - \frac{(bce-2ade+acf)\arctan\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}(e+fx^2)}{\sqrt{e}\sqrt{-de+cf}}\right)}{(-de+cf)^{3/2}}}{2e^{3/2}}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((Sqrt[e]*(b*e - a*f)*x*Sqrt[c + d*x^2])/((d*e - c*f)*(e + f*x^2)) - ((b*c
*e - 2*a*d*e + a*c*f)*ArcTan[(-(f*x*Sqrt[c + d*x^2]) + Sqrt[d]*(e + f*x^2)
)/(Sqrt[e]*Sqrt[-(d*e) + c*f])])/(-(d*e) + c*f)^(3/2))/(2*e^(3/2))
```

3.98.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \ 402$$

$$\frac{\int -\frac{bce-2ade+acf}{\sqrt{dx^2+c}(fx^2+e)}dx}{2e(de-cf)}+\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}$$

$$\Big\downarrow \ 25$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\int \frac{bce-2ade+acf}{\sqrt{dx^2+c}(fx^2+e)}dx}{2e(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\int \frac{1}{\sqrt{dx^2+c}(fx^2+e)}dx}{2e(de-cf)}$$

$$\Big\downarrow \ 291$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\int \frac{1}{e-\frac{(de-cf)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2e(de-cf)}$$

$$\Big\downarrow \ 221$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\text{arctanh}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

input
```
Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - ((b*c*e -
2*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(
2*e^(3/2)*(d*e - c*f)^(3/2))
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.98.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{(af-be)\sqrt{d\,x^{2}+c}\,x}{f\,x^{2}+e}-\frac{(acf-2ade+bce)\arctan\left(\frac{e\sqrt{d\,x^{2}+c}}{x\sqrt{(cf-de)e}}\right)}{\sqrt{(cf-de)e}}}{2(cf-de)e}$
default	$\frac{(-af+be)\left(-\frac{f\sqrt{d\left(x+\frac{\sqrt{-ef}}{f}\right)^{2}-\frac{2d\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{cf-de}{f}}}{(cf-de)\left(x+\frac{\sqrt{-ef}}{f}\right)}-\frac{d\sqrt{-ef}\,\ln\left(\frac{\frac{2cf-2de}{f}-\frac{2d\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{cf-de}{f}}\sqrt{d\left(x+\right.}}{x+\frac{\sqrt{-ef}}{f}}\right.}{(cf-de)\sqrt{\frac{cf-de}{f}}}\right.}{4e\,f^{2}}$

input
```
int((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*f-d*e)/e*((a*f-b*e)*(d*x^2+c)^(1/2)*x/(f*x^2+e)-(a*c*f-2*a*d*e+b*c*
e)/((c*f-d*e)*e)^(1/2)*arctan(e*(d*x^2+c)^(1/2)/x/((c*f-d*e)*e)^(1/2)))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.

Time = 1.11 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.54

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,(bde^3+acef^2-(bc+ad)e^2f)\sqrt{dx^2+c}x-(acef+(bc-2\,ad)e^2+(acf^2+(bc-2\,ad)ef)x^2)\sqrt{de^2-}}{8\,(d^2e^5-2\,cde^4f+c^2e^3f^2+(d^2e^4f-2\,c}\right.$$

input
```
integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x - (a*c
*e*f + (b*c - 2*a*d)*e^2 + (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(d*e^2 -
 c*e*f)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^
2 - 3*c^2*e*f)*x^2 + 4*((2*d*e - c*f)*x^3 + c*e*x)*sqrt(d*e^2 - c*e*f)*sqr
t(d*x^2 + c))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e
^3*f^2 + (d^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2), 1/4*(2*(b*d*e^3 +
 a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x + (a*c*e*f + (b*c - 2*a*
d)*e^2 + (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(-d*e^2 + c*e*f)*arctan(1/
2*sqrt(-d*e^2 + c*e*f)*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)/((d^2*e^2
 - c*d*e*f)*x^3 + (c*d*e^2 - c^2*e*f)*x)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e^
3*f^2 + (d^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2)]
```

3.98.6 Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

3.98.7 Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.

Time = 0.94 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.91

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\left(bc\sqrt{d}e - 2\,ad^{\frac{3}{2}}e + ac\sqrt{d}f\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 f+2\,de-cf}{2\,\sqrt{-d^2e^2+cdef}}\right)}{2\,\sqrt{-d^2e^2+cdef}(de^2-cef)} + \frac{2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bd^{\frac{3}{2}}e^2 - \left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc\sqrt{d}ef - 2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 ad^{\frac{3}{2}}ef + \left(\sqrt{d}x - \right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 f + 4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 de - 2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 cf \right.}$$

input
```
integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*c*sqrt(d)*e - 2*a*d^(3/2)*e + a*c*sqrt(d)*f)*arctan(1/2*((sqrt(d)*x
 - sqrt(d*x^2 + c))^2*f + 2*d*e - c*f)/sqrt(-d^2*e^2 + c*d*e*f))/(sqrt(-d^
2*e^2 + c*d*e*f)*(d*e^2 - c*e*f)) + (2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*d
^(3/2)*e^2 - (sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*e*f - 2*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d^(3/2)*e*f + (sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c
*sqrt(d)*f^2 + b*c^2*sqrt(d)*e*f - a*c^2*sqrt(d)*f^2)/(((sqrt(d)*x - sqrt(
d*x^2 + c))^4*f + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*d*e - 2*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*c*f + c^2*f)*(d*e^2*f - c*e*f^2))
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

3.99 $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

3.99.1 Optimal result

Integrand size = 33, antiderivative size = 359

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\sqrt{c}\sqrt{d}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}(b^2ce+a^2df)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

output
```
1/2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*EllipticE(x*d^(1/2)
/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1/2)*(f*x^2+e)^(1/
2)/a/b/(-d*x^2+c)^(1/2)/(1+f*x^2/e)^(1/2)+1/2*(a^2*d*f+b^2*c*e)*EllipticPi
(x*d^(1/2)/c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/a^2/b^2/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/2*(a*f+
b*e)*EllipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a/b^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{cex}{a+bx^2} - \frac{dex^3}{a+bx^2} + \frac{cfx^3}{a+bx^2} - \frac{dfx^5}{a+bx^2} + \frac{ic\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{b} - \frac{ic\sqrt{-\frac{d}{c}}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}{}}{}$$

input
```
Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) - (d*
f*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b - (I*c*Sqrt
[-(d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x],
-((c*f)/(d*e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)
]*x], -((c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

3.99.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {423, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{423}$$

$$-\frac{df\int \frac{a-bx^2}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{(af+be)\int \frac{1}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{f}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{323}$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{\frac{fx^2}{e}+1}(af+be)\int \frac{1}{\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{323}$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{321}$$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{331}$$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{fx^2+e}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{330}$$

3.99. $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\int\frac{\sqrt{\frac{fx^2}{e}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$\big\downarrow$ 327

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$\big\downarrow$ 413

$$\frac{\sqrt{1-\frac{dx^2}{c}}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{fx^2+e}}dx}{2\sqrt{c-dx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$\big\downarrow$ 413

$$\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{2\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$\big\downarrow$ 412

3.99. $\int\frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}dx$

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}$$

input
```
Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
(x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) - (d*f*(-((b*Sqrt[c]
*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]]
, -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e])) + (Sqr
t[c]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[
(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sqrt[e +
 f*x^2])))/(2*a*b^2) + (Sqrt[c]*((c*e)/a + (a*d*f)/b^2)*Sqrt[1 - (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((c*f)/(d*e))])/(2*a*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

3.99.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

3.99. $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

3.99.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)(f\,x^2+e)}\left(\frac{x\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}{2a\left(b\,x^2+a\right)}-\frac{df\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2b^2\sqrt{\frac{d}{c}}\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}-\frac{de\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1 \cdots}\right.}{2ab\sqrt{\frac{d}{c}}\sqrt{-df\,x^4+cf\,x^2-de\,x^2 \cdots}}\right.}{\sqrt{- \cdots}}$
default	$-\frac{\sqrt{-d\,x^2+c}\,\sqrt{f\,x^2+e}\left(\sqrt{\frac{d}{c}}\,a\,b^2df\,x^5+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a^2bdf\,x^2+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a\,b \cdots\right.}{\cdots}$

input
```
int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(-d
*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(b*x^2+a)-1/2*d*f/b^2/(d/c)^(1/2)*(1-d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Ellipti
cF(x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))-1/2*d/a/b*e/(d/c)^(1/2)*(1-d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*d/a/b*e/(d/c)^(1/2)*(1-d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticE
(x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2/b^2/(d/c)^(1/2)*(1-d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(
d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d*f+1/2/a^2/(d/c)^(1/2)*(1-d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Ellip
ticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*c*e)
```

3.99.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="frica
s")
```

output
```
Timed out
```

3.99.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c - d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

3.99.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.99.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac"
)
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx=\int \frac{\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.100 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

3.100.1 Optimal result

Integrand size = 32, antiderivative size = 381

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx &= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &+\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2b^2c\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &+\frac{\sqrt{-c}(b^2ce-a^2df)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-1/2*f*x*(d*x^2+c)^(1/2)/a/b/(f*x^2+e)^(1/2)+1/2*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(
1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/b/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/
(f*x^2+e)^(1/2)+1/2*d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*
f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^
2+c)^(1/2)/b^2/c/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*x*(d*
x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*(-a^2*d*f+b^2*c*e)*EllipticPi
(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2
)*(1+f*x^2/e)^(1/2)/a^2/b^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{cex}{a+bx^2}+\frac{dex^3}{a+bx^2}+\frac{cfx^3}{a+bx^2}+\frac{dfx^5}{a+bx^2}+\frac{ic\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{b}-\frac{ic\sqrt{\frac{d}{c}}(be+af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\text{Ellipti}}{b^2}}{2a}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) + (d*
f*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b - (I*c*Sqrt[d/c]*(b*
e + a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/b^2 - (I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/
c]) + (I*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticP
i[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

3.100.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 423}$$

$$\frac{df\int \frac{a-bx^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 406$$

$$\frac{df\left(a\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 320$$

$$\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 388$$

$$\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 313$$

$$\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 413$$

3.100. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{2\sqrt{c+dx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{2\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

3.100. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

output
```
(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (d*f*(-(b*((x*Sqrt
[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))) + (a*Sqrt[e]*Sqrt[c + d*x^2]*Elliptic
F[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(2*a*b^2) + (Sqrt[-c]*((c*e)/a
- (a*d*f)/b^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a*Sqrt[d]*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])
```

3.100.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

3.100.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{x\sqrt{dfx^4+cfx^2+dex^2+ce}}{2a(bx^2+a)}+\frac{df\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2b^2\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}+\frac{de\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\cdots}\right.}{2ab\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+\cdots}}\cdots\right.}{\sqrt{\cdots}}$
default	$\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}ab^2dfx^5+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2bdfx^2+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab^2dex\cdots\right.}{\cdots}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(d*f*
x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^2+a)+1/2*d*f/b^2/(-d/c)^(1/2)*(1+d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*d/a/b*e/(-d/c)^(1/2)*(1+d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2*d/a/b*e/(-d/c)^(1/2)*(1+d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2/b^2/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*d*f+1/2/a^2/(-d/c)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*c*e)
```

3.100.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.100.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

3.100.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.100.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.101 $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

3.101.1 Optimal result

Integrand size = 33, antiderivative size = 426

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a(bc+ad)\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}(b^2ce-3a^2df+ab(2de-2cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

output
```
1/2*b^2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(b*x^2+a
)+1/2*b*EllipticE(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d
*x^2/c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(-d*x^2+c)^(1/2)/(1+f
*x^2/e)^(1/2)+1/2*(b^2*c*e-3*a^2*d*f+a*b*(-2*c*f+2*d*e))*EllipticPi(x*d^(1
/2)/c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/a^2/(a*d+b*c)/(-a*f+b*e)/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
-1/2*EllipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a/(a*d+b*c)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \frac{-\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}-\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}-ibc\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)+ic\sqrt{-\frac{d}{c}}(b}{}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(-((b^2*c*e*x)/(a + b*x^2)) + (b^2*d*e*x^3)/(a + b*x^2) - (b^2*c*f*x^3)/(a
 + b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) - I*b*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d
*e))] + I*c*Sqrt[-(d/c)]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + (I*b^2*c*e*Sqrt[
1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sq
rt[-(d/c)]*x], -((c*f)/(d*e))])/(a*Sqrt[-(d/c)]) - (2*I)*b*c*Sqrt[-(d/c)]*
e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*Arc
Sinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + ((2*I)*b*d*f*Sqrt[1 - (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -
((c*f)/(d*e))])/(-(d/c))^(3/2) + (3*I)*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*
x], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[
e + f*x^2])
```

3.101.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {424, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 424$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\int \frac{bx^2+a}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \; 399$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{f}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \; 323$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{\frac{fx^2}{e}+1}(be-af)\int \frac{1}{\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \; 323$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \; 321$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \; 331$$

3.101. $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{fx^2+e}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$\downarrow$ 330

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\int \frac{\sqrt{\frac{fx^2}{e}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$\downarrow$ 327

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$\downarrow$ 413

$$\frac{\sqrt{1-\frac{dx^2}{c}}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{fx^2+e}}dx}{2a\sqrt{c-dx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$\downarrow$ 413

3.101. $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}dx$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\operatorname{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a +
b*x^2)) + (d*f*((b*Sqrt[c]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sq
rt[1 + (f*x^2)/e]) - (Sqrt[c]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f
*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])))/(2*a*(b*c + a*d)*(b*e - a*f)) + (Sqrt[
c]*(b^2*c*e - 3*a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*
f)/(d*e))])/(2*a^2*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e
+ f*x^2])
```

3.101. $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(369) = 738$.

Time = 6.06 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.32

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)(f x^2+e)}\left(-\frac{b^2 x\sqrt{-df x^4+cf x^2-de x^2+ce}}{2\left(a^2 df+acfb-abde-b^2 ce\right)a\left(b x^2+a\right)}-\frac{df\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2\left(a^2 df+acfb-abde-b^2 ce\right)\sqrt{\frac{d}{c}}\sqrt{-df x^4+cf x^2-de x^2+ce}}+\frac{bde\sqrt{1-\frac{d}{}}}{2\left(a^2 df+acfb-\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/2*b^2/(a
^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*x*(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(
b*x^2+a)-1/2*d*f/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/(d/c)^(1/2)*(1-d*x^2/c)
^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(x*
(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f+a*b*c*f-a*b*d*e-b^2
*c*e)/a*e/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^
2-d*e*x^2+c*e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))-1/2
*b*d/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*e/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(
1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticE(x*(d/c)^(1
/2),(-1-(c*f-d*e)/e/d)^(1/2))+3/2/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/(d/c)^
(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(
1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d*f+1/(a^
2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*b/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*
c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*c*f-1/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*
b/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^
2+c*e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d
*e-1/2/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a^2*b^2/(d/c)^(1/2)*(1-d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(
d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*c*e)
```

3.101.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.101.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\,\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^2\,\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

3.101.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\,\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.101.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^2\,\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(b\,x^2+a)^2\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.102.1 Optimal result

Integrand size = 32, antiderivative size = 485

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2c(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{-c}(b^2ce+3a^2df-2ab(de+cf))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc-ad)(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

output
```
-1/2*b*f*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/2*b*(
1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/(-a*d+b*c)
/(-a*f+b*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/2*d*(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)/(-a*f+
b*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*b^2*x*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)+1/2*(b^2*c*e+3*a^2*
d*f-2*a*b*(c*f+d*e))*EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/
2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a^2/(-a*d+b*c)/(-a*f+b*
e)/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}+\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}+ibc\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ic\sqrt{\frac{d}{c}}(be-af)\sqrt{}}{}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x^3)/(a +
b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*c*
Sqrt[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1
 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
)/(a*Sqrt[d/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + ((2*I
)*b*c*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (3*I)*a*c*Sqrt[d/c]*f*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(2*a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

3.102.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 424$$

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\Big\downarrow$ 406

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(a\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\Big\downarrow$ 320

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{a\sqrt{e}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\Big\downarrow$ 388

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{a\sqrt{e}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\Big\downarrow$ 313

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\Big\downarrow$ 413

3.102. $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{2a\sqrt{c+dx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\big\downarrow$ 413

$$\frac{\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{2a\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\big\downarrow$ 412

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

3.102. $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

output
```
(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a +
b*x^2)) - (d*f*(b*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sq
rt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (a*Sqrt[e]
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(2*a*(
b*c - a*d)*(b*e - a*f)) + (Sqrt[-c]*(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*
f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin
[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*
f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.102.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

3.102.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.01

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b^2 x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{2\left(a^2 df-acfb-abde+b^2ce\right)a\left(b\,x^2+a\right)}-\frac{df\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2\left(a^2df-acfb-abde+b^2ce\right)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{bde\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{}}{2\left(a^2df-acfb-abde\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*b^2/(a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^
2+a)-1/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*d*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/a*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2*b
*d/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/
2),(-1+(c*f+d*e)/e/d)^(1/2))+3/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(-d/c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*d*f-1/(a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*b/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*c*f-1/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a
*b/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))
*d*e+1/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a^2*b^2/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*c*e)
```

3.102.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.102.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.102.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.102.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 \sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.103 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

3.103.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

output
```
Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.103.2 Mathematica [N/A]

Not integrable

Time = 18.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

3.103.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {434}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

3.103.3.1 Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

3.103.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.103.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.103.6 Sympy [N/A]

Not integrable

Time = 12.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

3.103.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.103.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.103.9 Mupad [N/A]

Not integrable

Time = 5.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.104 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

3.104.1 Optimal result

Integrand size = 34, antiderivative size = 545

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx \\
&= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{b\sqrt{e}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{c\sqrt{e}(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/2*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/f/(d*x^2+c)^(1/2)+1/2*b*(-c*f+d*e)
*EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*
f+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/d/f
/(-a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*c*(-
a*d*f-b*c*f+b*d*e)*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d
*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*
(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)/a/d/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/2*EllipticE(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x
^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(-c*f+d*e)^(1/2)*(
b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)/f/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(f*x^2+e)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{\frac{x\sqrt{a+bx^2}(c+dx^2)}{\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{-de+cf}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{bce-acf}{ade-acf}\right)}{f\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(be-2af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\mathrm{Ellipti}}{\sqrt{e}f^2\sqrt{be-af}\sqrt{\frac{a(}{e(}}}}{2\sqrt{c+dx}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) +
c*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSi
n[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*
e - a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d
*e - c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[
ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*
c*e - a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b
*x^2))]) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e)
+ a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(S
qrt[a]*f^2*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*S
qrt[c + d*x^2])
```

3.104.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {430, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{ 430}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{bc(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2df}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 427}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2df\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 428}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{2adf\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-afx}}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 412$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-afx}}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 429$$

$$-\frac{\sqrt{a+bx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-afx}}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 327$$

3.104. $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

$$\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqr
t[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*Ellipti
cE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e
)/(a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f
*x^2]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
], ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d
*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)
/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b
*c - a*d)*e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

3.104.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

3.104. $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

3.104.4 Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.104.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.104.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

3.104.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.104.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.105 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$

3.105.1 Optimal result

Integrand size = 34, antiderivative size = 163

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(
-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d
*x^2+c))^(1/2)/a/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e
)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),\frac{(-bc+ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticP
i[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])]
, ((-(b*c) + a*d)*e)/(a*(d*e - c*f))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

3.105.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {428, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{428}$$

$$\frac{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{a\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{412}$$

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticP
i[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])]
, -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

3.105.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.105.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.105.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.105.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*sqrt(e + f*x**2)), x)
```

3.105.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.105.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.106 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

3.106.1 Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
EllipticE(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f
+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/a/(-
a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

3.106.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{429}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}}{\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{a\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

3.106.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.106.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{f\,x^{2}+e}}dx$$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

3.106.5 Fricas [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f*x^6 + (b^2
*e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^2), x)
```

3.106.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

3.106.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.106.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}\sqrt{f\,x^2+e}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.107 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.107.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

output
```
Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.107.2 Mathematica [N/A]

Not integrable

Time = 20.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

3.107.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {434}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.107.3.1 Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

3.107.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.107.5 Fricas [N/A]

Not integrable

Time = 54.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*
f*x^2 + e^2), x)
```

3.107.6 Sympy [N/A]

Not integrable

Time = 17.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

3.107.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.107.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.107.9 Mupad [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 34, antiderivative size = 484

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = {} & -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
& + \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
& - \frac{c^{3/2}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
& + \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

output
```
-c^(3/2)*(-a*f+b*e)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+
d*e)/c/(f*x^2+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/
2)/(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2)
)*(b*x^2+a)^(1/2)/a/e/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)+(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/
c/(f*x^2+e))^(1/2)*EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1
+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(
1/2)*(-c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)/e/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)-(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/f/(d*x^2+c)^(1/2)/(f*x^2+e)
^(1/2)+b*c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f
+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^
2+e)/e/(d*x^2+c))^(1/2)/a/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(f*x^2+e)^(1/2)
```

3.108.2 Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

3.108.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {432, 428, 412, 429, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{432}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$\downarrow$ 428

$$\frac{bc\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{af\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$\downarrow$ 412

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$\downarrow$ 429

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\int \frac{\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 324

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\int \frac{1}{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}+\frac{(de-cf)\int \frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 320

3.108. $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$

$$\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\int\frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{c}\ldots}\right.\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}\sqrt{\frac{c\left(x^{\ldots}\right.}{a\left(x^{\ldots}\right.}}}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 388

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$

$$\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}}-\frac{a\int\frac{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}}{\left(\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1\right)^{3/2}}d\frac{x}{\sqrt{fx^2+e}}}{be-af}\right)}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{Elli}\ldots}{\sqrt{de-cf}\sqrt{\ldots}}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 313

3.108. $\int\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}dx$

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} -$$

$$\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}} + \frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-c}{c\left(e+fx^2\right.}}}\right.}{}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*(((d*
e - c*f)*((a*x*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))])/((b*e - a*f)*S
qrt[e + f*x^2]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]) - (a*Sqrt[c]*S
qrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*EllipticE[ArcTan[(Sqrt[d*e - c*
f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b
*e - a*f)*Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*
(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f
*x^2))])))/c + (Sqrt[c]*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*Ellipt
icF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e +
 f*x^2)))/(a*(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x
^2)/(c*(e + f*x^2))])))/(e*f*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))])) + (b*c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x
^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sq
rt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*f*Sqrt[d*e - c*f]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

3.108. $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 432
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Simp[b/f   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sq
rt[e + f*x^2]), x], x] - Simp[(b*e - a*f)/f   Int[Sqrt[c + d*x^2]/(Sqrt[a +
 b*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.108.4 Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.108.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*
x^2 + e^2), x)
```

3.108.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

3.108.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.108.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.109 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(e+fx^2)^{3/2}}\,dx$

3.109.1 Optimal result

Integrand size = 34, antiderivative size = 319

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx &= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}\\ &\quad - \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &\quad + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
c^(3/2)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^
2+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-
c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*(b*x^2+a)^
(1/2)/a/e/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
-(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(
1/2)*EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-c*f+d*e
)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(1/2)*(-c*f+d*e
)^(1/2)*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)+(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*x^
2+e)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(-de+cf)}{c(-be+af)}\right)}{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*EllipticE[A
rcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*(-(d*e) + c*f)
)/(c*(-(b*e) + a*f))])/(e*Sqrt[-(b*e) + a*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))])
```

3.109.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {429, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{429}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\int \frac{\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\int \frac{1}{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}+\frac{(de-cf)\int \frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\int \frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}}\right.}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}}-\frac{a\int \frac{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}}{\left(\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1\right)^{3/2}}d\frac{x}{\sqrt{fx^2+e}}}{be-af}\right)}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\text{arcta}\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}}\right.}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 313

3.109. $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(e+fx^2)^{3/2}}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}}+\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}}-\frac{a\sqrt{c}\cdots}{(b\cdots}\right.}{\cdots}\right.}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*(((d*e - c*f)*((a*x
*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]])/((b*e - a*f)*Sqrt[e + f*x^2]
*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]]) - (a*Sqrt[c]*Sqrt[1 + ((b*e
- a*f)*x^2)/(a*(e + f*x^2))]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]
*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b*e - a*f)*Sqrt
[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*(c + ((d*e - c
*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]])))/c +
 (Sqrt[c]*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*EllipticF[ArcTan[(Sq
rt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c
*f)))])/(Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*(
c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*
x^2))]])))/(e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))])
```

3.109.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.109.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.109.5 Fricas [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*f^2*x^6 + (2*b
*e*f + a*f^2)*x^4 + a*e^2 + (b*e^2 + 2*a*e*f)*x^2), x)
```

3.109.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

3.109.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.109.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.110 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

3.110.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}},x\right)$$

output
```
Unintegrable((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.110.2 Mathematica [N/A]

Not integrable

Time = 21.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

3.110.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {434}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.110.3.1 Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

3.110.4 Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d\,x^{2}+c}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.110.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f^2*x^8 + 2*
(b^2*e*f + a*b*f^2)*x^6 + (b^2*e^2 + 4*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 +
2*(a*b*e^2 + a^2*e*f)*x^2), x)
```

3.110.6 Sympy [N/A]

Not integrable

Time = 11.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

3.110.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.110.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.110.9 Mupad [N/A]

Not integrable

Time = 6.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.111 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

3.111.1 Optimal result

Integrand size = 34, antiderivative size = 541

$$\begin{aligned}
&\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx \\
&= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} \\
&+ \frac{(bc-ad)\sqrt{e}(2be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&- \frac{a(adf-b(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)-1/2*EllipticE(x*(-a*
d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*c^
(1/2)*(-a*d+b*c)^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)/b/(
d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)-1/2*a*(a*d*f-b*(c*f+d*e))*E
llipticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a
*f+b*e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/
2)/b^2/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^
(1/2)+1/2*(-a*d+b*c)*(-a*f+2*b*e)*EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*
x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a
*(f*x^2+e)/e/(b*x^2+a))^(1/2)/b^2/c/(-a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)/(f*x^2+e)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 3.48 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(b^2c\sqrt{bc-ad}x\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}(e+fx^2)-bc\sqrt{bc-ad}\sqrt{e}\sqrt{be-af}\sqrt{a+bx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\Big(\text{a}\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(b^2*c*Sqrt[b*c - a
*d]*x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(e + f*x^2) - b*c*Sqrt[b*c - a
*d]*Sqrt[e]*Sqrt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b
*x^2))]*EllipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (
b*c*e - a*d*e)/(b*c*e - a*c*f)] + Sqrt[b*c - a*d]*(2*b*c - a*d)*Sqrt[e]*Sq
rt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Ellipt
icF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)
/(b*c*e - a*c*f)] - a*Sqrt[c]*(a*d*f - b*(d*e + c*f))*Sqrt[a + b*x^2]*Sqrt
[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*d), ArcSin[(Sq
rt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (b*c*e - a*c*f)/(b*c*e - a*d*
e)]))/(2*a*b^2*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.111.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {431, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 431$$

$$\frac{(bc-ad)(2be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}+\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 427$$

$$\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}+\frac{\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 321$$

$$\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 428$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{429}$$

$$-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\int \frac{\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{327}$$

3.111. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}dx$

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+$$
$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$
$$\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*
c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*
c - a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))])
 + ((b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/
(e*(a + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*
x^2])], ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a
*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) + (a*(b*d*e + b*c*f - a*d*
f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

3.111.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 431
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Simp[e*((b*e - a*f)/(2*f))   Int[Sqrt[c + d*x^2]/(Sqrt[a +
b*x^2]*(e + f*x^2)^(3/2)), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*f^2)
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[(b*e -
 a*f)*((d*e - 2*c*f)/(2*f^2))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt
[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c
]
```

3.111. $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

3.111.4 Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

3.111.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.111.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

3.111.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

3.111.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.112.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\text{Int}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

output
```
Unintegrable((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.112.2 Mathematica [N/A]

Not integrable

Time = 5.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

3.112.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {434}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

3.112.3.1 Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

3.112.4 Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.112.5 Fricas [N/A]

Not integrable

Time = 60.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f*x^4 + (d*e
 + c*f)*x^2 + c*e), x)
```

3.112.6 Sympy [N/A]

Not integrable

Time = 9.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.112.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.112.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.112.9 Mupad [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.113 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.113.1 Optimal result

Integrand size = 34, antiderivative size = 159

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
a*EllipticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*
(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(
1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

3.113.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {428, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{428}$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \text{412}$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

3.113.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.113.4 Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.113.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.113.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.113.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.113.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.114 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.114.1 Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f
+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/c/(-
a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

3.114.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {427, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \; 427$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

3.114.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.114.4 Maple [F]

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.114.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*d
*e + (b*c + a*d)*f)*x^4 + a*c*e + (a*c*f + (b*c + a*d)*e)*x^2), x)
```

3.114.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.114.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.114.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.115 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.115.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Int}\left(\frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

output
```
Unintegrable(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.115.2 Mathematica [N/A]

Not integrable

Time = 10.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

3.115.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {434}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

3.115.3.1 Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

3.115.4 Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.115.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.74

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f*x^8 + (b
^2*d*e + (b^2*c + 2*a*b*d)*f)*x^6 + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*
d)*f)*x^4 + a^2*c*e + (a^2*c*f + (2*a*b*c + a^2*d)*e)*x^2), x)
```

3.115.6 Sympy [N/A]

Not integrable

Time = 7.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.115.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.115.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.115.9 Mupad [N/A]

Not integrable

Time = 9.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```